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A note on Perng's unified field theory*

by

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ABSTRACT. — Based on Perng's assumptions and a fundamental property of a symmetric affine connection, a simple argument shows the inconsistency of Perng's theory.

Mr. Jeng J. Perng recently published a paper with the title *A generalization of metric tensor and its application* in this periodical [1]. Some of the basic assumptions of this paper have been found to be inconsistent, or even worse, lead to the fantastic conclusion that there is no electromagnetic field. The details will be shown below.

One of Perng's assumptions is that the tensor A_{ik} satisfies the equation

(1) $$A_{ik,m} - \Gamma^h_{im} A_{hk} - \Gamma^h_{km} A_{ih} = 0.$$

By interchanging indices, we have

(2) $$A_{im,k} - \Gamma^h_{ik} A_{hm} - \Gamma^h_{mk} A_{ih} = 0.$$

and

(3) $$A_{mk,i} - \Gamma^h_{mi} A_{hk} - \Gamma^h_{ki} A_{mh} = 0.$$

If we subtract (1) from the sum of (2) and (3), and use Perng's assumption that the affine connection Γ^h_{im} is symmetric, we immediately get

(4) $$(A_{mh} + A_{hm}) \Gamma^h_{ik} = A_{mk,i} + A_{im,k} - A_{ik,m},$$

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and then, by interchanging indices,

$$(5)\qquad (A_{mh} + A_{hm})\,\Gamma^h_{kl} = A_{mi,k} + A_{km,i} - A_{ki,m}.$$

From (4) and (5), it is esay to see that

$$(6)\quad \varepsilon_{mh}\,\Gamma^h_{ik} = \frac{1}{2}(\varepsilon_{mk,i} + \varepsilon_{mi,k} - \varepsilon_{ik,m}), \qquad \text{with} \quad \varepsilon_{mh} \overset{\text{def.}}{=} \frac{1}{2}(A_{mh} + A_{hm}).$$

If the tensor ε^{km} in Peng's paper is defined by [later we will show, based on Perng's assumption, that this must be this case !] :

$$(7)\qquad \varepsilon^{km}\,\varepsilon_{mh} = \delta^k_h,$$

then from (6) and (7) we conclude that [remember ε^{jk} must be symmetric according to (7)] :

$$(8)\quad \Gamma^j_{ik} = \frac{1}{2}\varepsilon^{jm}(\varepsilon_{mk,i} + \varepsilon_{mi,k} - \varepsilon_{ik,m}) = \frac{1}{2}\varepsilon^{mj}(\varepsilon_{mk,i} + \varepsilon_{mi,k} - \varepsilon_{ik,m}).$$

Perng specifies $\left\{ {\sigma \atop \mu\nu} \right\}$ as the Christoffel symbol constructed by $\varepsilon_{\mu\nu}$. Therefore it is safe to conclude that

$$(9)\qquad \Gamma^j_{ik} = \left\{ {j \atop ik} \right\}.$$

Now according to assumption (*i*) of Perng's paper, the symmetric affine connection Γ^j_{ik} is defined as

$$(10)\qquad \Gamma^j_{ik} = \left\{ {j \atop ik} \right\} + 2\text{æ}\,\varepsilon_{ik}\,\varepsilon^{js}\,\varphi_s.$$

From (9) and (10) we conclude that

$$(11)\qquad 2\,\text{æ}\,\varepsilon_{ik}\,\varepsilon^{js}\,\varphi_s = 0.$$

If $\text{æ} = 0$, then Perng's whole theory is empty. If $\text{æ} \neq 0$, then $\varepsilon_{ik}\,\varepsilon^{js}\,\varphi_s = 0$, and because $\det(\varepsilon^{js}) \neq 0$ [$\varepsilon^{\sigma\rho}\,\varepsilon_{\rho\mu} = \delta^\sigma_\mu$!] we must conclude $\varphi_s = 0$, i. e. there is no electromagnetic field !

