ANNALES DE L'I. H. P., SECTION B

W. SMOLENSKI

Linear Lusin-measurable functionals in case of a continuous cylinder measure

Annales de l'I. H. P., section B, tome 19, nº 4 (1983), p. 311-321

<http://www.numdam.org/item?id=AIHPB_1983__19_4_311_0>

Ann. Inst. Henri Poincaré,
Vol. XIX, n° 4-1983, p. 311-321.

Section B :
Calcul des Probabilités et Statistique.

Linear Lusin-measurable functionals in case of a continuous cylinder measure

by

W. SMOLENSKI
Institute of Mathematics, Warsaw Technical University,
00-661 Warsaw, Poland

RÉSUMÉ. — Soit μ une mesure cylindrique sur un e. v. t. E. On donne des résultats sur l'adhérence de E′ pour la topologie de convergence en μ.

SUMMARY. — Let μ be a cylinder measure on a linear topological space E. Some results concerning the closure of E′ in the topology of the convergence in μ are given.

1. INTRODUCTION

Let μ be a tight probability measure on a complete locally convex space E. A measurable linear functional is called Lusin-measurable if for every positive ε there exists a convex and compact set K such that $\mu(K) > 1 - \varepsilon$ and the functional restricted to K is continuous (Slowikowski [*9*]). Lusin-measurable functionals form the closure of E′ in $L_0(E, \mu)$ [*9*]. In general not every linear measurable functional is Lusin-measurable (Kanter [*6a*]), see also Urbanik [*15*] and theorem 5.6 below). Using a notion of a pre-support introduced by Slowikowski [*9*] we define Lusin-measurable functionals in case when μ is a continuous cylinder measure. We obtain results similar to the case when μ is tight. We use extensively a notion of a kernel introduced by Hoffmann-Jorgensen [5] and Borell [2].

The paper is nearly self-contained. In paragraph 2 we recall some definitions and facts concerning linear topological spaces and cylinder measures. In paragraphs 3 and 4 we prove some propositions about pre-supports and kernels of cylinder measures. Some of them are known; for a survey of results about kernels and pre-supports see [3] or [4]. Paragraph 5 contains main results of this paper.

2. PRELIMINARIES

By a locally convex space we will understand a linear space with a fixed locally convex topology. So, if we say for instance that a set is compact we mean compactness in this original fixed topology. The letter E will be reserved to denote a locally convex space. E′ and E^a will denote its topological and algebraical duals respectively. If Z is a subset of E then Z^0 denotes the polar set of Z i. e.

$$Z^0 = \{ f \in E' : \forall e \in Z \, | \langle e, f \rangle | \leqslant 1 \}$$

2.1. Definition. — Let U be a linear subspace of E, and let h be a linear functional defined on U. If h restricted to any compact and convex subset of U is continuous then we will say that it is almost uniformly continuous on U. A topology on E′ given by polars of compact and convex subsets of U will be called the topology of almost uniform convergence on U and will be denoted by τ_U.

2.2. Definition. — A linear subspace is called standard if it is a union of countably many compact convex sets. It is called quasi-standard if these sets are closed and convex only.

The following theorem is a version of Grothendieck's Completness Theorem (cf. [6], p. 248).

2.3. Theorem. — Let U be a dense standard subspace of a locally convex space E. Let U* be the space of linear functionals almost uniformly continuous on U. Then U* is the completion of E′ in τ_U.

2.4. Remark. — Since U is standard τ_U coincides with the Mackey topology $\tau(E', U)$.

A subset Z of E is called a cylinder set if it is of the form $Z = T^{-1}(B)$, where T is a continuous linear map from E into R^n and B is a Borel subset of R^n. A positive normed set function μ on the algebra of cylinder sets is a cylinder measure if for every T as above $\mu \circ T^{-1}$ is a σ-additive Borel measure on R^n.

