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Scattering of a plane gravitational wave by a magnetic dipole field in the Schwarzschild metric

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ABSTRACT. — The equations governing the propagation of high-frequency coupled gravitational and electromagnetic waves are integrated for the case of a plane gravitational wave impinging onto a Schwarzschild space-time equipped with a magnetic dipole field. The generated electromagnetic wave is focused due to the gravitational lens effect. An expression is derived for the intensity of the electromagnetic wave in the focal region. In the case of alignment of incidence direction and magnetic axis no rotation of the polarization plane occurs. The relevance of the effect for the exterior region of neutron stars is discussed.

1. INTRODUCTION

As is well known, electromagnetic and gravitational perturbations cannot be treated separately in regions of space-time occupied by a strong electromagnetic field. Rather, they lose their individuality and appear only in a coupled manner. This coupling leads to the effect of conversion of gravitational into electromagnetic waves (and *vice versa*). In the simplest case Minkowski space-time is taken as background and the conversion effect is studied by using external static or stationary electromagnetic fields [*1-10*], fields of moving charges [*11*, *12*] and rotating magnetic dipoles [*13*] or by investigating bremsstrahlung processes [*14*]. Similarly, the effect of photoproduction of gravitons in static electromagnetic fields is predicted by a quantized version of linearized general relativity [*15*, *16*].

If the background space-time is curved, the linearized Einstein-Maxwell equations exhibit a rather complicated structure. Therefore, one either chooses a special background metric or assumes the waves to be of small wavelength. As far as the former possibility is concerned, several techniques were developed to treat perturbations of the Reissner-Nordström solution, among them Zerilli's approach *[17]*, Moncrief's method *[18]* based on a Hamiltonian formalism as well as the approach employing the Newman-Penrose *[19]* formalism (see the recent review article by Bicák *[20]* containing a description and comparison of all these techniques). Using these methods, a number of authors *[21-28]* calculated the efficiency of the conversion effect in the Reissner-Nordström background. In particular, the conversion cross section of charged black holes can be derived *[26-28]*. On the other hand, high-frequency waves can be treated as perturbations away from arbitrary solutions of the Einstein-Maxwell equations by deriving propagation equations governing the change of the wave amplitudes along the rays *[29-33]*. The new effect which emerges is a Faraday-like rotation of the polarization plane of linearly polarized radiation with respect to a parallely propagated tetrad along the rays.

The present paper differs from previous research in two ways. First, we assume a somewhat more realistic situation by using a Schwarzschild solution equipped with a magnetic dipole field. In contrast to the commonly used electric monopole fields, this situation may be a good model for, e. g., the exterior region of a neutron star. Second, we take into account the focusing of the generated electromagnetic radiation due to the gravitational lens effect. In particular, the intensity in the focal region, which can be obtained by means of waveoptical methods *[34-38]*, is of some interest. The main question which arises is whether the rather pessimistic predictions of previous papers concerning the astrophysical significance of the conversion effect (see, e. g., *[10]*) may be changed in this case.

The following section outlines the basic equations governing the propagation of coupled gravitational and electromagnetic waves in the high-frequency limit as derived in *[33]*. This formalism is then used in sec. 3 and sec. 4 to investigate the scattering process described above. We use units with $c = 1$ and the signature $(+, +, +, -)$. $\varkappa$ denotes Einstein's gravitational constant.

2. PROPAGATION OF INTERACTING ELECTROMAGNETIC AND GRAVITATIONAL PERTURBATIONS IN THE LIMIT OF SMALL WAVELENGTHS

In the approximation of geometrical optics, electromagnetic and gravitational waves are conveniently studied by means of the null tetrad formalism introduced by Newman and Penrose *[19]*. Consider a solution of the Einstein-

Maxwell equations characterized by its Weyl tensor $C_{\alpha\beta\gamma\delta} + \varepsilon\widehat{C}_{\alpha\beta\gamma\delta}$ and its Maxwell tensor $F_{\alpha\beta} + \varepsilon\widehat{F}_{\alpha\beta}$. The parameter ε is supposed to be small in order to ensure that the field equations can be linearized. Let ϕ_A, $\widehat{\phi}_A$ and $\widehat{\Psi}_A$ be the tetrad components of $F_{\alpha\beta}$, $\widehat{F}_{\alpha\beta}$ and $\widehat{C}_{\alpha\beta\gamma\delta}$ with respect to a null tetrad defined in the background space-time. If the scale on which the perturbations vary is much smaller than the typical curvature radius of the background geometry, waves mainly propagate along null geodesics of the background metric, which allows calculations to be simplified by identifying the tetrad vector l_α with the ray vector and assuming the remainder of the tetrad to undergo parallel transport along the rays. Under these assumptions the perturbations are completely characterized by the two complex functions $\widehat{\Psi}_4$ and $\widehat{\phi}_2$, which take for nearly monochromatic waves the form