Next we want to prove that if the Christoffel symbol $\left\{ {\sigma \atop \mu\nu} \right\}$ is defined as

$$(12)\qquad \left\{ {\sigma \atop \mu\nu} \right\} = \frac{1}{2}\varepsilon^{\sigma\rho}[\varepsilon_{\rho\mu,\nu} + \varepsilon_{\rho\nu,\mu} - \varepsilon_{\nu\mu,\rho}],$$

with $\varepsilon^{\sigma\rho}$ any second rank contravariant tensor, and if $\Gamma^\sigma_{\mu\nu}$ is defined as in Equation (10), then $\varepsilon^{\sigma\rho}\,\varepsilon_{\rho\mu} = \delta^\sigma_\mu$.

Proof. — Under an arbitrary coordinate transformation, $\left\{ \begin{matrix} \sigma \\ \mu\nu \end{matrix} \right\}$ as defined in (12) will transform as

$$\left\{ \begin{matrix} \sigma \\ \mu\nu \end{matrix} \right\}' = \frac{\partial x'^\sigma}{\partial x^m} \left[\frac{\partial x^t}{\partial x'^\mu} \frac{\partial x^h}{\partial x'^\nu} \left\{ \begin{matrix} m \\ th \end{matrix} \right\} + \varepsilon^{ms} \varepsilon_{st} \frac{\partial^2 x^t}{\partial x'^\mu \partial x'^\nu} \right].$$

Since $2 \times \varepsilon_{\mu\nu} \varepsilon^{\sigma s} \varphi_s$ is a third rank mixed tensor, $\Gamma^\sigma_{\mu\nu}$ as defined in (10) will transform as

$$\Gamma^{\sigma'}_{\mu\nu} = \frac{\partial x'^\sigma}{\partial x^m} \left[\frac{\partial x^t}{\partial x'^\mu} \frac{\partial x^h}{\partial x'^\nu} \Gamma^m_{th} + \varepsilon^{ms} \varepsilon_{st} \frac{\partial^2 x^t}{\partial x'^\mu \partial x'^\nu} \right]. \tag{13}$$

Because $\Gamma^\sigma_{\mu\nu}$ is a symmetric affine connection, it should transform as

$$\Gamma^{\sigma'}_{\nu\mu} = \frac{\partial x'^\sigma}{\partial x^m} \left[\frac{\partial x^t}{\partial x'^\mu} \frac{\partial x^h}{\partial x'^\nu} \Gamma^m_{th} + \frac{\partial^2 x^m}{\partial x'^\mu \partial x'^\nu} \right]. \tag{14}$$

Subtracting (14) from (13) we have

$$\frac{\partial x'^\sigma}{\partial x^m} [\varepsilon^{ms} \varepsilon_{st} - \delta^m_t] \frac{\partial^2 x^t}{\partial x'^\mu x'^\nu} = 0, \qquad \text{and thus} \quad \varepsilon^{ms} \varepsilon_{st} - \delta^m_t = 0,$$

or

$$\varepsilon^{ms} \varepsilon_{st} = \delta^m_t.$$

Q. E. D.

If we define $\left\{ \begin{matrix} \sigma \\ \mu\nu \end{matrix} \right\} \overset{\text{def.}}{=} \frac{1}{2} \varepsilon^{\rho\sigma} [\varepsilon_{\rho\mu,\nu} + \varepsilon_{\rho\nu,\mu} - \varepsilon_{\nu\mu,\rho}]$ then we will conclude $\varepsilon^{sm} \varepsilon_{st} = \delta^m_t$ and because ε_{st} is symmetric, then ε^{sm} must be symmetric and thus we also get $\varepsilon^{ms} \varepsilon_{st} = \delta^m_t$.

We thus see that the symmetric affine connection Γ^i_{jk} is totally determined by the tensor ε_{ik} if ε_{ik} is nonsingular and if we postulate $A_{ik;l} = 0$. Also, as we have shown, an attempt to modify the definition of the tensor ε^{ik} but keep the form of the symmetric affine connection proposed by Perng is doomed to be a failure.

REFERENCE

[1] JENG J. PERNG, *Ann. Inst. H. Poincaré*, t. XIII, n° 4, 1970, p. 287.

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