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A characterization of harmonic measure and Markov processes whose hitting distributions are preserved by rotations translations and dilatations


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A CHARACTERIZATION OF HARMONIC MEASURE AND MARKOV PROCESSES WHOSE HITTING DISTRIBUTIONS ARE PRESERVED BY ROTATIONS, TRANSLATIONS AND DILATATIONS

by B. ØKSENDAL and D. W. STROOCK (1)

0. Introduction.

A famous result by P. Lévy states that if $B_t$ is Brownian motion in the complex plane $\mathbb{C}$ starting at $x \in V$ (open) and $\varphi : V \rightarrow \mathbb{C}$ is analytic and non-constant, then $\varphi(B_t)$ is—up to the exit time of $V$—Brownian motion starting at $\varphi(x)$, except for a change of time scale. See [7] for a proof. This result can be extended (see [2]) to a characterization of the functions $\varphi : W \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^p$ which preserve the paths of Brownian motion in this way, i.e. which are Brownian path preserving (BPP). In particular, the functions $\varphi : V \subseteq \mathbb{C} \rightarrow \mathbb{C}$ which are BPP are exactly the analytic and the conjugate analytic functions.

Also, if $n > 2$ a function $\varphi : W \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^n$ is BPP if and only if it is an affine function of the form

\[ \varphi(x) = \lambda A x + b, \]

where $\lambda > 0$, $A \in \mathbb{R}^{n \times n}$ is an orthogonal matrix and $b \in \mathbb{R}^n$.

In view of the many applications of the Lévy theorem in complex analysis, it is natural to ask if there are processes other than Brownian motion in $\mathbb{C}$ whose paths are preserved (in the sense above) by analytic

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functions. We answer this question in the negative. More precisely, we prove the following converse of the Lévy theorem (Theorem 3):

Let $X_t$ be a continuous path Markov process in $\mathbb{R}^n$ with probability laws $P^y, y \in \mathbb{R}^n$. Assume that for all $\varphi$ of the form given in (1), the exit distributions of $\varphi(X_t)$ under $P^y$ coincide with those of $X_t$ under $P^y(\cdot)$. Then $X_t$ is, up to a change of time scale, Brownian motion on $\mathbb{R}^n$.

This result will be proved in Section 2, as an application of a characterization, proved in Section 1, of harmonic measure (or more generally the exit distribution for a strong, continuous Markov process) as a weak star limit of successive spherical sweepings of the unit point mass (Theorem 1). Theorem 1 also implies that two strong, continuous Markov processes on $\mathbb{R}^n$ which have the same exit distribution from balls when starting from the center, have the same exit distribution from all open sets, provided they both exit a.s. from bounded sets (Theorem 2).

Finally, as a third application of Theorem 1 we give in Section 3 a converse of the mean value property for harmonic functions: A function $h$ on an open set $U$ which at each point $x \in U$ satisfies the mean value property for at least one sphere of radius $r(x)$ centered at $x$ is necessarily harmonic in $U$, under certain conditions on $h$ and $r(x)$. (See Corollary 3). Results of this kind have been obtained earlier by many authors. See for example [1], [6], [7], [11] and [12].

1. A characterization of exit distribution.

Let $X_t(\omega); t > 0, \omega \in \Omega$ be a strong Markov process in $\mathbb{R}^n$ with probability laws $P^y, y \in \mathbb{R}^n$. Assume that the paths of $X_t$ are continuous. If $E$ is an open set we let

$$T_E = T_\omega = \inf \{t > 0; X_t \notin E\}$$

be the (first) exit time from $E$.

The exit distribution for $E$ (with respect to $X_t$), starting from $y \in \mathbb{R}^n$, is the measure $\mu$ on the boundary $\partial E$ of $E$ defined by

$$\mu(G) = \mu_\mathbb{R}^n(G) = P^y[\tau_E < \infty], \ G \text{ Borel set.}$$

In the special case when $X_t$ is the Brownian motion $B_t$ and $E$ is a bounded open set $V \subset \mathbb{R}^n$, then $\mu_\mathbb{R}^n(\partial V) = 1$ and $\mu_\mathbb{R}^n$ coincides with the
A CHARACTERIZATION OF HARMONIC MEASURE 223
classical harmonic measure at \( y \) with respect to \( V \), which we will denote by \( \lambda^y_V \). For a general open set \( V \), when \( X_t = B_t \), we will adopt (*) as our definition of harmonic measure \( \lambda^y_V \):

\[
\lambda^y_V(G) = P^y[B_{\tau_V} \in G, \tau_V < \infty], \quad G \text{ Borel set}
\]
(see [9] or [10] for an account of probabilistic potential theory). In particular, if \( V \) is an open ball with center \( y \), then the exit distribution from \( V \) starting at \( y \) is the uniform distribution of mass 1 on \( \partial V \).

