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Liquid light and accelerated frames

by

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SUMMARY. — Ideas of condensed matter physics are introduced into relativity.

In 1859 Gustav Kirchoff formulated a principle about heat radiation, which in paraphrased form reads as follows: « A volume of black body radiation of temperature T *has properties which are all together independent of the material* composition of the enclosing chamber. » The world of physics has never been the same once workers such as Planck and Einstein started some forty years later to understand the puzzle behind this observation.

In 1973 John A. Wheeler formulated a principle [*1*] which in paraphrased form reads as follows: « A black hole of mass M, charge Q, and angular momentum S *has properties which are all together independent of the material* from which it is formed. » This, roughly, is also known as the « no hair » principle.

Whether there turns out to be a historical parallel between these two remains to be seen. Our present task is to make a number of quantum mechanical observations about space-time with an event horizon.

Instead of focussing on curved space-time with a compact event horizon, we shall focus on flat space-time with the infinite event horizon of an accelerated coordinate frame. The geometry in the form of curvature has thus been lost, but the normal mode spectrum becomes much simpler [*2*], and many of the quantum mechanical essentials appear to remain the same.

The system we shall consider is a relativistic wave field, the Klein Gordon field, partitioned into two subsystems, say I and II, by the two Rindler coordinate charts [3] (also called I and II) of a pair of accelerated coordinate frames.

2. Our point of departure is the analysis, first done by Unruh [4] and other workers [5], [6], of an accelerated detector interacting with the quantized wave field. They consider « elementary » interactions between the detector and the wave field. By « elementary » we mean that only finitely many (in their case one or two) quanta are involved in the interaction process. Their central result is that the quantum mechanical description of an elementary process is equivalent for inertial and accelerated coordinate frames.

Let us assume that the normal mode spectrum of the quantized field is discrete. This discreteness is achieved by confining the field to a pair of semi-infinite cavities accelerating symmetrically into opposite directions. Let the transverse cross sectional areas be L^2. Let both cavities have finite bottoms ($b > 0$). Thus their respective world lines are

$$x = \pm b \cosh g\tau ; \qquad t = b \sinh g\tau .$$

Then the inner product of *a*) the familiar Minkowski vacuum $| O_M \rangle$ of the quantized field with *b*) a Rindler vacuum $| O_R \rangle$, the ground state relative to the pair of accelerated cavities I and II, is [7]

$$\langle O_R | O_M \rangle = \exp\left\{ - \left(\frac{L}{b}\right)^2 \frac{1}{720\pi} \right\}.$$

One can readily see what happens in the limit of a continuous mode spectrum, the « thermodynamic » limit. It is obtained by removing both bottoms (i. e. $b \to 0$) of the accelerated frames. In that limit

$$\langle O_R | O_M \rangle = 0 ,$$

i. e. the Rindler vacuum is orthogonal to the Minkowski vacuum. It follows that the Rindler vacuum is orthogonal to *every* quantum state considered by these workers in their analysis of accelerated detectors. In fact, in the thermodynamic limit, the whole Hilbert space generated from a Rindler vacuum is disjoint from that generated from the Minkowski vacuum.

3. A Rindler vacuum is a peculiar state. It has a large number of photons, but they occupy pairs of opposite traveling modes in such a systematic way, that each pair is not only 100 % correlated, but each mode is also in a highly squeezed vacuum state [8].

If the Klein Gordon (KG) field is charged, then the photons get replaced by Minkowski mesons and the paired correlated modes carry opposite charge [9].

4. The Rindler vacuum for a given pair of accelerated coordinate frame

is one which minimizes [9] the expected moment of mass-energy of each KG subsystem I and II. Expanded in terms of the Minkowski Bessel modes the moment for I is [10]

$$H_I = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\omega d^2k[H_{I\omega k} + H_{\omega k}^{\text{correl.}}]$$

where

$$H_{I\omega k} = \frac{\omega e^{\pi\omega}}{2 \sinh \pi\omega}\left(a_{\omega k}^{+}a_{\omega k} + \frac{1}{2}\right).$$

can be viewed as the moment of energy of an individual mode, and

$$H_{\omega k}^{\text{correl.}} = \frac{1}{2}\frac{\omega}{2 \sinh \pi\omega}(a_{\omega k}a_{-\omega -k} + a_{\omega k}^{+}a_{-\omega -k}^{+})$$

can be viewed as the correlation moment of energy between the given pair of modes. The total moment of mass energy of a pair is

$$H_{I\omega k}^{\text{tot.}} = H_{I\omega k} + H_{I-\omega-k} + 2H_{\omega k}^{\text{correl.}} \qquad (\omega > 0).$$

If the KG field is charged, the moment of energy gets augmented to include the amount for each type of charge. The correlation energy will be between mesons of opposite charge traveling into opposite directions [9].

