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An asymptotic bound for the tail of the distribution of the maximum of a Gaussian process

by

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ABSTRACT. — Let $X(t)$, $t \in T$, be a real Gaussian process with mean 0, continuous covariance function, and continuous sample paths, where T is a closed cube in R^n, $n \geqslant 1$. The main result is a new bound for the probability $P(\max_T X(t) > u)$, for $u > 0$. It is obtained by an extension of the original Fernique inequality. This bound is asymptotically smaller, for $u \to \infty$, than the bound that can be obtained directly from the original inequality.

Key words and phrases, Gaussian process, maximum.

RÉSUMÉ. — Soit $X(t)$, $t \in T$, un processus Gaussien réel centré, de trajectoires continues, où T est un cube fermé dans R^n, $n \geqq 1$. Le résultat principal consiste en une nouvelle borne de la probabilité $P(\max_T X(t) > u)$, pour $u > 0$. Elle est obtenue par une extension d'une inégalité de Fernique et est asymptotiquement plus petite lorsque $u \to \infty$, que la borne qui peut être obtenue directement de l'inégalité de Fernique.

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1. INTRODUCTION AND SUMMARY

One of the very useful tools of the theory of Gaussian processes is the inequality of Fernique [3] for the tail of the distribution of the supremum of the modulus of the sample function on a set T of parameter values: $P(\sup_{t\in T} |X(t)| > u)$. The bound for this probability, which is reviewed in Section 2, has been used in several subareas of Gaussian process theory, including extreme value theory (see [5], and the references to earlier work, particularly [6].) The general nature of the bound is the relation of the distribution of $\sup_T |X(t)|$ to the distribution of $X(t_0)$ for a particular $t_0 \in T$. Let ξ be a standard normal random variable, and put $\Psi(x) = P(\xi > x)$. If we define σ_T as $\sup_T \{ \operatorname{Var} X(t) \}^{1/2}$, then the Fernique inequality states that there is a functional $q > 0$, depending on $E(X(t) - X(s))^2$ and the set T, such that

$$P(\sup_T |X(t)| > u) \leqslant \mathit{constant}\, \Psi\left(\frac{u}{\sigma_T + q}\right), \tag{1.1}$$

for all $u \geqslant u_0$, for some $u_0 > 0$.

In extreme value theory, there is much interest in obtaining the asymptotic form of $P(\sup_T X(t) > u)$ for $u \to \infty$ and T fixed. One of the main steps in determining such a bound is the establishment of a bound of the form

$$P(\sup_T X(t) > u) \leqslant \mathit{constant}\; v(u)\Psi\left(\frac{u}{\sigma_T}\right) \tag{1.2}$$

for a suitable function $v(u)$. The most extensively studied case is that of a stationary Gaussian process with mean 0 and variance 1, and where the covariance function $r(t)$, satisfies $1 - r(t) \sim C|t|^\alpha$, $t \to 0$ for some $C > 0$, $0 < \alpha \leqslant 2$. In this example, the function v in (1.2) may be taken as a certain power of u, namely, $v(u) = u^{2/\alpha}$. It then follows from the form of Ψ that the bound in (1.2) is of much smaller order than that in (1.1).

The purpose of this paper is to show that the original inequality (1.1) can, in general, be extended to a version of (1.2) which leads directly to a sharper estimate in the case $u \to \infty$. This was done in particular cases in [1] and [2], namely for stationary, or « locally » stationary Gaussian processes. Here we give an extended version of Fernique's inequality which holds for nonstationary processes as well as random fields with several-dimensional time parameters. The extension is carried out by comparing the maximum of the sample function to its value at the point where the variance is largest, and conditioning by the value of the latter random variable. The original Fernique inequality is then applied to the conditioned process.

The extended inequality is stated in Theorem 3.1, and the asymptotic form in Theorem 3.2. In Section 4 it is shown that the asymptotic inequality in the latter theorem is, in cases of most interest, sharper than that obtainable directly from the original inequality.

2. FERNIQUE'S INEQUALITY AND ITS MODIFICATIONS

Let $X(t)$, $t \in [0, 1]^n$ be a real separable Gaussian process with mean 0 and continuous covariance function $\Gamma(s, t)$, s, $t \in [0, 1]^n$. We define the metric $\delta(s, t) = \max \{ | s_i - t_i | : i = 1, \ldots, n \}$ where (s_i) and (t_i) are the real components of s and t, respectively. Define the function $\varphi(h)$,

$$\varphi(h) = \max_{\substack{s,t \in [0,1]^n \\ \delta(s,t) \leqslant h}} [E(X(s) - X(t))^2]^{1/2} . \tag{2.1}$$

Fernique showed that if

$$\int_1^\infty \varphi(e^{-y^2}) dy < \infty$$

then the sample functions are almost surely continuous; and that, for every closed set T of δ-diameter at most h,

$$P\left(\max_T | X(s) | \geqslant x \left[\max \Gamma^{1/2} + (2 + \sqrt{2}) \int_1^\infty \varphi(hp^{-y^2}) dy \right] \right) \leqslant \lambda \Psi(x) , \tag{2.2}$$

for all integers $p \geqslant 2$ and all $x \geqslant (1 + 4n \log p)^{1/2}$, where

$$\lambda = \frac{5}{2} p^{2n} \sqrt{2\pi} \quad \text{and} \quad \Psi(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{1}{2} y^2} dy .$$

This is a slightly more general statement of [*4*], Theorem 4.1.1, and corresponds to the lemma stated in Section 4.1.3.

