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ROSS STREET

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CORRECTION TO "FIBRATIONS IN BICATEGORIES"
by Ross STREET

Given homomorphisms of bicategories $J: A \rightarrow \text{Cat}$ and $S: A \rightarrow K$, the bilimit $\{J, S\}$ of S indexed by J can be constructed using biproducts, cotensor biproducts and biequalizers. However, the construction described in Section 1 (1.24), page 120, of the paper "Fibrations in bicategories" (these *Cahiers*, XXI-2, 1980, 111-160) is wrong. I am grateful to Max Kelly for noticing this error. He also recognized that $\{J, S\}$ could be constructed if K admitted some further bilimits of a simple kind. In fact, no further bilimits are needed: I shall show that they can be constructed from those at hand.

Certain small categories *Iso*, *End*, *Aut*, *T* will be required. A functor from one of these into a category amounts to an isomorphism, endomorphism, automorphism, composable pair of isomorphisms, respectively, in the category. Notice that *Iso*, *T* are equivalent to 1 while *Aut* is not.

The *biequifier* of 2-cells

$$\theta, \phi: f \Rightarrow g: A \rightarrow B$$

in K is an arrow $h: X \rightarrow A$ which, for all objects X , induces an equivalence of categories between $K(K, X)$ and the full subcategory of $K(K, A)$ consisting of those $a: K \rightarrow A$ for which $\theta a = \phi a: fa \Rightarrow ga$. (If θ, ϕ are invertible this is the same as the *biequinverter* of θ, ϕ as used later (4.2) in the paper.) The *biidentifier* of an endo-2-cell

$$\psi: f \Rightarrow f: A \rightarrow B$$

is the *biequifier* of ψ and the identity of f . If ϕ is invertible, the *biequifier* of θ, ϕ is the *biidentifier* of $\phi^{-1}\theta$. So to construct *biequifiers* of invertible pairs it suffices to construct *biidentifiers* of auto-2-cells.

Consider an auto-2-cell $\psi: f \Rightarrow f: A \rightarrow B$. Let $\psi^-, f^-: A \rightarrow \mathbf{Aut}(B)$ be the arrows corresponding to the automorphisms $\psi, 1_\psi$ in $K(A, B)$. Let $h: H \rightarrow A, \sigma: \psi^- h \approx f^- h$, be the biequalizer of ψ^-, f^- . Let $k: K \rightarrow A, \tau: f k \approx f k$, be the biequalizer of f, f . Let $\alpha: \mathbf{Aut}(B) \rightarrow B$ be induced by the unique functor $1 \rightarrow \mathbf{Aut}$. There exist $l: H \rightarrow K$ and $v: kl \approx h$ rendering α isomorphic to τl by definition of K . Similarly, we obtain $d: A \rightarrow K$ and $kd \approx 1_k$ rendering 1_ψ isomorphic to τd . Now form the bipullback

$$\begin{array}{ccc}
 P & \xrightarrow{v} & H \\
 u \downarrow & = & \downarrow l \\
 A & \xrightarrow{d} & K
 \end{array}$$

of d, l ; this is just the biequalizer of the two arrows from the biproduct of A, H into K which use d, l and the projections. I claim $u: P \rightarrow A$ is the biidentifier of $\psi: f \Rightarrow f$. Since bilimits are defined representably, we only need to check the construction in Cat . Then K can be taken to be the category of pairs (a, τ) where a is an object of A and $\tau: fa \approx fa$ in B , while H can be taken to be the full subcategory of K consisting of the pairs (a, σ) with $\sigma \cdot \psi a = \sigma$. Since σ is invertible, the last equation implies $\psi a = 1$. Also l is the inclusion and d takes a to $(a, 1_\psi a)$. With this we see that the objects of P are pairs (σ, ρ) where

$$\rho: a \approx a' \text{ in } A, (a, \sigma) \in H \text{ and } f\rho \cdot \sigma = f\rho.$$

This last condition implies $\sigma = 1_\psi a$, and, since ψ is natural, $\psi a' = 1$. So P is equivalent to the full subcategory of A consisting of those a with $\psi a = 1$.

(The above construction with \mathbf{Aut} replaced by \mathbf{End} yields the biidentifier of any endo-2-cell ψ .)