In this section will characterize the exit distribution \( \mu \) of \( X_t \) for a class of open sets as a weak star limit of what could be called successive spherical sweepings of the unit point mass:

Let \( U \) be an open set in \( \mathbb{R}^n \) with closure \( \overline{U} \). Let \( r(x) \) be a measurable function on \( U \) such that

\[
0 < r(x) < \text{dist}(x, \partial U)
\]

and

\[
\text{inf} \{ r(x); \ x \in K \} > 0
\]

for all closed subsets \( K \) of \( U \) with \( \text{dist}(K, \partial U) > 0 \).

For each \( x \in U \) we let \( \Gamma_x \) denote the sphere centered at \( x \) with radius \( r(x) \).

Define a sequence of stopping times \( \tau_k = \tau^x_k \) for \( X_t \) by induction as follows:

\[
\tau_0 = 0
\]

\[
\tau_k = \inf \{ t \geq \tau_{k-1}; |X_t - X_{\tau_{k-1}}| \geq r(X_{\tau_{k-1}}) \}; \ k \geq 1.
\]

Associated with \( \tau_k \), \( X \) and \( y \in U \) we define the measure \( \nu_k = \nu^x_k \) by

\[
\int f d\nu_k = E^y[f(X_{\tau_k}), \tau_k < \infty]; \ f \in C_b(U), \ k = 0, 1, 2, \ldots,
\]

where \( C_b(U) \) denote the space of bounded continuous functions on \( U \).

Observe that

\[
\nu_0 = \delta_y, \text{ the unit point mass at } y,
\]
and by the strong Markov property

\begin{align*}
\int f \, d\nu_{k+1} &= E^y[f(X_{t_{k+1}}), \tau_{k+1} < \infty] = \int E^y[f(X_{t_{k+1}}), \tau_{k+1} < \infty | X_{t_k}] \, dP^y \\
&= \int E^{X_k}[f(X_{t_{k+1}}), \tau_{k+1} < \infty] \, dP^y \\
&= \int E^x[f(X_{T_x}), T_x < \infty | P^y[X_{t_k} \in dx, \tau_k < \infty] \\
&= \int E^x[f(X_{T_x}), T_x < \infty] \, d\nu_k(x); \quad k = 0, 1, 2, \ldots,
\end{align*}

where $T_x = \inf \{t > 0; |X_t - x| \geq r(x)\}$ is the exit time for $X_t$ from the ball centered at $x$ with radius $r(x)$. In other words, $\nu_{k+1}$ can be thought of as having been obtained from $\nu_k$ by a «point-wise spherical sweeping»: at each point $x$, $f(x)$ is replaced by the $X_t$-average of $f$ over the sphere $\Gamma_x$.

**Theorem 1.** — Let $U \subset \mathbb{R}^n$ be open and let $\tau_U$ be the exit time of $U$. Then, with $\tau_k$ and $\nu_k$ as above:

(i) $\lim_{k \to \infty} \tau_k = \tau_U$ a.s.,

(ii) if $P^y[\tau_U < \infty] = 1$ and $\mu_U$ is the exit distribution for $U$ with respect to $X_t$, then

\[ \int f \, d\nu_k \to \int f \, d\mu_u \text{ as } k \to \infty, \text{ for all } f \in C_b(U). \]

**Proof.** — (i): We have $\tau_{k+1} \geq \tau_k$ and $\tau_k \leq \tau_U$ for all $k$. So $\tau = \lim_{k \to \infty} \tau_k$ exists and $\tau \leq \tau_U$.

Assume $\tau(\omega) < \tau_U(\omega)$. Then there exists $\varepsilon > 0$ such that

$\text{dist} (X_{t_k}(\omega), \partial U) \geq \varepsilon$ for all $k$.

