Slightly more births at full moon
Supplementary Appendix

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Least squares

In the theory described by eq. (1), we take into account the maximum number of weekly and annual frequencies that a spectral analysis enables with daily data, i.e. $q = 1, 2, 3$ and $q' = 1, ..., 182$ respectively. For the interactions frequencies, based on the Fourier transform of the data (Fig. 1), for each $q = 1, 2, 3$ we take 50 components $q'' = 1, ..., 25$.

All the parameters, $T(t), a_k(t), b_k(t), A(t)$ for each day t, and α_p for each particular day p (mostly national holidays, see below), are put in a unique 'model' vector $\boldsymbol{m}$ whose dimension is ≈ 12.3 million. Eq. (1) can then be written $\boldsymbol{n} = g(\boldsymbol{m}) + \boldsymbol{r}$ where $\boldsymbol{n}$ and $\boldsymbol{r}$ are the vectors formed by the values $n(t)$ and $r(t)$ for every day t. Since the size of the data $\boldsymbol{n}$ is 18,263, the inverse problem consisting in inferring $\boldsymbol{m}$ that minimizes $\boldsymbol{r}$ is regularized by also minimizing the required model $\boldsymbol{m}$. Accordingly, we minimize the following classical cost function f (Tarantola & Valette, 1982):

$$f(\boldsymbol{m}) = ||\boldsymbol{r}||^2 + ||\boldsymbol{m}||^2 = \boldsymbol{r}^T C_n^{-1} \boldsymbol{r} + \boldsymbol{m}^T C_m^{-1} \boldsymbol{m} = (\boldsymbol{n} - g(\boldsymbol{m}))^T C_n^{-1} (\boldsymbol{n} - g(\boldsymbol{m})) + \boldsymbol{m}^T C_m^{-1} \boldsymbol{m} \quad (1)$$

where C_n is the covariance matrix of the data and C_m is the a priori covariance matrix of the parameters. The minimization algorithm is a Newton scheme applied to the gradient of f. It yields the iterative algorithm (Tarantola & Valette, 1982):

$$\hat{\boldsymbol{m}}_{i+1} = C_m\, G_i^T\, \left(C_n + G_i\, C_m\, G_i^T\right)^{-1} \Big(\boldsymbol{n} - g(\hat{\boldsymbol{m}}_i) + G_i\, \hat{\boldsymbol{m}}_i\Big) \tag{2}$$

where $G_i = \frac{\partial g}{\partial \boldsymbol{m}}(\hat{\boldsymbol{m}}_i)$. Only 4 or 5 iterations are sufficient for convergence. Since data are assumed to be independent and homoscedastic, C_n is proportional to identity, and the corresponding standard deviation is chosen as its unbiased least square estimate. Parameters are assumed to be independent, thus C_m is a block diagonal matrix, but each parameter on one day t is supposed to be correlated to the same parameter on another day t', thus each main-diagonal block is a non-diagonal covariance matrix. The correlation in time is chosen gaussian: the element of the covariance matrix for days t and t' reads $C_{m_{tt'}} = \sigma^2 \exp\left(-\frac{(t-t')^2}{2\tau^2}\right)$, where σ is a constant standard deviation and τ is a correlation length. In order to have slowly varying functions, these correlation times have been chosen to be long: 5 years, except for the trend for which it is several months (i.e. longer than a lunar month). The a priori standard deviations have been chosen after some trials in order to insure convergence and good statistical properties of the residuals: normality and independence, but also no significant variations within the week, the year or holidays or with the frequency.

Particular days

By inspecting the data, in particular the number of births in the mean year over the 50 years, we identified 22 particular days in the year: see Table 1.

Testing the hypotheses that a deficit of birth at these days (value in last column of the table) equals 0, using the normal distribution, gives p-values that need to be corrected for multiple testing among 366 days. The Bonferroni corrections give upper bounds of the FWER p-values that are all small (most are less than 10^{-10}, some are about 10^{-4}), except Valentine's day and switch to winter time. Thus, all values except these two can be considered to be significantly different from 0 with a very small probability of error. The number of each particular day in the data is between 9 (29 February) and 85 (Friday 13).

Since the switch time days have one hour more or less, they should theoretically have -4 % and +4 % births in relative, that is -85 and +85 births. As it should be, the

Table 1: Particular days in the year. The last column gives the mean value of the deficit or excess of birth over the 50 years for each day, that is α_p times the temporal mean of $(1 + A(t))$. The greatest deficit is for Christmas which is -23 % with respect to the mean global number of births. 'Extra long weekend' corresponds to weeks with a holiday on tuesday or thursday.

#	Day	Event	Average deficit of births
		Fixed date holidays	
1	1 January	New year	-466
2	1 May	Labour day	-389
3	8 May	Surrender of Germany (1945)	-457
4	14 July	National Day	-393
5	15 August	Assumption	-340
6	1 November	All Saints	-356
7	11 November	Armistice (1918)	-372
8	25 December	Christmas	-491
		Mobile holidays	
9		Easter Monday	-364
10		Ascension Thursday	-391
11		Whit Monday	-388
		Other particular days	
12	2 January	New Year's Next day	-115
13	22 December	D-3 before Christmas	-64
14	23 December	D-2 before Christmas	-107
15	24 December	Christmas Eve	-214
16	31 December	New Year's Eve	-149
17	Extra long weekend	Monday or Friday	-71
18	Friday 13	Superstition	-43
19	29 February	Leap day	-117
20	14 February	Valentine's Day	+23
21	Last Sunday in March	Switch to summer time	-59 (-3 %)
22	Last Sunday in October	Switch to winter time	+38 (+2 %)

differences with these theoretical values can not be considered as significant since the corresponding FWER p-values are greater than 2 %.