The next bilimit required is the *descent object* $\text{Desc}(X)$ of a truncated bicosimplicial diagram

$$\begin{array}{ccccc}
 & \xrightarrow{\delta_0} & & \xrightarrow{\delta_0} & \\
 X: & X_0 & \xleftarrow{1} & X_1 & \xrightarrow{\delta_1} & X_2 \\
 & \xrightarrow{\delta_1} & & \xrightarrow{\delta_2} & \\
 \delta_{i,j} : \delta_i \delta_{j-1} \approx \delta_j \delta_i & \text{ for } & i < j, & n_i : 1 \approx 1 \delta_i
 \end{array}$$

in a bicategory K . When K is Cat , the category $Desc(X)$ has objects pairs (x, θ) where x is an object of X_0 and $\theta: \delta_0 x \approx \delta_1 x$ in X_1 such that

$$\iota\theta = \mu_1\mu_0^{-1}, \quad \sigma_{12}.\delta_1\theta.\sigma_{01} = \delta_2\theta.\sigma_{02}.\delta_0\theta,$$

and has arrows $\chi: (x, \theta) \rightarrow (x', \theta')$ where $\chi: x \rightarrow x'$ is an arrow of X_0 such that $Q'.\delta_0\chi = \delta_1\chi.\theta$. For a general K , the descent object of X consists of an object D , an arrow $h: D \rightarrow X_0$ and an invertible 2-cell $\omega: \delta_0 h \approx \delta_1 h$ inducing an equivalence between $K(K, D)$ and $Desc(K(K, X))$. Notice that X can be regarded as a homomorphism from an appropriate category A into K and, if we take $J: A \rightarrow Cat$ to be the functor amounting to the diagram

$$\begin{array}{ccccc} & \longrightarrow & & \longrightarrow & \\ 1 & \longleftarrow & Iso & \longrightarrow & T \\ & \longrightarrow & & \longrightarrow & \end{array}$$

the bilimit $\{J, X\}$ is equivalent to $Desc(X)$.

The descent object can be constructed using biequalizers and biidentifiers of auto-2-cells. First, take the biequalizer

$$h: H \rightarrow X_0, \quad \theta: \delta_0 h \approx \delta_1 h,$$

of δ_0, δ_1 , then the biequalifier $k: K \rightarrow H$ of the two invertible 2-cells

$$\begin{array}{ccc} \iota\delta_0 h & \xrightarrow{\iota\theta} & \iota\delta_1 h \\ \mu_0^{-1} h \searrow & & \nearrow \mu_1 h \\ & h & \end{array}$$

and then, the biequalifier $m: M \rightarrow L$ of the two invertible 2-cells

$$\begin{array}{ccccc} & & \delta_1\delta_0 hk & \xrightarrow{\delta_1\theta k} & \delta_1\delta_1 hk \\ \sigma_{01} hk \nearrow & & & & \searrow \sigma_{12} hk \\ \delta_0\delta_0 hk & & & & \delta_2\delta_1 hk \\ \delta_0\theta k \searrow & & \delta_0\delta_1 hk & \xrightarrow{\sigma_{02} hk} & \delta_2\delta_0 h \\ & & & & \nearrow \delta_2\theta k \end{array}$$

Then $L, hkm, \theta km$ form $Desc(X)$.

The bilimit $\{J, S\}$ can be obtained as the descent object $Desc(X)$ where

$$X_0 = \prod_{\mathbf{A}} \{JA, SA\}, \quad X_1 = \prod_{\mathbf{A}, \mathbf{B}} \{A(A, B) \times JA, SB\},$$

$$X_2 = \text{III}_{A,B,C} (A \otimes B, C) \times A(A, B) \times JA, SC).$$

In (1.25) it was stated that indexed pseudo-limits in a 2-category could be constructed from cotensor products, products and equalizers. This is certainly true since pseudo-limits are particular indexed limits and all indexed limits can be so constructed (see [14], using the *Bibliography* of the paper). The proof outlined in (1.25) was a modification of (1.24). Using the corrected (1.24), we can squeeze out more from the method. Many naturally occurring 2-categories have *iso-inserters*. The iso-inserter of the diagram $f, g: A \rightrightarrows B$ is its *limit* (not pseudo-limit) indexed by the diagram

$$1 \rightrightarrows \text{Iso} \quad \text{in Cat.}$$

An iso-inserter is a biequalizer but not conversely. The *strict descent object* of a truncated simplicial object X (this time μ_i, σ_{ij} are identities) is defined as for the descent object except that we insist on an isomorphism between $K(K, D)$ and $\text{Desc}(K(K, X))$, not merely an equivalence. It can be constructed using an iso-inserter and identifiers of auto-2-cells (the latter are defined as were biidentifiers except that we ask for an isomorphism in the representation property). Then $\text{psdlim}(J, S)$ is the strict descent object for X as before with biproducts and cotensor biproducts replaced by their "non-bi" versions. However, it does not seem possible to construct identifiers of auto-2-cells using a "non-bi" version of the construction of biidentifiers. The object P we are led to does support an idempotent whose splitting gives the identifier; but this is already true of the iso-inserter of ψ, f . So *products, cotensor products, iso-inserters, and, either identifiers of auto-2-cells or splittings of idempotents, imply all indexed pseudo-limits.*

I would like to stress that I am currently using the word "weighted" in preference to "indexed" in this context.

Finally, there is a typographical error in (4.2) on page 140. The functors between 1 and Iso should have their directions reversed.

Department of Mathematics
Macquarie University
N.S.W. 2109
AUSTRALIA