Put

$\eta = \inf \{r(x); \text{dist} (x, \partial U) \geq \varepsilon\}$.

Then $\eta > 0$ by (1.2) and

$|X_{t_k} - X_{t_k-1}| \geq \eta$ for all $k$. 

Since $X_t$ has continuous paths this implies that $\tau = \infty$, contradicting $\tau < \tau_U$.

(ii): Let $f \in C_b(\bar{U})$. Then if $\tau_U(\omega) < \infty$, we have $\lim_{k \to \infty} f(X_{\tau_k}) = f(X_{\tau_U})$ from (i), so if we assume that $P^\tau[\tau_U < \infty] = 1$, we conclude that

$$\int f \, d\nu_k = E^\tau[f(X_{\tau_k})] \to E^\tau[f(X_{\tau_U})] = \int f \, d\mu_U.$$ 

This completes the proof of Theorem 1.

One consequence of this result is that the exit distribution for balls starting from the center to a large extent determine the exit distributions for general open sets. For example, we have the following: (For $y \in \mathbb{R}^n$ and $r > 0$ we put $\Delta_r(y) = \{x \in \mathbb{R}^n; |x - y| < r\}$.)

**Theorem 2.** — Let $X_t$ and $Y_t$ be strong, continuous Markov processes in $\mathbb{R}^n$. Suppose there exists a sequence $r_m \downarrow 0$ such for all $y \in \mathbb{R}^n$ and all in the exit distributions of $X_t$ and $Y_t$ from $\Delta_r(y)$ starting at $y$ coincide.

Moreover, suppose that the exit times from balls are finite, a.s. for both processes.

Then $X_t$ and $Y_t$ have the same exit distributions for all open sets.

**Proof.** — Let $U$ be a bounded open set in $\mathbb{R}^n$ and fix $y \in U$. For $x \in U$ define

$$r(x) = \max \{r_m; r_m \leq \text{dist} (x, \partial U)\}.$$ 

If $K$ is a closed subset of $U$ with $\text{dist} (K, \partial U) > \varepsilon$, then for all $x \in K$ we have

$$r(x) \geq \max \{r_m; r_m \leq \varepsilon\}.$$ 

Therefore $r(x)$ satisfies conditions (1.1) and (1.2). Let $\tau_k$, $\tilde{\tau}_k$ be the corresponding sequences (1.3) of stopping times for $X_t$, $Y_t$, respectively and let $\nu_k$, $\tilde{\nu}_k$ be the associated measures (1.4).

Then from (1.6) we have

$$\int f \, d\nu_{k+1} = \int E^x[f(X_{\tau_k})] \, d\nu_k(x).$$
(1.8)

$$\int f \, d\tilde{\nu}_{k+1} = \int \hat{E}^x[f(Y_{t_x})] \, d\tilde{\nu}_k(x) \text{ for } f \in C_0(U), \quad k = 0, 1, 2, \ldots,$$

where $\hat{E}$ denotes expectation with respect to the probability law $\hat{P}^x$ for $Y_t$ and $\hat{t}_x = \inf\{t > 0; |Y_t - x| \geq r(x)\}$.

By assumption,

(1.9) \[ E^x[f(X_{\tau_U})] = \hat{E}^x[f(Y_{\hat{t}_x})], \]

and since $\nu_0 = \hat{\nu}_0 = \delta_x$, we conclude from (1.7)-(1.9) that

$$\nu_k = \hat{\nu}_k \text{ for all } k.$$

So from Theorem 1 we conclude that

(1.10) \[ E'[f(X_{\tau_U})] = \lim_{k \to \infty} E'[f(X_{\tau_U})] = \lim_{k \to \infty} \int f \, d\nu_k = \lim_{k \to \infty} \int f \, d\tilde{\nu}_k = \lim_{k \to \infty} \hat{E}^y[f(Y_{\hat{t}_x})], \]

where $\tau_U$ denotes the first time that $Y_t$ exits from $U$. This proves the result when $U$ is bounded.

Finally, let $U$ be any open set in $\mathbb{R}^n$.