With every cylinder measure we can associate a linear map T_μ from E' into $L_0(\Omega, \mathscr{M}, P)$ (so called « adjoint linear stochastic process », cf. [1]). We say that a cylinder measure μ is continuous if T_μ is i. e. if, on E', τ_E is stronger than the topology s_μ of the convergence in μ. A cylinder measure μ is full if for every non-zero element f of E' $\mu(f^{-1}(\{0\})) < 1$. If μ is full and U is a dense linear subspace of E then μ is also a full cylinder measure on U endowed with the induced topology.

3. PRE-SUPPORTS OF A CYLINDER MEASURE

Let μ be a full cylinder measure on a locally convex space E and let the dual space E' be endowed with the topology τ_E.

3.1. DEFINITION. — A linear subspace U of E is a pre-support of μ if $\forall \varepsilon > 0\ \exists K_\varepsilon \subset U$, convex and compact such that

$$(*) \quad \forall f \in E' \qquad f \in K_\varepsilon^0 \Rightarrow \mu(e \in E : |\langle e, f \rangle| \leqslant 1) \geqslant 1 - \varepsilon$$

A symmetric convex and compact set which fulfills (*) will be called a set of (up to) ε-concentration.

3.2. REMARK. — Let U be a dense subspace of E. The following conditions are equivalent:

i) U is a pre-support of μ;

ii) μ is a continuous cylinder measure on U;

iii) U contains a standard subspace R such that on E' the topology τ_R is stronger than the topology s_μ of convergence in μ.

3.3. PROPOSITION. — The intersection of countably many pre-supports is a pre-support.

Proof. — Let $K = K_1 \cap K_2$, where K_1 and K_2 are sets of ε_1- and ε_2- concentration respectively. We have:

$$K^0 = \overline{\bigcup_{\lambda \in (0,1)}}^{\tau(E',E)} \lambda K_1^0 + (1-\lambda) K_2^0$$

It results that $\mu(e \in E : |\langle e, f \rangle| \leqslant 1) \geqslant 1 - (\varepsilon_1 + \varepsilon_2)$.

Hence K is a set of $(\varepsilon_1 + \varepsilon_2)$-concentration.

Now let (U_n) be a sequence of pre-supports and fix $\varepsilon > 0$.

For every n let K_n be a set of ε_n-concentration contained in U_n, where $\Sigma\varepsilon_n = \varepsilon/2$. We shall show that $K = \cap K_n$ is a set of ε-concentration. Suppose it is not so. Then for some $f \in K^0$ and for some $\delta > 0$ $\mu(e \in E : |\langle e, f \rangle| \geqslant 1 + \delta) > \varepsilon$.

Put $C_n = \bigcap_{i=1}^{n} K_i$. For each n C_n is a set of $\varepsilon/2$ concentration and the sequence (C_n) decreases to K.

By a standard topological argument there exists a number n_0 such that

$$C_{n_0} \subset \{ e \in E : |\langle e, f \rangle| < 1 + \delta \}$$

But this is contradictory to the fact that C_{n_0} is a set of $\varepsilon/2$-concentration. The proof is completed.

3.4. Corollary. — If μ is continuous then pre-supports and $\sigma(E, E')$-pre-supports are the same.

Proof. — Obviously a pre-support in a stronger topology is a pre-support in a weaker one. Conversely, let U be a $\sigma(E, E')$-pre-support. We may assume that U is $\sigma(E, E')$-standard. Since μ is continuous there exist a a standard pre-support $\tilde{U}$. By Proposition 3.3 $\tilde{U} \cap U$ is a $\sigma(E, E')$-pre-support. But $\tilde{U} \cap U$ is a standard subspace. Thus U is a pre-support (in the original topology).

3.5. Proposition. — If μ is σ-additive then every standard pre-support equals to the intersection of all measure one quasi-standard linear subspaces which contain it.

