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On relativistic kinetic theory of transport processes, in particular of neutrino systems

by

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El diámetro del Aleph sería de dos o tres centímetros, pero el espacio cósmico estaba ahí, sin disminución de tamaño (Jorge Luis Borges, *El Aleph*).

RÉSUMÉ. — Les coefficients de transport pour systèmes, régis par des interactions diverses de particules, sont obtenus à partir d'une équation de transport relativiste et quantique. Pour des systèmes de neutrinos, dominés par l'interaction faible, la conductivité thermique, les viscosités et les coefficients de diffusion sont calculés. En outre, un mécanisme est indiqué pour *la séparation de neutrinos et d'antineutrinos* par diffusion thermique. Ces résultats revêtent une importance cosmologique, puisqu'ils s'appliquent au gaz leptonique qui existait de 10^{-4} à 10 secondes après l'explosion primordiale.

ABSTRACT. — From a transport equation, incorporating both relativistic and quantum aspects, transport coefficients have been obtained for various particle interactions. For neutrino systems, dominated by the weak interaction, viscosities, heat conductivities and diffusion coefficients have been calculated as functions of the state parameters. Moreover, a mechanism for *the separation of neutrinos and antineutrinos* through thermal diffusion is indicated. These results constitute data of cosmological importance, since they apply to the primordial gas of the lepton era, which lasted from 10^{-4} to 10 seconds after the hot big bang.

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1. KINETIC THEORY

To arrive from the microscopic level of physics at the macroscopic description, various theoretical devices have been developed. The kinetic theory of gases is one of them. From the knowledge of the particle interactions and by means of an appropriate averaging procedure a « transport equation » for the probability distribution function of the system may be set up. With its help the macroscopic conservation laws, the entropy law and linear laws for the transport processes may be construed. The theory culminates in the calculation of the characteristic transport coefficients in terms of the (microscopic) particle parameters and the (macroscopic) state variables.

2. HISTORICAL REMARKS

The kinetic theory of gases in its original—nonrelativistic and classical—form is due to Boltzmann [*1*]. Its extension to quantum theory is due to Waldmann [*2*] and Snider [*3*]. The first relativistically covariant equation —for the classical case—was proposed by Lichnerowicz and Marrot [*4*]. Finally, an equation, incorporating both the relativistic and the quantum aspects, was derived by de Boer and van Weert [*5*]. On its basis one may derive transport properties for systems of particles interacting according to the various Lagrangian densities of the elementary forces.

3. ON THE RELATIVISTIC KINETIC THEORY OF QUANTUM SYSTEMS [*6*]

The microscopic starting point of the relativistic theory of transport processes in quantum systems is the description of particle collisions by means of quantum field theory.

To arrive at the macroscopic level of the theory one needs a condition of a statistical nature. It has been shown by de Boer and van Weert that Bogoliubov's condition [*7*] of the absence of initial correlations is sufficient for the purpose. This condition is much weaker than Boltzmann's original hypothesis of molecular chaos (« Stosszahlansatz »), postulated to be valid at each moment in the course of time. For the systems to be studied presently, a limitation to binary collisions is employed. Moreover, the systems are such that their macroscopic properties change slowly over the spatial and temporal ranges of the particle interactions (for instance atomic diameters or Compton wavelengths, and corresponding time intervals).

In this way one arrives at a transport equation for the quantum-mechanical distribution function, the Wigner function, which in general carries spin indices. This equation, which is covariant, contains a « streaming member », involving spatial and temporal derivatives, and a « collision member », which contains a transition operator, depending upon the particle interaction Lagrangian.

4. REDUCED TRANSPORT EQUATION

The Wigner distribution function reduces to a single-component function for classical systems, and also for neutrino systems, because the neutrinos have a fixed polarisation of their spins. The transport equation for a neutrino-antineutrino system reads:

$$p^{\mu}\partial_{\mu}f^{(i)}(x, p) = \sum_{j=1}^{2} \mathrm{C}^{(ij)}(x, p) \quad , \quad (i = 1, 2) \tag{1}$$

where $f^{(i)}(x, p)$ is the distribution of the neutrinos ($i = 1$) or the antineutrinos ($i = 2$). It is a function of the time-space coordinates $x: = (ct, x, y, z)$ and the particle energy-momentum $p: = (E/c, p_x, p_y, p_z)$. Furthermore ∂_{μ} stands for $\partial/\partial x^{\mu}$. The collision terms of (1) are given by