$$\begin{aligned} \widehat{\Psi}_4 &= \omega\overset{(0)}{\Psi}{}_4^+ e^{i\omega S} + \omega\overset{(0)}{\Psi}{}_4^- e^{-i\omega S} \\ \widehat{\phi}_2 &= \overset{(0)}{\phi}{}_2^+ e^{i\omega S} + \overset{(0)}{\phi}{}_2^- e^{-i\omega S} \end{aligned} \tag{2.1}$$

with $l_\alpha = S_{,\alpha}$. ω is a large parameter ensuring that the wave oscillates on a small enough scale. Its upper limit is determined by the assumption that the leading contribution to the curvature tensor of the perturbed space-time originates in the background metric. The amplitudes describing right-handed $(\overset{(0)}{\Psi}{}_4^+, \overset{(0)}{\phi}{}_2^+)$ and left-handed $(\overset{(0)}{\Psi}{}_4^-, \overset{(0)}{\phi}{}_2^-)$ polarized waves are invariant with respect to coordinate gauge transformations owing to the high-frequency assumption (1). The propagation equations

$$\begin{aligned} (D + \rho)\overset{(0)}{\Psi}{}_4^+ + i\varkappa\phi_0^* . \overset{(0)}{\phi}{}_2^+ &= 0 \\ (D + \rho)\overset{(0)}{\phi}{}_2^+ + i\phi_0 . \overset{(0)}{\Psi}{}_4^+ &= 0, \end{aligned} \tag{2.2}$$

which determine the rate of change of the amplitudes along the rays, are coupled by the tetrad component ϕ_0 of the background Maxwell field. In the following we confine ourselves to right-handed polarized waves, since the corresponding equations describing left-handed polarized waves do not differ essentially from those referring to the helicity labelled +. The system (2.2) can be decoupled by introducing normal modes $\overset{(0)}{\chi}{}_{1/2}^{\pm}$:

$$\begin{aligned} \overset{(0)}{\Psi}{}_4^+ &= T_{11}^+ \overset{(0)}{\chi}{}_1^+ + T_{12}^+ \overset{(0)}{\chi}{}_2^+ \\ \sqrt{\varkappa}\overset{-(0)}{\phi}{}_2^+ &= T_{21}^+ \overset{(0)}{\chi}{}_1^+ + T_{22}^+ \overset{(0)}{\chi}{}_2^+ \end{aligned} \tag{2.3}$$

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(1) Rather than project both background parts and perturbations of the tensor fields onto a background null tetrad, we can define the variables (2.1) equally well by splitting the tetrad components (formed with a perturbed null tetrad) into unperturbed parts and perturbations. In this case the amplitudes are also invariant with respect to tetrad gauge transformations and, in particular, identical with the above-constructed amplitudes.

Inserting (2.3) into (2.2) we obtain the system

$$\begin{aligned} (D + \rho + i\sqrt{\varkappa}\,|\,\phi_0\,|)\overset{(0)}{\chi}{}_1^{+} &= 0 \\ (D + \rho - i\sqrt{\varkappa}\,|\,\phi_0\,|)\overset{(0)}{\chi}{}_2^{+} &= 0, \end{aligned} \tag{2.4}$$

which can immediately be solved, and a system for the elements of the (unitary) matrix T^+. Introducing the new variables η and $\tilde{y}$ by

$$\phi_0 = |\,\phi_0\,|.\exp(i\eta) \quad , \quad \tilde{y}(y) = i\sqrt{\varkappa}.\int |\,\phi_0\,|\,dy \tag{2.5}$$

(y affine parameter), we can write the latter in the simple form

$$\begin{aligned} dT_{11}^{+}/d\tilde{y} - T_{11}^{+} + \exp(-\,i\eta).T_{21}^{+} &= 0 \\ dT_{21}^{+}/d\tilde{y} - T_{21}^{+} + \exp(i\eta).T_{11}^{+} &= 0 \\ dT_{12}^{+}/d\tilde{y} + T_{12}^{+} + \exp(-\,i\eta).T_{22}^{+} &= 0 \\ dT_{22}^{+}/d\tilde{y} + T_{22}^{+} + \exp(i\eta).T_{12}^{+} &= 0. \end{aligned} \tag{2.6}$$

Although (2.6) is in general a complicated system, it has simple solutions in important special cases [*29-33*].