Proof. — Let U be a standard pre-support spanned by a decreasing (with the increase of epsylon) family of symmetric convex and compact sets $\{ K_\varepsilon \}_{0 < \varepsilon < 1}$, where for each ε K_ε is a set of ε-concentration. Let $e_0 \in E \backslash U$. Take a decreasing sequence (ε_n) of positive numbers such that $\Sigma\varepsilon_n = 1$. Fix $n \geqslant 1$. For every positive integer k take a linear functional f_k^n such that $\langle e_0, f_k^n \rangle = k$ and $f_k^n \in K^0_{\delta(n,k)}$, where $\delta(n, k) = 2^{-k}\varepsilon_n$. Put

$$W_n = \bigcap_{k=1}^{\infty} \{ e \in E : |\langle e, f_k^n \rangle| \leqslant 1 \}.$$

Let $W = \bigcup_{m=1}^{\infty} \bigcap_{n=m}^{\infty} W_n$. It is easy to see that span (W) is a measure one quasi-standard subspace not containing e_0.

3.6. — **Proposition.** — If μ is continuous and σ-additive then every measure one quasi-standard linear subspace is a pre-support.

Proof. — The proof can be done just like the proof of Proposition 3.3. So we omit it.

4. THE KERNEL OF A CONTINUOUS CYLINDER MEASURE

From now on we make an assumption that μ is continuous.

4.1. **Definition.** — The intersection of all pre-supports is called the kernel of a continuous cylinder measure μ and will be denoted by J_μ. Let us denote by $\tilde{\mu}$ the probability measure on E'^a (= the algebraic dual of E′) which corresponds to μ in a natural way (cf. [*1*]). Since μ is continuous $\tilde{\mu}$ is continuous too.

4.2. **Proposition.** — *a*) $J_\mu = J_{\tilde{\mu}}$.
b) $J_{\tilde{\mu}}$equals to the intersection if all quasi-standard $\tilde{\mu}$-measurable linear subspaces of E'^a of measure $\tilde{\mu}$ one.

Proof. — If a linear subspace U of E is pre-support of μ it is also a pre-support of $\tilde{\mu}$. By the continuity of $\tilde{\mu}$ and proposition 3.3 we get that $J_\mu = J_{\tilde{\mu}}$. The second assertion follows directly from Propositions 3.5 and 3.6.

We give now two useful characterizations of the kernel J_μ.

4.3. **Proposition.** — Let $F_\varepsilon = \{ f \in E' : \mu(e \in E : |\langle e, f \rangle| \leqslant 1) \geqslant 1 - \varepsilon$. Let $B_\varepsilon = \bigcap_{f \in F_\varepsilon} \{ e \in E : |\langle e, f \rangle| \leqslant 1 \}$. Then $J_\mu = \operatorname{span}(\{ B_\varepsilon \}_{0 < \varepsilon < 1})$.

4.4. **Proposition.** — $J_\mu = (E', s_\mu)'$.

Proof. — We will prove that $\operatorname{span}(\{ B_\varepsilon \}) \subset (E', s_\mu)' \subset J_\mu \subset \operatorname{span}(\{ B_\varepsilon \})$. Fix $0 < \varepsilon < 1$ and $\bar{e} \in B_\varepsilon$. Let $(f_n) \in E'$ converges to zero in s_μ. We have

$$\forall \delta > 0 \quad \exists n_0 \forall n \geqslant n_0 \quad \mu(e \in E : |\langle e, f_n \rangle| \leqslant \delta) \quad 1 - \varepsilon$$

Thus

$$\forall n \geqslant n_0 \quad \delta^{-1} f_n \in B_\varepsilon^0 .$$

It follows that $(\langle \bar{e}, f_n \rangle)$ tends to zero. Since (f_n) was an arbitrary sequence converging to zero in s_μ this implies that $\bar{e} \in (E', s_\mu)'$. This proves the first inclusion.