These results suggest that superstition may affect the number of births since a deficit of birth appears in some days when the medical staff is a priori as numerous as the other days (Friday 13, Leap day). This can be noticed in the US for Valentine's day and Halloween, where the differences in the number of births (respectively +5 % and -11 %) are statistically significant (Levy & Chung, 2011).

Outliers

By inspecting the residuals after a first application of the least squares algorithm, we isolated 12 residuals that were greater than 3 standard deviations and for which we were able to find a sociological event that might have influenced the number of births on that day. These outliers have thus been corrected with the corresponding residual values, and the least squares algorithm has been applied again to the "corrected raw" data.

The largest outlier is 19 March, 2001: there has been a deficit of -339 births (correction+final residual). It was the beginning of a general midwives strike movement. The day before is also an outlier: there has been +203 births more than usual. Undoubtly, midwives anticipated the strike. Two other important days were noticed: 11 August, 1999 when there has been a total solar eclipse in France (-237 births) and 31 December, 1999, which was the Millennium New Year's Eve (-213 births). These two days, the medical staff was probably less numerous than usual. The other days are mainly those for which people could have a longer week-end or longer holidays.

Testing the hypotheses that these values equal 0, using the normal distribution with the population standard deviation, gives p-values that mostly do not permit a conclusion. Indeed, individual p-values are very small but they need to be corrected for multiple testing among 18,263 days. On the other hand, the Bonferroni correction gives only upper bounds of the FWER p-values and with such a high number of days, the corrections are large. Consequently, FWER p-values are smaller than 10^{-2} only for the 5 differences greater than 237 births:
- the start of midwives strike movement;
- Tuesday, 30 June, 1987, the day before summer holidays (+288 births);
- Saturday, 14 July, 2007, National holiday after a Friday 13 (+263);
- Friday, 6 January, 1989, Weekend vigil and return from vacation on the 5th (+254);

- the solar eclipse day.

Likelihood ratio test

The residuals $r(t)$ obtained from equation (1) are divided in 30 classes, of respective lengths $n_1, \dots n_{30}$, according to their corresponding lunar day $i = 1, \dots, 30$, and are modeled by the classical one-way model (Miller, 1986, Ch. 3):

$$r_{ij} = \mu_i + \epsilon_{ij} \text{ , with } i = 1, \dots 30,\ j = 1, \dots, n_i,\ N = \sum_{i=1}^{30} n_i = 18\,263 \text{ ,}$$

where ϵ_{ij} are assumed to be independent and identically distributed as a gaussian variable with mean 0 and unknown variance, and μ_i is the mean of r_{ij}. Within this model, the hypothesis H_0 : "$\mu_1 = \cdots = \mu_{30} = 0$" can be tested against the alternative H_1 : "There exists $i \in \{1, \dots, 30\} : \mu_i \neq 0$" thanks to the likelihood-ratio test (Lehmann & Romano, 2005, Ch. 5 and 12). The associated test statistic reduces to $-2 \log T$, where $T = \left(\sum_{i,j} (r_{ij} - \bar{r}_i.)^2 / \sum_{i,j} r_{ij}^2 \right)^{N/2}$, in terms of $\bar{r}_i. = (1/n_i) \sum_{j=1}^{n_i} r_{ij}$. This statistic is asymptotically distributed as a χ^2 variable with 30 degrees of freedom.

Chi-square test

One can be tempted to perform χ^2 tests on the repartition of births within the 30 days of the lunar moon to test the hypothesis that the probability of each birth to appear in one given day of the lunar month is uniform in the month. The number of degrees of freedom is 29. For our raw data, the χ^2 statistic is 216, which yields a p-value $< 10^{-30}$. However, as explained in the text, the statistic is very high because the trend and seasonality component have important fluctuations, and the raw data do not respect the conditions necessary for the application of the χ^2 test. This error is often made and responsible for most of (false) detections of correlation between lunar phases and births (Rotton & Kelly, 1985, Rotton, Kelly & Frey, 1983).

On the contrary, our residuals are closer to the conditions for satisfying for the application of the χ^2 test. In particular, their standard deviation (= 46.8) is close to that of a multinomial of the total number of births distributed over the days of the series (= 46.1). The χ^2 statistic is 72, which yields a p-value $= 2 \times 10^{-5}$: the hypothesis that the days in the lunar month are equiprobable can be rejected. When we don't take into account FM and FM+1 days, the χ^2 statistic is 33, the number of degrees of freedom is 27, which yields a p-value = 2 %. This does not seem small enough to reject equiprobability between the days of the lunar month. When we do not take into account the FM, FM+1 and FM+11 days, the χ^2 statistic is 23, the number of degrees of freedom is 26, which yields a p-value = 60 %. Again, it shows that only FM and FM+1 can be quite safely considered as responsible for the significant variations of births in the lunar month.

References

See References section in the paper, and :

Levy BR, Chung PH, Slade MD, 2011, Influence of Valentine's Day and Halloween on Birth Timing, *Social Science & Medicine* **73**, 1246-1248. https://www.sciencedirect.com/science/article/abs/pii/S0277953611004485.