Put

$$U_m = \{x \in U; |x| \leq m\}; \quad m = 1, 2, \ldots.$$

Then if $G$ is open and bounded, we have

$$\{\omega; X_{\tau_U} \in G, \tau_U < \infty\} = \bigcup_m \{x; X_{\tau_U} \in G\}.$$

Therefore,

$$P^y[X_{\tau_U} \in G, \tau_U < \infty] = \lim_{m \to \infty} P^y[X_{\tau_{U_m}} \in G],$$

and similarly for $Y_t$. 
Thus, the general case follows from the case in which $U$ is bounded.

In particular, letting $Y_t$ be the Brownian motion process we obtain:

**Corollary 1.** — Let $X_t$ be a strong Markov process in $\mathbb{R}^n$ with continuous paths. Suppose there exists a sequence $r_m \downarrow 0$ such that for all $y \in \mathbb{R}^n$ the exit distribution of $X_t$ from $\Delta_{r_m}(y)$ starting at $y$ is uniform. Moreover, assume that the exit times for $X_t$ for balls are finite, a.s.

Then the exit distribution of $X_t$ from an arbitrary open set coincides with the harmonic measure for the set.

**Remark.** — It follows from a theorem of Blumenthal, Getoor and McKean ([3], [4]) that since the processes $X_t$ and $Y_t$ in Theorem 2 have the same exit distributions, one can be obtained from the other through a change of time scale. Similarly, in Corollary 2 we may conclude that $X_t$ is the Brownian motion with changed time scale.

2. A converse of the Lévy theorem.

We are now ready to prove the converse of the Lévy theorem stated in the introduction:

**Theorem 3.** — Let $X_t$ be a non-constant, strong Markov process on $\mathbb{R}^n$ with continuous paths and probability laws $P_y$, $y \in \mathbb{R}^n$. Assume that for all affine functions $\varphi$ on the form

$$\varphi(x) = \lambda Ax + b,$$

with $\lambda > 0$, $A \in \mathbb{R}^{n \times n}$ a rotation matrix (i.e. orthogonal with determinant 1) and $b \in \mathbb{R}^n$, the exit distributions of $\omega(X_t)$ with respect to $P_y$ coincide with the exit distribution of $X_t$ with respect to $P_{\varphi(y)}$. Then $X_t$ is the Brownian motion process in $\mathbb{R}^n$, possibly with a changed time scale.

**Proof.** — A point $y \in \mathbb{R}^n$ is called a trap for the process $X_t$ if

$$P_y(X_t = y) = 1 \text{ for all } t > 0.$$

Since $X_t$ is not constant, there exists a point $y_0 \in \mathbb{R}^n$ which is not a trap for $X_t$. By applying the function $\varphi(x) = x - y_0 + y$ we see that no points $y \in \mathbb{R}^n$ are traps for $X_t$.

Fix a point $y \in \mathbb{R}^n$. Put $\sigma_0 = \tau_{\{y\}}$. Then $\sigma_0$ is a stopping time.
We have \( \{\sigma_0 > s + t\} = \{\sigma_0 > s\} \cap \{\sigma_0 \circ \theta_s > t\} \), where \( \theta_s \) is the time shift operator. Thus, by the Markov property

\[
(2.1) \quad P^t[\sigma_0 > s + t] = E^t[P^{X_t}[\sigma_0 > t], \sigma_0 > s] = P^t[\sigma_0 > s]. P^t[\sigma_0 > t]
\]
since \( X_s = y \) on \( \{\sigma_0 > s\} \).

On the other hand, by the strong Markov property and continuity of paths we have, with \( S = \{\sigma_0 \leq t\} \),

\[
(2.2) \quad P^t[\sigma_0 \leq t] = \int_S \chi_s(\omega) \ dP^t(\omega) = \int_S E^t[\chi_S|\mathcal{F}_{\sigma_0}](\omega) \ dP^t(\omega)
\]

\[
= E^t[P^{X_{\sigma_0(\omega)}}[\sigma_0 \leq t - \sigma_0(\omega)] \cdot \chi_s(\omega)]
\]

\[
E^t[P^t[\sigma_0 \leq t] \cdot \chi_s(\omega)] = (P^t[\sigma_0 \leq t])^2,
\]

where \( \mathcal{F}_{\sigma_0} \) is the intersection of all \( \sigma \)-algebras \( \mathcal{F}_{\sigma_0 + \epsilon} \) of events depending on behaviour up to time \( \sigma_0 + \epsilon \), for \( \epsilon > 0 \).