Since μ is continuous $(E', s_\mu)'$ is contained in E. Take $\bar{e} \in E \setminus J_\mu$. By the definition of J_μ there exists a pre-support U such that $\bar{e} \notin U$. For $0 < \varepsilon < 1$ let K_ε be a set of ε-concentration contained in U. There exists a sequence (f_n) of continuous linear functionals such that $nf_n \in K^0_{1/n}$ and $\langle \bar{e}, f_n \rangle = 1$. It follows that (f_n) converges to zero in s_μ. Hence $\bar{e} \notin (E', s_\mu)'$ and the second inclusion is proved. Let $\bar{e} \in E \backslash \text{span}(\{B_\varepsilon\})$. We want to show that $\bar{e}$ is not an element of J_μ. By proposition 4.2 it is enough to show the existence of a quasi-standard linear subspace E_0 of E'^a such that $\tilde{\mu}(E_0) = 1$ and $\bar{e} \notin E_0$. Let $\tilde{B}_\varepsilon$ denote the analogue of B_ε defined for $\tilde{\mu}$. It is easy to see that $\tilde{B}_\varepsilon \cap E = B_\varepsilon$. Thus $\bar{e} \in E'^a \backslash \text{span}(\{\tilde{B}_\varepsilon\})$. By the definition of $\{\tilde{B}_\varepsilon\}$ for every $\varepsilon > 0$ there exists a sequence $(f_n) \in E'$ such that $\langle \bar{e}, f_n \rangle = n$ and $\tilde{\mu}(e \in E'^a : |\langle e, f_n \rangle| \leqslant 1) \geqslant 1 - \varepsilon/2^n$. Now it is enough to use an argument from the end of the proof of Proposition 3.5.

4.5. REMARK. — It is clear that sets B_ε which appeared in Proposition 4.3 are closed, absolutely convex and they increase when epsylon decreases. They are also compact because each B_ε is contained in every set of ε-concentration. Conversely, the intersection of all sets of ε-concentration is contained in $B_{\varepsilon/2}$.

4.6. COROLLARY. — The kernel is a standard subspace.

The kernel is defined as the intersection of all pre-supports. By Proposition 3.3 an intersection of countably many pre-supports is a pre-support. The following theorem gives a necessary and sufficient condition to ensure that the kernel is a pre-support.

4.7. THEOREM. — Let μ be a full and continuous cylinder measure on a locally convex space E. The following conditions are equivalent:

i) the kernel J_μ of μ is a pre-support of μ;
ii) E′ is locally convex in the topology s_μ of the convergence in μ.

Proof. — If J_μ is a pre-support then τ_{J_μ} is stronger than s_μ. On the other hand by Proposition 4.4 s_μ is stronger than $\sigma(E', J_\mu)$. It follows (bornology argument) that s_μ is stronger than $\tau(E', J_\mu)$. Since J_μ is standard this ends the proof that (*i*) implies (*ii*).

Conversely, if (E', s_μ) is locally convex then since it is metrisable and since $J_\mu = (E', s_\mu)'$ it follows that $(E', s_\mu) = (E', \tau(E', J_\mu))$. By Corollary 3.4 the last equality implies that J_μ is a pre-support.

4.8. REMARK. — It can be shown (cf. [*8*]) that (E', s_μ) is nuclear if and only if μ is σ-additive and $\mu(J_\mu) = 1$.

5. LINEAR LUSIN-MEASURABLE FUNCTIONALS

Let μ be a full and continuous cylinder measure on a locally convex space E. Let D be a standard pre-support of μ and let h be a linear functional almost uniformly continuous on D. We will denote by X the space of such pairs (h, D) factored by the following equivalence relation:

$$(h_1, \mathrm{D}_1) \sim (h_2, \mathrm{D}_2) \qquad \text{if there exists} \qquad (h_3, \mathrm{D}_3)$$

such that

$$\mathrm{D}_3 \subset \mathrm{D}_1 \cap \mathrm{D}_2 \qquad \text{and} \qquad h_1 |_{\mathrm{D}_3} = h_2 |_{\mathrm{D}_3} = h_3 .$$

By Proposition 3.3 X is a linear space.

5.1. DEFINITION. — Elements of X will be called linear Lusin-measurable functionals. They will be denoted by x or by (h, D).