3. A PLANE GRAVITATIONAL WAVE IMPINGING ONTO A SCHWARZSCHILD SPACE-TIME

In the remainder of this paper the underlying space-time is assumed to be described by the Schwarzschild metric

$$ds^2 = \frac{r}{r - 2M}\,dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta\,d\varphi^2) - \frac{r - 2M}{r}\,dt^2. \tag{3.1}$$

Since (3.1) represents a vacuum solution of the Einstein equations, we neglect the back reaction of both background Maxwell field and electromagnetic wave on the metric. As already pointed out by Zeldovich [*10*], this leads to no inconsistencies if the change of the beating phase (defined below) along a ray traversing the space-time domain considered is small compared with unity. Otherwise one encounters instabilities indicating that the back reaction of $F_{\alpha\beta}$ on $g_{\alpha\beta}$ cannot be neglected. Moreover, in order to be compatible with the linearization of the field equations as described above, the typical values of the background Maxwell field $F_{\alpha\beta}$ are, in turn, supposed to be large compared with the typical values of the wave $\varepsilon\widehat{F}_{\alpha\beta}$.

Consider a bundle of null geodesics each of which lies in a hypersurface $\varphi = \text{const}$ of the metric (3.1). An appropriate null tetrad which undergoes parallel propagation along these rays is given by ($\alpha = 1, 2, 3, 4$ corresponds to r, ϑ, φ, t)

$$\begin{aligned} l^\alpha &= \left(\varepsilon_1\sqrt{1 - a^2(r - 2M)/r^3},\ \varepsilon_2 a/r^2,\ 0,\ r/(r - 2M)\right) \\ n^\alpha &= \tilde{n}^\alpha + |b|^2 l^\alpha + b\tilde{m}^{\alpha *} + b^*\tilde{m}^\alpha \\ m^\alpha &= \tilde{m}^\alpha + b l^\alpha \end{aligned} \tag{3.2 a}$$

with the abbreviations

$$\begin{aligned} \tilde{n}^\alpha &= 1/2\cdot\left(-\varepsilon_1(r - 2M)/r\cdot\sqrt{1 - a^2(r - 2M)/r^3},\ -\varepsilon_2 a(r - 2M)/r^3,\ 0,\ 1\right) \\ \tilde{m}^\alpha &= 1/(\sqrt{2}r)\cdot\left(-\varepsilon_1 a(r - 2M)/r,\ \varepsilon_2\sqrt{1 - a^2(r - 2M)/r^3},\ i/\sin\vartheta,\ 0\right) \\ b &= aM/\sqrt{2}\cdot\int[1 - a^2(r - 2M)/r^3]^{-1/2}r^{-3}dr \\ \varepsilon_1 &= \pm 1,\ \varepsilon_2 = \pm 1. \end{aligned} \tag{3.2 b}$$

The congruence is completely characterized if the impact parameter a is known as a function of r and ϑ. Although the spin coefficients associated with (3.2 a, b) are rather lenthy expressions, we easily succeed in solving the propagation equations since only ρ is needed:

$$\rho = \varepsilon_1/r\cdot[1 - a^2(r - 2M)/r^3]^{-1/2}\cdot\left[1 - a^2(r - M)/(2r^3) - a(r - 2M)/(2r^2)\cdot\frac{\partial a}{\partial r}\right] + \varepsilon_2/(2r^2)\cdot\left(a\cot\vartheta + \frac{\partial a}{\partial \vartheta}\right) \tag{3.3}$$