Put \( g(t) = P^t[\sigma_0 > t] \). Then \( 0 \leq g(t) \leq 1 \) so by (2.2) we have for each \( t \) that \( g(t) = 0 \) or \( 1 \).

If \( g \) is not identically equal to 1, let \( t_0 = \inf \{t; g(t) = 0\} \). Then by

\[
(2.1) \quad g\left(\frac{t_0}{2}\right)^2 = g(t_0) = 0 \quad \text{by right-continuity. So} \quad t_0 = 0, \quad \text{i.e.} \quad g \equiv 0.
\]

Since \( y \) is not a trap we conclude that

\[
P^t[\sigma_0 = 0] = 1.
\]

Therefore, for all \( \epsilon > 0 \) there exists \( r > 0 \) such that

\[
P^t[\sigma < \infty] > 1 - \epsilon,
\]

where \( \sigma_r = \inf \{t > 0; |X_t - y| > r\} \).

By applying the affine function

\[
\phi(x) = \frac{x - y}{r} + y
\]

to \( X_t \) we get that

\[
P^t[\sigma_1 < \infty] > 1 - \epsilon.
\]
Since \( \varepsilon \) was arbitrary,

\[
P^e[\sigma_1 < \infty] = 1.
\]

Further, by applying \( \varphi(x) = R(x-y) + y \) to \( X_t \) we obtain that

\[
(2.3) \quad P^e[\sigma_R < \infty] = 1 \quad \text{for all} \quad R > 0.
\]

Finally, by applying affine functions

\[
\varphi(x) = A(x-y) + y,
\]

with \( A \in \mathbb{R}^{n \times n} \) a rotation matrix, we obtain that the exit distribution \( \mu^x_B \) of \( X_t \) from the ball \( B = \{ x; |x-y| < r \} \) starting at \( y \) satisfies

\[
(2.4) \quad \mu^x_B(\varphi(E)) = P^e[X_{t_B} \in \varphi(E)] = P^{e[u]}[X_{t_B} \in \varphi(E)]
\]

\[
= P^e[\varphi(X_{t_B}) \in \varphi(E)] = P^e[X_{t_B} \in E] = \mu^x_B(E).
\]

From (2.3) and (2.4) we conclude that \( \mu^x_B \) coincide with the uniform distribution of unit mass on \( \partial B \). Applying Corollary 1, we see that the exit distribution of \( X_t \) from any open set coincides with \( \lambda^x_U \). Thus, as remarked earlier, it follows from a theorem of Blumenthal, Getoor and McKean ([3], [4]) that \( X_t \) can be obtained from Brownian motion through a change of time scale. This completes the proof.

**Corollary 2.** — The only continuous strong Markov process in the complex plane whose hitting distributions are preserved by analytic functions is the Brownian motion, possibly with a changed time scale.

### 3. A converse of the mean value property for harmonic functions.

We now apply Theorem 1 to be the special case when \( X_t \) is the Brownian motion \( B_t \). First observe that in this case we could have defined the measures \( \nu_k = \nu^B_k \) inductively as follows:

\[
(3.1) \quad \nu_0 = \delta_y,
\]

\[
(3.2) \quad \int f \, d\nu_{k+1} = \int \left( \int_{\Gamma_x} f(u) \, d\rho_x(u) \right) d\nu_k(x), \quad k = 0, 1, 2, \ldots,
\]
where $\rho_x$ is the uniform distribution of mass 1 on $\Gamma_x$, the sphere centered at $x$ with radius $r(x)$ satisfying (1.1) and (1.2). We obtain the following characterization of harmonic measure:

**Theorem 4.** Let $U \subset \mathbb{R}^n$ be open, $y \in U$ and $\lambda_y^U$ the harmonic measure for $U$ at the point $y$. Assume $\lambda_y^U(\mathbb{R}^n) > 0$. Then

$$v_k \rightarrow \lambda_y^U \quad \text{as} \quad k \rightarrow \infty$$

weak star in the dual of $C_0(U)$.