5.2. THEOREM. — The above constructed space X is the complection of E′ in the topology s_μ of convergence in μ : $\mathrm{X} = \overline{(\mathrm{E}', s_\mu)}$. More precisely:

a) for every linear Lusin-measurable functional (h, D) there exists a Cauchy sequence in (E', s_μ) converging to h almost uniformly on D;

b) every Cauchy sequence in (E', s_μ) contains a subsequence which converges almost uniformly on some pre-support D;

c) the following conditions are equivalent:

i) $(h_1, \mathrm{D}_1) \sim (h_2, \mathrm{D}_2)$

ii) if for $i = 1, 2$ (f_n^i) is a sequence of elements of E′ converging to h_i almost uniformly on D_i then $(f_n^1 - f_n^2)$ converges to zero in s_μ.

Proof. — *a*) Follows immediatly from Theorem 2.3 and from the definition of pre-support.

b) Let (f_n) be a Cauchy sequence in (E', s_μ). Thanks to Egoroff's theorem there exists a subsequence (f_{n_k}) of (f_n) and an increasing sequence (F_m) of closed subsets of E'^a such that for every m (f_{n_k}) *converges uniformly* on F_m and $\tilde{\mu}(\mathrm{F}_m) > 1 - \dfrac{1}{m}$. Evidently, F_m can be replaced by its closed absolutely convex hull F'_m. Let U be a standard pre-support of μ. $\mathrm{U} = \bigcup_{m=1}^{\infty} \mathrm{K}_m$, where K_m is a set of $\dfrac{1}{m}$-concentration and $\mathrm{K}_m \subset \mathrm{K}_{m+1}$. Let us put

$$\mathrm{D} = \mathrm{U} \cap \operatorname{span}\left(\{ \mathrm{F}'_m \}_{m=1}^{\infty} \right).$$

By Propositions 3.3 and 3.6 D is a pre-support of μ. On the other hand for every m (f_{n_k}) converges uniformly on $C_m = m(K_m \cap F'_m)$ and $D = \bigcup_{m=1}^{r} C_m$. This proves ($b$).

c) (i) implies (ii) by the definition of pre-support. The reversed implication can be proved in the same way as (b).

5.3. REMARK. — Every linear Lusin-measurable functional is almost uniformly continuous on J_μ. However, it is not true in general that every linear functional defined and almost uniformly continuous on J_μ can be extended to a Lusin-measurable one. (For instance if $E = R^\infty$ and μ is an infinite product of p-stable laws, $0 < p < 1$, then $J_\mu = l^\infty$, but $X = l^p$ not l^1). On the other hand it can happen that $J_\mu = \{ 0 \}$ (cf. [*12*]).

The following theorem is a completion of Theorem 4.7.

5.4. THEOREM. — The following condition are equivalent:

i) every linear functional almost uniformly continuous on J_μ has a unique extension to a Lusin-measurable one;

ii) $(X, s_\mu) = \overline{(E', \tau(E', J_\mu))}$

iii) (X, s_μ) is locally convex

iv) J_μ is a pre-support of μ.

Proof. — Obviously we only have to prove that (i) implies (ii). Let J_μ^* denote the space of functionals almost uniformly continuous and linear on J_μ. By (i) J_μ is dense in E, so by Theorem 2.3 and Remark 2.4

$$(J_\mu^*, \tau_{J_\mu}) = \overline{(E', \tau(E', J_\mu))}$$

Let I denote the map from X into J_μ^* that associates with every Lusin-measurable functional its restriction to J_μ. By (i), Theorem 5.2.b and Corollary 4.6. I is a continuous linear bijection from (X, s_μ) onto (J_μ^*, τ_{J_μ}). Thus, by the Open Mapping Theorem of Banach, I^{-1} is also continuous. This finishes the proof.

Let us call a pre-support hilbertien if it is of the form span K, where K is a compact absolutely convex set and the Minkowski functional of K can be induced by a scalar product. There are, of course, continuous cylinder measures which have no hilbertien pre-support. However, every tight probability measure on a Frechet space has « sufficiently rich » family of hilbertien pre-supports.