What is the nature of the photon dynamics as determined by the total boost generator? For comparative purposes, consider the Hamiltonian of a dilute hard core Bose gas [11] below its lambda temperature. It is remarkable that not only is its Hamiltonian the same as the boost generator, but its ground state has the same form as a Rindler vacuum. In condensed matter physics the ground state manifests itself macroscopically as a superfluid. We shall now ask whether the Rindler vacuum yields a similar macroscopic manifestation, in which case the photon configuration would be condensed liquid light.

5. Taking our cue from condensed matter physics, we look for oscillations in an order parameter when the system is not exactly in the condensed vacuum state. An order parameter is a quantity that, among other things, becomes non-zero when the system is cooled below a critical temperature. For a superfluid the order parameter is the density of the condensate below the lambda temperature, while for a ferromagnet the parameter is a non-zero magnetization below the Curie temperature. Slightly above $T = 0°K$ one has oscillations in this parameter, « first » sound waves in a superfluid and spin waves in the ferromagnet. The quanta of these disturbances in the order parameter are called quasi-particles. For a superfluid they are known as phonons; for a ferromagnet they are known as magnons. For an acceleration-partitioned KG field the quasi-particles are the Fulling particles [12]. They occupy the Rindler modes whose spectrum, like Gold-

stone modes, extends all the way to zero boost frequency. A tentative candidate for an order parameter is the renormalized moment of energy. It is the sum total of the contributions from all pairs of correlated Minkowski Bessel modes. The contribution from a single pair occupied by $N_{\omega k}$ quasi-particles in Rindler sector I is

$$\langle N_{\omega k} | H^{\text{tot.}}_{I\omega k} | N_{\omega k} \rangle - \langle O_M | H^{\text{tot.}}_{I\omega k} | O_M \rangle = \omega [N_{\omega k} - (\exp 2\pi\omega - 1)^{-1}].$$

At $T = 0°K$ in accelerated frame I, $N_{\omega k} = 0$. Consequently, the order parameter (= moment of mass energy) has its maximum negative value. It is not difficult to see that the presence of quasi-particles ($N_{\omega k} \neq 0$) has a tendency to decrease the magnitude of the order parameter. In fact, suppose the KG system is heated to temperature T in the Rindler frame I. Then the quasi-particle number will have its Planckian thermal mean. The corresponding value of the order parameter [*14*] is obtained from

$$\omega [\langle N_{\omega k} \rangle - (\exp 2\pi\omega - 1)^{-1}] = \omega \left[\frac{1}{\exp (\hbar\omega/kT) - 1} - \frac{1}{\exp 2\pi\omega - 1} \right]$$

for each pair of Minkowski Bessel modes. One sees that the Davies-Unruh temperature

$$T = \frac{1}{k} \frac{\hbar}{2\pi} \frac{g}{c}$$

is the critical temperature at which the order parameter variables.

6. Candelas and Sciama [*13*] in a generally unappreciated calculation consider an *inertial* detector passing through Rindler sector I when the acceleration partioned field is in the condensed vacuum state. They assume single quantum processes. They find that modulo « transients » the detector makes no transitions.

This is a significant result because, even though plenty of photons are present, the detector passes through the condensed vacuum as if there were no interaction. Like a superfluid, the photons let the detector pass through unimpeded. Their result can be derived from (moment of) energy conservation as follows: consider a free particle (a « detector ») whose wave equation is the KG equation. Its quantum states are the Minkowski Bessel wave states. These are the stationary states for which the moment of mass energy of the particle is a constant of motion. Now let this particle interact with the quantized wave field in a Lorentz boost invariant way. It follows that the moment of energy of the total system, particle plus field in Rindler sector I, is conserved. If the field is in the condensed (« Rindler ») vacuum state of least moment of energy, then the particle can extract no more energy from the field by making an absorptive single quantum transition. Thus, at least to first order perturbation theory, the particle passes through the field without friction.

7. The quantum mechanical implementation of Einstein's edict [*15*] « *All Gaussian coordinate systems are essentially equivalent for the laws of nature* », demands that under parallel translation a zero-particle (= ground) state go over into a zero particle state, and in general an n-particle state go over into an n-particle state.

However, a calculation shows [*8*] that in the thermodynamic limit (« bottomless » accelerated frames) a Rindler vacuum state relative to one pair of accelerated frames cannot be related by a unitary transformation to another Rindler vacuum state relative to a parallel translated pair of accelerated frames. One concludes, therefore, that a KG field can be prepared in anyone of an infinite number of condensed vacuum states, all lying in different Hilbert spaces. This is expressed by saying that translation symmetry is broken by a Rindler vacuum state. Einstein's edict cannot be implemented quantum mechanically.

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[*14*] A definition similar to this one was introduced in the 1987 OSU preprint « Liquid Light » by U. H. Gerlach. That definition, as well as the one given here, must be considered tentative. This is because above the critical Davies-Unruh temperature the quantity defined here does not stay zero.

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