$$\overline{\mathrm{C}}^{(ij)}(x, p) = \left(1 - \frac{1}{2}\delta_{ij}\right) \int \frac{d^3p_1}{p_1^0} \frac{d^3p'}{p'^0} \frac{d^3p_1'}{p_1'^0} \mathrm{W}^{(ij)}(p', p_1' \mid p, p_1)$$
$$[f^{(i)}(x, p')f^{(j)}(x, p_1') - f^{(i)}(x, p)f^{(j)}(x, p_1)] \quad , \quad (i, j = 1, 2) \tag{2}$$

for collisions $p + p_1 \rightarrow p' + p_1'$. The transition rates $\mathrm{W}^{(ij)}$ have the form

$$\mathrm{W}^{(ij)}(p', p_1' \mid p, p_1) = \frac{(2\pi)^{10}\hbar^8}{c^2} \delta^{(4)}(p + p_1 - p' - p_1') \mid \langle \vec{p}', \vec{p}_1' \mid t \mid \vec{p}, \vec{p}_1 \rangle^{(ij)} \mid^2 \tag{3}$$

with t the transition operator, which in lowest approximation is connected to the interaction Lagrangian density $\mathrm{L_I}(x)$ as

$$\langle \vec{p}', \vec{p}_1' \mid t \mid \vec{p}, \vec{p}_1 \rangle = - \langle \vec{p}', \vec{p}_1' \mid \mathrm{L_I}(0) \mid \vec{p}, \vec{p}_1 \rangle, \tag{4}$$

where the particular Lagrangian density at the right-hand side follows from the general one by the relation.

$$\mathrm{L_I}(x) = \exp\left(\frac{i}{\hbar}\mathrm{P}.x\right)\mathrm{L_I}(0) \exp\left(-\frac{i}{\hbar}\mathrm{P}.x\right). \tag{5}$$

Before inserting the Lagrangian density for the neutrino system, the general linear laws will be introduced.

5. LINEAR THEORY

The distribution functions may be developed around the solution $f^{(0)}(x, p)$ of the homogeneous transport equation, *i. e.*, the equations with vanishing collision member. One then writes:

$$f(x, p) = f^{(0)}(x, p)[1 + \varphi(x, p)] \tag{6}$$

If the theory is now developed up to terms linear in the function $\varphi(x, p)$ one arrives at linear laws for the transport processes. These laws connect the « fluxes » and the driving « thermodynamical forces ». The fluxes are the heat flow I_q^μ, the diffusion flow I_1^μ, the symmetric traceless part of the viscous pressure tensor $\Pi^{\mu\nu}$ and the trace of the viscous pressure tensor Π. The thermodynamical forces are proportional to the temperature gradient $\nabla^\mu T$ (with $T(x)$ the temperature field and ∇^μ the covariant gradient operator which has the time-space components $(0, -\partial/\partial x, -\partial/\partial y, -\partial/\partial z)$ in the proper frame), the concentration gradient $\nabla^\mu x_1$ (with x_1 the neutrino fraction; $x_2 = 1 - x_1$ is the antineutrino fraction), the symmetric and traceless gradient $\langle \nabla^\mu U^\nu \rangle$ and the divergence $\nabla^\mu U_\mu$ of the hydrodynamic velocity field $U^\mu(x)$ (the temperature gradient is in fact accompanied by a term $-\nabla^\mu p/4nk$ with $p = nkT$ the pressure, n the particle density and k Boltzmann's constant. It is omitted here for brevity). Since the system is spatially isotropic, one obtains linear laws, connecting fluxes and thermodynamic forces, of the form [*8*]:

$$I_q^\mu = \lambda \nabla^\mu T + D'_T nkT^2 \nabla^\mu x_1, \tag{7}$$

$$I_1^\mu = D_T x_1 x_2 n \nabla^\mu T + Dn \nabla^\mu x_1, \tag{8}$$

$$\Pi^{\mu\nu} = 2\eta \langle \nabla^\mu U^\nu \rangle, \tag{9}$$

$$\Pi = -\eta_v \nabla^\mu U_\mu. \tag{10}$$

The transport coefficients which appear here are the heat conductivity λ, the Dufour coefficient D'_T, the thermal diffusion coefficient D_T, the diffusion coefficient D, the shear viscosity η and the volume viscosity η_v.