5.5. THEOREM. — Let μ be a tight probability measure on a Frechet space E. Then there exists a family $(U_\sigma)_{\sigma\in\Sigma}$ of pre-supports of μ with the following properties:

i) U_σ is hilbertien for every $\sigma \in \Sigma$;

ii) for every Lusin-measurable functional x there exists $\sigma \in \Sigma$ such that x admits a representation (h, U_σ);

iii) $\bigcap_{\sigma\in\Sigma} U_\sigma = J_\mu$.

Proof. — By a result of Kuelbs (cf. [7]) there exists a Banach space E_0 continuously embeded in E such that μ is a tight measure on E_0. Thus without loss of generality we can assume that E is a separable Banach space. Let Σ be the set of bounded Borel functions σ on E, μ-almost everywhere positive and such that $\int_E \| e \|^2 \sigma(e) d\mu(e)$ is finite. For every $\sigma \in \Sigma$ let T_σ be the identity operator from E′ into $L_2(\sigma d\mu)$. T_σ is compact (cf. [*14*], Proposition 3*a*).

The adjoint operator T′ is given by the Bochner integral:

$$L_2(\sigma d\mu) \ni g \overset{T'_\sigma}{\to} \int_E g(e)\sigma(e) e d\mu(e) \in E .$$

We put $U_\sigma = T'_\sigma(L_2(\sigma d\mu))$. From Chebyshev Inequality and from the compactness of T'_σ it follows that U_σ is a pre-support. Obviously U_σ is hilbertien. Let x be a Lusin-measurable functional and let (f_n) be a sequence of elements of E′ converging μ-almost surely to x. Then for

$$\sigma(e) = \min \left(\| e \|^{-2}, \quad (1 + \sup_n \langle e, f_n \rangle^2)^{-1} \right)$$

(f_n) converges almost uniformly on U_σ. This proves (*ii*).

Finally let $e \in \bigcap_{\sigma\in\Sigma} U_\sigma$ and let $(f_n) \in E'$ converge to zero in s_μ. To finish the proof we have to show that $(\langle e, f_n \rangle)$ converges to zero. Suppose it is not so. Taking, if necessary, a subsequence we can assume that (f_n) converges to zero μ-almost surely but $\langle e, f_n \rangle > \varepsilon > 0$. This contradicts the fact that $e \in U_\sigma$, where σ is constructed as above. This finishes the proof.

At the end of this paragraph we give an example of an infinite dimensional probability measure with interesting properties. A construction of this example is based on the following theorem of S. Mazur:

THEOREM. — (S. Mazur) [1]. Let (p_n) be an increasing sequence of integers

[1] To appear.

with $p_0 = 0$ such that $p_{n+1}(p_n)^{-1} > 1 + \varepsilon$, $\varepsilon > 0$, $\varepsilon(1+\varepsilon)^{1+\varepsilon^{-1}} > 2^{-1}$, $n = 1, 2, \ldots$ Let (f_k), $k = 1, 2, \ldots$ be a sequence of functions on the interval $[0, 1)$ of the form $f_k(t) = \sum_{n=0}^{\infty} c_{k,n} t^{p_n}$. Suppose that (f_k) converges in Lebesgue measure to some function f. Then

1) for every n $(c_{k,n})$ converges to c_n;

2) $f(t) = \sum_{n=0}^{\infty} c_n t^{p_n}$;

3) (f_k) converges to f uniformly on every subinterval $[0, r]$, $r < 1$.

5.6. THEOREM. — Let E be an infinite dimensional Frechet space. Then there exists a tight probability measure μ on E with the following properties:

1) every μ-measurable functional is a μ-measurable linear functional;

2) there are μ-measurable linear functionals which are not Lusin-measurable;

3) $\mu(J_\mu) = 1$.

Proof. — Assume first that $E = R^\infty$. Let T be a map from the unit interval $[0, 1]$ into R^∞ given by $T(t) = (t^{p_n})$, where (p_n) is a above.