Using (3.3) we obtain

$$\int \rho dy = \ln r + 1/4\cdot\ln[1 - a^2(r - 2M)/r^3] + 1/2\cdot\ln\sin\vartheta + 1/2\cdot\ln\left|\frac{\partial^2 S}{\partial a^2}\right| + f(a), \tag{3.4}$$

where y is an affine parameter defined along the rays. As mentioned above, we are interested in an incident plane gravitational wave coming from the direction $\vartheta = \pi$. Hence the yet unspecified function $f(a)$ has to be appropriately fixed: $f(a) = -1/2\cdot\ln a$. Taking into account only waves with positive helicity we obtain with the help of (2.1), (2.3), (2.4) and (3.4) the perturbations

$$\begin{aligned} \hat{\Psi}_4 &= \omega[r^4 - a^2 r(r - 2M)]^{-1/4}\cdot\left(\sin\vartheta\left|\frac{\partial^2 S}{\partial a^2}\right|\right)^{-1/2}\cdot a^{1/2} \\ &\quad\cdot[C_1^+T_{11}^+\exp(-iS_b) + C_2^+T_{12}^+\exp(iS_b)]\cdot\exp(i\omega S) \\ \hat{\phi}_2 &= [r^4 - a^2 r(r - 2M)]^{-1/4}\cdot\left(\varkappa\sin\vartheta\left|\frac{\partial^2 S}{\partial a^2}\right|\right)^{-1/2}\cdot a^{1/2} \\ &\quad\cdot[C_1^+T_{21}^+\exp(-iS_b) + C_2^+T_{22}^+\exp(iS_b)]\cdot\exp(i\omega S). \end{aligned} \tag{3.5}$$

The coupling of the two kinds of waves is determined by the appearance of the beating phase $S_b = \sqrt{\varkappa}\int |\phi_0|\,dy$. The constants of integration $C^{+}_{1/2}$ should not depend on the impact parameter a, which corresponds to the boundary condition of an incident plane wave. Moreover, once a solution of (2.6) has been fixed, the constants $C^{+}_{1/2}$ must be chosen such that the incoming wave contains no electromagnetic part.

Consider a space-time domain where the ray vector l_α exhibits a positive radial component ($\varepsilon_1 = +1$), i. e., points off the gravitating mass (this appears to be the case in the region $0 \leq \vartheta < \pi$ behind the mass). In this domain the phase of the wave is given by

$$\begin{aligned} S = r + 2M\cdot\ln(r/(2M)-1) - t - \int_{r_T}^{\infty}\left(1-\sqrt{1-a^2(r-2M)/r^3}\right)\cdot r/(r-2M)dr \\ - \int_{r_T}^{r}\left(1-\sqrt{1-a^2(r-2M)/r^3}\right)\cdot r/(r-2M)dr - 2r_T \\ - 4M\ln(r_T/(2M)-1) \pm (a\vartheta - \pi/(4\omega)), \qquad (3.6) \end{aligned}$$

where r_T denotes the classical turning point in the outer part of the effective potential of a zero-rest-mass particle. Ignoring secondary rays, which revolve the deflecting mass in the vicinity of the circle $r = 3M$ before they return to infinity, we have to consider the two main rays passing the mass at opposite sides. The yet unspecified sign in front of the last term in (3.6) depends on which of these two rays is considered. Therefore, the perturbations (3.5) are in general sums of two contributions associated with these rays.

As mentioned in the Introduction, we are interested in the amplification of the generated electromagnetic radiation due to the gravitational lens effect. Treatments of the gravitational focusing of high-frequency radiation by the Schwarzschild field [*34-38*] suggest that the maximum intensity is to be expected in a narrow focal region around the axis of symmetry $\vartheta = 0$. In particular, the focal region is characterized by the condition $r\vartheta^2 \ll 2M$ in the case of an incident plane wave. Unfortunately, this condition just describes the range in which the expressions (3.5) fail to be applicable. This is not surprising since all rays which lie in various $\varphi = \text{const}$ surfaces and which have practically the same impact parameter interfere in the vicinity of the axis $\vartheta = 0$. In order to circumvent this difficulty, we now remember the exact waveoptical treatment of the field in the focal region involving an expansion in terms of spherical harmonics [*36-38*]. For very small ϑ, the contributions (3.5), (3.6) associated with the two rays differ only in a ϑ-dependent factor. The sum of these factors yields a cos function. As shown in previous papers, the actual perturbations are obtained in the focal region by making the simple substitution

$$(\sin\vartheta)^{-1/2}.\cos(\omega a\vartheta - \pi/4) \to (\pi/2.\omega a)^{1/2}.J_0(\omega a\vartheta), \qquad (3.7)$$

where J_0 denotes a Bessel function of the first kind.