6. WEAK INTERACTION

The unified theory of weak and electromagnetic interaction, due to Weinberg [*9*], Salam [*10*], 't Hooft [*11*] and Veltman [*12*], provides a reliable form for the weak interaction Lagrangian density of electrons, neutrinos and their antiparticles. They interact *via* charged vector bosons W^μ and neutral vector bosons Z^μ, both of which have a rest energy mc^2 of the order of 10 to 100 GeV. The expression reads:

$$\begin{aligned} L_I(x) = \frac{g}{2\sqrt{2}} \bar{e}(x)\gamma_\mu(1 + \gamma_5)\nu(x)W^\mu(x) + \text{h. c.} \\ + \frac{1}{4}(g^2 + g'^2)^{1/2}\bar{\nu}(x)\gamma_\mu(1 + \gamma_5)\nu(x)Z^\mu(x) \\ - \frac{1}{4}(g^2 - g'^2)(g^2 + g'^2)^{-1/2}\bar{e}(x)\gamma_\mu(1 + \gamma_5)e(x)Z^\mu(x) \\ + \frac{1}{2}g'^2(g^2 + g'^2)^{-1/2}\bar{e}(x)\gamma_\mu(1 - \gamma_5)e(x)Z^\mu(x). \quad (11) \end{aligned}$$

Here $e(x)$ and $\nu(x)$ are the electron and neutrino field operators, γ^μ and γ_5 Dirac matrices and g and g' two interaction constants. The latter are related to the elementary electric charge e and the Weinberg angle θ_W as

$$g = \frac{e}{\sin\theta_W} \quad , \quad g' = \frac{e}{\cos\theta_W} . \quad (12)$$

Since we shall be concerned with neutrinos of energies between 1 and 100 MeV, which are thus much smaller than the rest energies of the vector bosons, an effective Lagrangian density may be used valid in the relatively low energy range. It has been extracted from (11) by Siskens and van Weert [13] and reads:

$$\begin{aligned} L_I(x) = - \frac{G}{4\sqrt{2}} :\bar{\nu}(x)\gamma^\mu(1 + \gamma_5)\nu(x)\bar{\nu}(x)\gamma_\mu(1 + \gamma_5)\nu(x) : \\ - \frac{G}{2\sqrt{2}} :\bar{\nu}(x)\gamma^\mu(1 + \gamma_5)\nu(x)\bar{e}(x)\gamma_\mu(1 + C + \gamma_5)e(x):, \quad (13) \end{aligned}$$

where the weak interaction constant G and the Weinberg constant C appear. They are related to the previously introduced constants g and θ_W as:

$$\frac{G}{\sqrt{2}} = \frac{g^2\hbar^2}{8m_W^2c^2} \quad , \quad C = 4\sin^2\theta_W, \quad (14)$$

where m_W is the mass of the charged vector bosons, $\hbar$ Planck's quantum constant and c the speed of light. The numerical values of these constants are

$$G = 1.44\ 10^{-49}\ \text{g cm}^5\text{s}^{-2} \quad , \quad C = 0.90, \quad (15)$$

values to be used in the numerical calculations of the transport coefficients.

If only neutrinos and antineutrinos, but no electrons and antielectrons, enter into play, the expression (13) reduces to

$$L_I(x) = - \frac{G}{4\sqrt{2}} :\bar{\nu}(x)\gamma^\mu(1 + \gamma_5)\nu(x)\bar{\nu}(x)\gamma_\mu(1 + \gamma_5)\nu(x):, \quad (16)$$

an expression to be employed in the next two sections, dealing with neutrino systems. If it is inserted into (3)-(4) one obtains the transition rates for collision between identical and different particles respectively [*14*, *15*]:

$$W^{(ii)}(p', p_1' \mid p, p_1) = \frac{8G^2}{(2\pi\hbar^2 c)^2}(p\cdot p_1)(p'\cdot p_1')\delta^{(4)}(p + p_1 - p' - p_1') \qquad (17)$$
$$(i = 1, 2),$$
$$W^{(ij)}(p', p_1' \mid p, p_1) = \frac{8G^2}{(2\pi\hbar^2 c)^2}(p\cdot p_1')(p_1\cdot p')\delta^{(4)}(p + p_1 - p' - p_1') \qquad (18)$$
$$(i, j = 1, 2 ; i \neq j).$$

These expressions form the basis for the calculation of the transport properties.