We put $\tilde{\mu} = \lambda \circ T^{-1}$, where λ is the Lebesgue measure. Since $\tilde{\mu}$ is supported by a linearly independent subset $\{ (t^{p_n})_{n=1}^{\infty} \}_{0<t<1}$ of R property 1 is clearly fulfilled. Properties 2 and 3 follow directly from Mazur's theorem.

For general E let K be an infinite dimensional symmetric convex compact subset of E (such K exists in every complete infinite dimensional linear metric space by Mazur's argument (cf. [1*a*], p. 268)). There exists an affine homeomorphism G from the subset $[-1, 1]^\infty$ of R^∞ on to a subset of K such that $G(0) = 0$ (cf. [*16*], p. 321). It is easy to see that $\mu = \tilde{\mu} \circ G^{-1}$ has properties 1, 2, 3.

ACKNOWLEDGMENTS

An idea taken from [*11*] is used in the proof of theorem 5.5.

REFERENCES

[*1*] A. BADRIKIAN, Séminaire sur les fonctions aléatoires linéaires et les mesures cylindriques, *L. N. M.*, 139, Springer, 1970.

[*1a*] C. BESSAGA, A. PELCZYNSKI, *Selected topics in infinite-dimensional topology*, PWN, Warszawa, 1975.

[2] C. BORELL, Random linear functionals and subspaces of probability one, *Arkiv for matematik*, t. **14**, 1976, p. 79-92.
[3] S. CHEVET, Kernel associated with a cylindrical measure, Probability in Banach spaces III, Proceedings, Medford 1980, *L. N. M.*, 860, Springer, 1981.
[4] A. CLAVILIER, *Noyau, présupports et supports d'une mesure cylindrique*, Thèse, Université de Clermont-Ferrand, 1981.
[5] J. HOFFMANN-JORGENSEN, Integrability of semi-norms, the 0-1 law and the affine kernel for product measures, *Studia Math.*, t. **61**, 1977, p. 137-159.
[6] J. HORVATH, *Topological vector spaces and distributions*, t. **I**, Addison-Wesley, 1966.
[6 a] M. KANTER, Linear spaces and stable processes, *J. Functional Anal.*, t. **9**, 1972, p. 441-459.
[7] J. KUELBS, Some results for probability measures on linear topological space with an application to Strassen's loglog law, *J. Functional Anal.*, t. **14**, 1973, p. 28-43.
[8] S. KWAPIEN, W. SMOLENSKI, On the nuclearity of dual space with convergence in probability topology, *Z. Wahrsch. verw. Gebiete*, t. **59**, 1982, p. 197-201.
[9] W. SLOWIKOWSKI, Pre-supports of linear probability measures and linear Lusin-measurable functionals, *Dissertationes Math. (Rozprawy Matematyczne)*, t. **93**, 1972, p. 1-43.
[10] W. SLOWIKOWSKI, Concerning pre-supports of linear probability measures, Measure Theory, Proceedings, Oberwolfach 1979, *L. N. M.*, 794, Springer, 1980.
[11] A. V. SKOROHOD, Linear and almost linear functionals on a measurable Hilbert space, *Theor. Prob. Appl.*, t. **23**, 1978, p. 380-385.
[12] W. SMOLENSKI, Pre-supports and kernels of probability measures in Frechet spaces, *Demonstration Math.*, t. **10**, 1977, p. 751-762.
[13] W. SMOLENSKI, Pewne aspekty teorii miar cylindrycznych na przestrzeniach liniowych, Praca doktorska, Warszawa, 1978.
[14] V. TARIELADZE, N. VAKHANIA, Covariances operators of probability measures in locally convex spaces, *Theor. Prob. Appl.*, t. **23**, 1978.
[15] K. URBANIK, Random linear functionals and random integrals, *Colloq. Math.*, t. **33**, 1975, p. 255-263.
[16] H. V. WEIZSACKER, A note on infinite dimensional convex sets, *Math. Scand.*, t. **38**, 1976, p. 321-324.

(Manuscrit reçu le 25 février 1982)