In order to calculate the energy flux associated with both electromagnetic and gravitational perturbations, the corresponding energy-momentum tensors are needed. Adopting Isaacson's [39] reasonable definition of effective energy and momentum carried by a short gravitational wave we get the expressions

$$\begin{aligned} T^{elm}_{\alpha\beta} &= 2\varepsilon^2\Big[\big|\overset{(0)}{\phi}{}^+_2\big|^2 + \big|\overset{(0)}{\phi}{}^-_2\big|^2 + (\overset{(0)}{\phi}{}^-_2)^*\overset{(0)}{\phi}{}^+_2 e^{2i\omega S} + (\overset{(0)}{\phi}{}^+_2)^*\overset{(0)}{\phi}{}^-_2 e^{-2i\omega S}\Big] l_\alpha l_\beta \\ \varkappa T^{grav}_{\alpha\beta} &= 2\varepsilon^2\Big[\big|\overset{(0)}{\Psi}{}^+_4\big|^2 + \big|\overset{(0)}{\Psi}{}^-_4\big|^2 + 3(\overset{(0)}{\Psi}{}^-_4)^*\overset{(0)}{\Psi}{}^+_4 e^{2i\omega S} + 3(\overset{(0)}{\Psi}{}^+_4)^*\overset{(0)}{\Psi}{}^-_4 e^{-2i\omega S}\Big] l_\alpha l_\beta, \end{aligned} \tag{3.8}$$

which are invariant with respect to coordinate gauge transformations in this approximation. Using (3.5)-(3.8) one finally arrives at an expression for the intensity (magnitude of the time-averaged, three-dimensional Poynting vector) of the electromagnetic wave in the focal region normalized to the intensity of the incident gravitational wave:

$$\begin{aligned} \langle \bar{S}^{elm} \rangle / \langle \bar{S}^{grav} \rangle &= 2\pi a^2 \omega r^{-2} \sqrt{r/(r-2M)} \left| \frac{\partial^2 S}{\partial a^2} \right|^{-1} J_0^2(a\omega\vartheta) \\ &\cdot \left| C_1^+ T_{21}^+ e^{-iS_b} + C_2^+ T_{22}^+ e^{iS_b} \right|^2 \cdot \left| C_1^+ (T_{11}^+)^{in} + C_2^+ (T_{12}^+)^{in} \right|^{-2} \end{aligned} \tag{3.9}$$

$(T^+)^{in}$ indicates that the values of the matrix T^+ have to be taken for the incident plane wave.

4. SCATTERING BY A MAGNETIC DIPOLE FIELD

So far, nothing has been assumed about the electromagnetic background field $F_{\alpha\beta}$, which enters into the beating phase S_b. For the sake of simplicity, we restrict ourselves to the simplest non-spherically symmetric multipole, i. e., a magnetic dipole. In this case the exact solution of the Maxwell equations in the metric (3.1) is given by

$$\begin{aligned} F_{\vartheta\varphi} &= 2m/r \cdot \sin\vartheta(\sin\vartheta\cos\varphi\sin\chi - \cos\vartheta\cos\chi)F(1, 3, 4, 2M/r) \\ F_{r\vartheta} &= -\sin\varphi\sin\chi \frac{d}{dr} m/r \cdot F(1, 3, 4, 2M/r) \\ F_{\varphi r} &= \sin\vartheta(\cos\vartheta\cos\varphi\sin\chi + \sin\vartheta\cos\chi)\frac{d}{dr} m/r \cdot F(1, 3, 4, 2M/r). \end{aligned} \tag{4.1}$$

m is the magnitude of the (time-independent) dipole moment, and χ denotes the angle between the direction from which the incident gravitational wave comes in ($\vartheta = \pi$) and the magnetic axis. F is a hypergeometric function:

$$\left[z(1-z)\frac{d^2}{dz^2} + (4-5z)\frac{d}{dz} - 3\right]F(1, 3, 4, z) = 0 \tag{4.2}$$