7. THE PURE NEUTRINO GAS

For the pure neutrino gas ($x_1 = 1$, $x_2 = 0$) the linear laws (7)-(10) simplify: in (7) only the first term at the right-hand side remains, and (8) is altogether absent.

In the procedure to solve transport equations solutions are obtained by successive approximations. The first order results for the heat conductivity and the two viscosities of the pure neutrino gas are [*14*]:

$$\lambda = \frac{3}{80}\frac{\pi\hbar^4 c^5}{kT^2G^2}, \qquad (19)$$

$$\eta = \frac{3}{184}\frac{\pi\hbar^4 c^3}{kTG^2}, \qquad (20)$$

$$\eta_v = 0. \qquad (21)$$

In the case of neutrino systems, the approximations may be obtained up to arbitrary order, and exact values were obtained by means of extrapolation procedures. The latter give rise to numerical values somewhat different from the first order results (19) and (20): the factors then become 0.20 and 0.026 respectively. The volume viscosity vanishes in all orders for systems of particles with zero mass.

During the lepton era of the universe, which lasted from 10^{-4} to 10 seconds after the hot big bang, the temperature dropped from 10^{12} Kelvin to 10^{10} Kelvin. One gets an idea of the cosmological numbers involved by inserting for the temperature T the value of 10^{11} Kelvin. Then one obtains for the (exact values of) the heat conductivity and the shear viscosity:

$$\lambda = 5\ 10^{35}\ \text{g cm s}^{-3}\ \text{K}^{-1}, \qquad (22)$$
$$\eta = 9.3\ 10^{24}\ \text{g cm}^{-1}\ \text{s}^{-1}. \qquad (23)$$

One may appreciate the magnitude of these numbers by comparing them with « terrestrial » values for ordinary gases, where one has 10^{-2} and 10^{-5} (same units as above) as orders of magnitude for λ and η respectively.

8. NEUTRINO-ANTINEUTRINO SYSTEMS : TRANSPORT COEFFICIENTS

For mixtures of neutrinos and antineutrinos we have the linear laws (7)-(10) for the transport processes. Owing to the inherent symmetry between the two components of the mixture, hardly to be surpassed by any other system, the first order results for the transport coefficients have a particularly elegant appearance. They are found to be [*15*]: heat conductivity

$$\lambda = \frac{3}{80} \frac{97 + 112x_1x_2}{97 + 12x_1x_2} \frac{\pi\hbar^4c^5}{kT^2G^2}, \tag{24}$$

thermal diffusion and Dufour coefficients

$$D_T = D'_T = \frac{3}{80} \frac{24(x_1 - x_2)}{97 + 12x_1x_2} \frac{\pi\hbar^4c^5}{nk^2T^3G^2}, \tag{25}$$

diffusion coefficient

$$D = \frac{3}{80} \frac{95 - 12x_1x_2}{97 + 12x_1x_2} \frac{\pi\hbar^4c^5}{nk^2T^2G^2}, \tag{26}$$

shear viscosity

$$\eta = \frac{3}{184} \frac{119 + 184x_1x_2}{119 + 52x_1x_2} \frac{\pi\hbar^4c^3}{kTG^2}, \tag{27}$$

volume viscosity

$$\eta_v = 0. \tag{28}$$

The formulae show the dependence upon the state variables: the temperature T, the particle density n and the neutrino fraction x_1 (the antineutrino fraction x_2 is equal to $1 - x_1$), and upon the constants of nature $\hbar$, c and G. The equality of the coefficients D_T and D'_T is an example of a relativistic Onsager relation arising from the motion reversal invariance property obeyed by the (microscopic) equations of motion of the particles. The symmetry with respect to x_1 and x_2 may be worded by saying that the « direct » coefficients λ, η and D remain invariant under the interchange of x_1 and x_2, while the « cross-coefficients » D_T and D'_T change sign under this operation. In the next section more attention will be paid to the phenomenon of thermal diffusion.

Again approximations can be obtained in all orders and exact results from extrapolation of these. It turns out that λ and η are of roughly the same magnitude as for the pure neutrino gas; in fact, these coefficients depend only slightly upon the composition x_1. For the case of $x_1 = 3/4$ one obtains for the products of the first two factors in the thermal diffusion coefficient (25) and the diffusion coefficient (26) values of 0.017 and 0.005 respectively. For equal amounts of neutrinos and antineutrinos ($x_1 = x_2 = 1/2$) these factors are 0 and 0.005 respectively. In fact, while D_T depends of course critically upon x_1, the coefficient D is again almost the same for all compositions.