In fact,

$$\int f \, dv_k \rightarrow \int f \, d\lambda_y^U$$

for all bounded, measurable functions $f$ on $U$, vanishing at $\infty$, which are continuous a.e. with respect to $\lambda_y^U$. In particular, if $\lambda_y^U(\mathbb{R}^n) = 1$, then the condition that $f$ vanishes at $\infty$ may be dropped.

**Proof.**—Theorem 1 (ii) gives that $v_k \rightarrow \lambda_y^U$ weak star if $U$ satisfies $E^y[\tau_U < \infty] = 1$.

So assume $0 < E^y[\tau_U < \infty] < 1$. Then necessarily $n \geq 3$ and $B_t$ is transient ([9], Prop. 2.12), i.e.

$$|B_t| \rightarrow \infty \quad \text{as} \quad t \rightarrow \infty, \quad \text{a.s.}$$

So if $f \in C_0(U)$ we have from Theorem 1 (i) that

$$\lim_{k \rightarrow \infty} f(B_{\tau_k}) = \begin{cases} 0 & \text{if} \quad \lim_k \tau_k = \infty \\ f(B_{\tau_k}) & \text{if} \quad \lim_k \tau_k < \infty. \end{cases}$$

Hence

$$\int f \, d\lambda_y^U = E^y[f(B_{\tau_k}) \chi[\tau_U < \infty]] = \lim_{k \rightarrow \infty} E^y[f(B_{\tau_k})] = \lim_{k \rightarrow \infty} \int f \, dv_k,$$

where

$$\chi[\tau_U < \infty](\omega) = \begin{cases} 0 & \text{if} \quad \tau_U(\omega) = \infty \\ 1 & \text{if} \quad \tau_U(\omega) < \infty. \end{cases}$$

The last assertion follows from the weak star convergence by standard arguments.
As an application of Theorem 4 we obtain a converse of the spherical mean value property for harmonic functions. Compared to the results in [1], [6] and [12] our condition on the function (continuity a.e. \( \lambda^U \)) is stronger, but our condition on the radii \( r(x) \) of the spheres \( \Gamma_x \) is weaker. In [7] the condition (1.2) on the radii is dropped, but on the other hand the function is required to be continuous on \( U \cup R \), where \( R \) is the set of regular points (a classical theorem of Kellogg states that \( \lambda^U_y(\partial U \setminus R) = 0 \)). In [11] the open set \( U \) is assumed to be a bounded Lipschitz domain.

We now state and prove our result.

**Corollary 3.** — Let \( U \subset \mathbb{R}^n \) be open such that \( \lambda_y^U(\mathbb{R}^n) > 0 \) for some \( y \in U \). Let \( h \) be a bounded measurable function on \( \overline{U} \), vanishing at \( \infty \) and continuous \( \lambda^U_y \) — a.e. on \( \overline{U} \). Suppose that for all \( x \in U \) we can find a sphere \( \Gamma_x \) centered at \( x \) with radius \( r(x) \) satisfying (1.1) and (1.2) such that \( h \) satisfies the mean value on \( \Gamma_x \):

\[
h(x) = \int_{\Gamma_x} h(u) \, d\rho_x(u).
\]

Then \( h \) is harmonic in \( U \).

**Proof.** — It follows from Proposition 2.1 in [11] that we may assume that \( r(x) \) is measurable.

Choose \( y \in U \) and let \( v_k \) be the successive spherical sweepings defined by (3.1) and (3.2) with respect to the radii \( r(x) \). Then by (3.2) and our hypothesis on \( h \)

\[
\int h \, dv_{k+1} = \left( \int_{\Gamma_x} h(u) \, d\rho_x(u) \right) dv_k(x) = \int h \, dv_k, \quad k \geq 0.
\]

Therefore by Theorem 4

\[
\int h \, d\lambda^U_y = \lim_{k \to \infty} \int h \, dv_k = \int h \, dv_0 = h(y).
\]

Since this holds for all \( y \in U \), \( h \) is harmonic in \( U \).

**Added in proof:**

A version of Theorem 1 and Corollary 3 valid for \( \beta \)-harmonic spaces has been obtained by J. Vesely in « Sequence solutions of the Dirichlet problem » (Časopis pro pěstování matematiky, roč. 106 (1981), 84-93) and « Restricted mean value property in axiomatic potential theory ». (Preprint).
BIBLIOGRAPHY


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