Projection of (4.1) onto the null tetrad (3.2 *a*, *b*) yields the tetrad component

$$\begin{aligned}\phi_0 = &- \varepsilon_1\varepsilon_2/(\sqrt{2}r)\cdot m \sin\varphi \sin\chi \frac{d}{dr} r^{-1}\mathrm{F}(1, 3, 4, 2\mathrm{M}/r) \\ &+ i/(\sqrt{2}r)\cdot m\Big[\varepsilon_2\cdot 2a/r^3(\sin\vartheta\cos\varphi\sin\chi - \cos\vartheta\cos\chi)\mathrm{F}(1, 3, 4, 2\mathrm{M}/r) \\ &- \varepsilon_1\sqrt{1 - a^2(r - 2\mathrm{M})/r^3}\cdot(\cos\vartheta\cos\varphi\sin\chi + \sin\vartheta\cos\chi) \\ &\qquad \frac{d}{dr} r^{-1}\mathrm{F}(1,3,4,2\mathrm{M}/r)\Big] \end{aligned} \tag{4.3}$$

which couples the perturbations in the shortwave approximation. The argument η of ϕ_0 is in general a rather complicated function of the affine parameter y along a ray. Consequently, the polarization plane of linearly polarized radiation does not undergo parallel transport, but rotates along a ray while energy is partly transferred from gravitational to electromagnetic waves. Unfortunately, the system (2.6) does not seem to be solvable analytically.

In order to get an estimate of the conversion effect despite this difficulty, we now introduce two further simplifications. First, we assume alignment of incidence direction and magnetic axis ($\chi = 0$). In this special case the argument η of ϕ_0 takes the constant value of $\pi/2$, and (2.6) is solved by a constant matrix:

$$\mathrm{T}^+ = \begin{pmatrix} \exp(-i\pi/4) & \exp(-i\pi/4) \\ \exp(i\pi/4) & -\exp(i\pi/4) \end{pmatrix} \tag{4.4}$$

Since the incoming plane wave was assumed to contain no electromagnetic part, we have to put $C_1^+ = C_2^+$. Consequently, the polarization plane of linearly polarized waves does not rotate along the rays. Second, we assume $r \gg 2\mathrm{M}$ in (3.9), which implies that the impact parameters of the two rays are large compared with 2M as well. In this approximation the null geodesic equation is solved by

$$1/r = -\varepsilon_2 \sin\vartheta/a + \mathrm{M}(1 + \cos\vartheta)^2/a^2. \tag{4.5}$$

Using the expansion

$$\mathrm{F}(1, 3, 4, 2\mathrm{M}/r) = 1 + 3\mathrm{M}/(2r) + \mathrm{O}(\mathrm{M}^2/r^2) \tag{4.6}$$

of the hypergeometric function one obtains the beating phase

$$\mathrm{S}_b = \sqrt{\varkappa}m/(3\sqrt{2}\mathrm{M}r)\cdot[1 + \mathrm{O}(\sqrt{\mathrm{M}/r})] \tag{4.7}$$

on the axis of symmetry $\vartheta = 0$. Assuming the rate of conversion of gravitational wave energy into electromagnetic wave energy to be small (i. e., $\mathrm{S}_b \ll 1$) we get from (3.9)

$$\langle \bar{\mathrm{S}}^{\mathrm{elm}} \rangle / \langle \bar{\mathrm{S}}^{\mathrm{grav}} \rangle = 2\pi/9\cdot\varkappa\omega m^2/(\mathrm{M}r^2)\cdot[1 + \mathrm{O}(\sqrt{\mathrm{M}/r})] \tag{4.8}$$

for $\vartheta = 0$. Thus the intensity of the induced electromagnetic wave decreases on the axis of symmetry in radial direction as r^{-2}, since the strength of the magnetic dipole field falls off with increasing distance from the mass acting both as gravitational lens and magnetic dipole.

Unfortunately, even under favourable astrophysical conditions the expected effect is rather small. Consider a neutron star of one solar mass, with a radius of $2 \cdot 10^4$ m and a dipole-like magnetic field of 10^{12}G at the poles on the surface of the star. Due to the shadow effect of the deflector [*36*], the focal region starts at a distance of $6.9 \cdot 10^4$m from the centre of the star. If the intensity of the generated electromagnetic wave amounts there to one-tenth of the intensity of the incident gravitational wave, a gravitational wavelength of about $2 \cdot 10^{-6}$ m is required, which is extremely small unlike the electromagnetic case. Though situations with much smaller rates of conversion are also important owing to the stronger coupling of electromagnetic radiation to matter, other phenomena influence both the conversion effect [*10*] and the focusing of the radiation [*40*, *41*].

Nevertheless, it would be of some interest to treat the case of a magnetic axis which is inclined relative to the incidence direction and the case of a rotating magnetic dipole field [*42*, *43*] generated by a pulsar. In these cases the system (2.6) has to be solved numerically.

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