Cosmological numbers may be judged by inserting typical values pertaining to the lepton era: $T = 10^{11}$ Kelvin and $n = 10^{35}$ cm^{-3}. Then one obtains for $x_1 = 3/4$ the (exact) numbers:

$$D_T = D'_T = 4.3 \ 10^4 \text{ cm}^2 \text{ s}^{-1} \text{ K}^{-1}, \tag{29}$$

$$D = 1.3 \ 10^{16} \text{ cm}^2 \text{ s}^{-1}, \tag{30}$$

and for equal amounts of neutrinos and antineutrinos ($x_1 = x_2 = 1/2$):

$$D_T = D'_T = 0, \tag{31}$$

$$D = 1.2 \ 10^{16} \text{ cm}^2 \text{ s}^{-1}, \tag{32}$$

showing qualitatively the same features as discussed above for the first order results. Numerically the thermal diffusion coefficient (29) is few (say 7) orders of magnitude greater than its terrestrial counterpart. The diffusion coefficient however is several orders of magnitude greater than 1 cm^2 s^{-1}, which is a typical terrestrial value.

9. SEPARATION EFFECTS IN NEUTRINO-ANTINEUTRINO SYSTEMS [*15*]

The most striking result of the preceding section concerns the thermal diffusion coefficient D_T, as given by formula (25). As the linear law (8) shows, thermal diffusion is responsible for a flow of matter and thus a separation effect is produced if a temperature gradient is present in the system. The expression (25) shows that it vanishes if the number of neutrinos happens to be equal to the number of antineutrinos ($x_1 = x_2$). But in the general case of unequal amounts of neutrinos and antineutrinos ($x_1 \neq x_2$) it follows from the linear law (8) in conjunction with the expression (25) that temperature gradients will tend to enhance the concentration of the more abundant component in the relatively colder parts of the system. A measure for the separation provoked by thermal diffusion may be obtained from the condition of vanishing diffusion flow I_1^μ. Then one has from (8) as a quantity determining the separation

$$-\frac{|\nabla^\mu x_1|/x_1 x_2}{|\nabla^\mu T|/T} = \frac{D_T T}{D} =: \alpha, \tag{33}$$

which is called the « thermal diffusion factor » α. If the first order results (25) and (26) are inserted into the last member of (33), one obtains a simple and characteristic expression for the thermal diffusion factor:

$$\alpha = \frac{24(x_1 - x_2)}{95 - 12x_1 x_2}. \tag{34}$$

It is seen to depend uniquely upon the composition.

Again it is interesting to compare celestial and terrestrial values of the thermal diffusion factor. For $x_1 = 3/4$ the expression (34) yields a value of 0.33 for the thermal diffusion factor. This is only slightly smaller than most typical terrestrial values. So if the primordial neutrino-antineutrino gas is not thermally uniform, appreciable separation effects might well arise.

An important point however is whether the mechanism proposed here to explain separation of matter and antimatter (*in casu* neutrinos and antineutrinos) fits into the temporal and spatial dimensions available during the lepton era. To this end we need to know the characteristic time θ in which a concentration gradient may be built up (or destroyed) over a spatial extension of a certain magnitude A. From the solution of the equations which govern the phenomena it turns out that the time θ is equal to $A^2/\pi^2 D$. If we now choose as characteristic times θ within the lepton period 0.1, 1 and 10 seconds, we obtain for the dimension A, with the help of the value of D given above, a magnitude of the order of 10^6 to 10^7 cm. These values must be notably smaller than the radius of the universe at the time of the lepton era. And this is indeed the case since we may say that it was then of the order of 10^{16} cm.

10. ELECTRON-NEUTRINO SYSTEMS [*16*]

For a mixture of neutrinos and electrons we need the full weak interaction Lagrangian (13) and the ensuing transition rates. The transport processes are not appreciably affected by the electromagnetic interaction [*17*]. One finds ultimately, again for a temperature T equal to 10^{11} kelvin, heat conductivities and shear viscosities of the same order of magnitude as those given in section 7. But now the bulk viscosity does not vanish: one finds a value of the order of 10^{15} g cm^{-1} s^{-1}. Furthermore the thermal diffusion coefficient of the neutrinos with respect to the electrons is such that the neutrinos move to the relatively hotter parts of the system. The thermal diffusion factor (34) is found to be of the same order of magnitude as for terrestrial systems.

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