We note this simple consequence of the Cauchy-Schwartz inequality:

$$\max_{s,t \in T} \Gamma(s, t) = \max_{s \in T} \Gamma(s, s) . \tag{2.3}$$

We note also that (2.2) remains valid of we put

$$u = x \left[\max \Gamma^{1/2} + (2 + \sqrt{2}) \int_1^\infty \varphi(hp^{-y^2}) dy \right] , \tag{2.4}$$

and take u to satisfy

$$u \geqslant (1 + 4n \log p)^{1/2}\left[\max \Gamma^{1/2} + (2 + \sqrt{2}) \int_1^\infty \varphi(hp^{-y^2})dy\right]. \quad (2.5)$$

Put

$$Q(h) = \varphi(h) + (2 + \sqrt{2}) \int_1^\infty \varphi(hp^{-y^2})dy. \quad (2.6)$$

Now we establish two modified versions of (2.2), one for the process $X(t) - X(c)$, where $c \in T$, and another for $X(t) - E(X(t) \mid X(c))$.

LEMMA 2.1. — *For* $T \subset [0, 1]^n$ *and of diameter at most h, and any* $c \in T$,

$$P(\max_T \mid X(t) - X(c) \mid \geqslant u) \leqslant \lambda\Psi\left(\frac{u}{Q(h)}\right) \quad (2.7)$$

for all $u \geqslant (1 + 4n \log p)^{1/2} Q(h)$.

Proof. — The φ-function for the process $X(t) - X(c)$ is identical with the one for the original process $X(t)$, defined by (2.1). Furthermore, by (2.3), the covariance of the process $X(t) - X(c)$ is dominated by the maximum of the function $E(X(t) - X(c))^2$, $t \in T$, which is at most equal to $\varphi^2(h)$. The assertion of the lemma now follows from (2.2), (2.4) and (2.6).

LEMMA 2.2. — *For any* $T \subset [0, 1]^n$ *of diameter at most h, and any* $c \in T$,

$$P(\max_T \mid X(t) - E(X(t) \mid X(c)) \mid \geqslant u) \leqslant \lambda\Psi\left(\frac{u}{Q(h)}\right), \quad (2.8)$$

for all $u \geqslant (1 + 4n \log p)^{1/2} Q(h)$.

Proof. — The proof is similar to that of Lemma 2.1. The only modification is to note that the variances of $X(t) - E(X(t) \mid X(c))$ and $[X(t) - E(X(t) \mid X(c))] - [X(s) - E(X(s) \mid X(c))]$ are, as conditional variances, less than or equal to the corresponding unconditional variances.

Finally we need the following elementary result:

LEMMA 2.3. — *Let* $c \in T$ *be a point such that*

$$EX^2(c) = \max_T EX^2(t); \quad (2.9)$$

then, for $t \in T$ *and* $y \geqslant 0$,

$$E(X(t) \mid X(c) = y) \leqslant y. \quad (2.10)$$

Proof. — By definition, we have

$$E(X(t) \mid X(c) \mid = y) = \frac{EX(t)X(c)}{EX^2(c)} y.$$

If $EX(t)X(c) \leqslant 0$, the result is trivial. If $EX(t)X(c) > 0$, then the result follows upon application of the Cauchy-Schwarz inequality.

3. THE MAIN RESULTS

We use $\Phi(z)$ to represent the standard normal distribution function, and put $\phi = \Phi'(z)$ and $\Psi(z) = 1 - \Phi(z)$. We note the inequality

$$\phi(x + y) \leqslant \phi(x)e^{-xy} \tag{3.1}$$

and the well known relations,

$$\Psi(x) \sim x^{-1}\phi(x), \qquad \text{for} \qquad x \to \infty \tag{3.2}$$

and

$$\lim_{x\to\infty} \frac{\Psi(x + y/x)}{\Psi(x)} = e^{-y}, \qquad -\infty < y < \infty . \tag{3.3}$$

The number $(1 + 4n \log p)^{1/2}$ which appears in Section 2 and in the rest of this work will, for simplicity, be denoted as γ:

$$\gamma = (1 + 4n \log p)^{1/2} . \tag{3.4}$$

Our first result is an extension of the inequality (2.2); here $\max X$ is used in the place of $\max |X|$.

THEOREM 3.1. — *For any closed set* T, *define*

$$\sigma_T^2 = \max_T EX^2(t) . \tag{3.5}$$

If $T \subset [0, 1]^n$ *and* $\operatorname{diam}(T) \leqslant h$, *then*

$$P(\max_T X(t) > u) \leqslant \Psi\left(\frac{u - \gamma Q(h)}{\sigma_T}\right) + \lambda \Psi\left(\frac{u}{Q(h)}\right) + \lambda \exp\left\{\frac{u^2 Q^2(h)}{2\sigma_T^4}\right\} \frac{\sigma_T}{u} \phi\left(\frac{u}{\sigma_T}\right), \tag{3.6}$$

for all $u \geqslant \gamma Q(h)$.

Proof. — Let c be a point in T satisfying (2.9). The event $\max_T X(t) > u$ is included in the union of the three events,

$$X(c) \geqslant u - \gamma Q(h) \tag{3.7}$$

$$\max_T (X(t) - X(c)) > u \tag{3.8}$$

and

$$0 \leqslant X(c) \leqslant u - \gamma Q(h), \qquad \max_T X(t) > u . \tag{3.9}$$

This can be verified by a direct enumeration of possibilities.

The probability of (3.7) is, by definition, equal to the first term on the right hand side of (3.6). By Lemma 2.1, the probability of (3.8) is at most equal to the second term on the right hand side of (3.6).

In order to complete the proof, we show that the probability of (3.9) is at most equal to the last member of (3.6). By Lemma 2.3, $E(X(t) \mid X(c)) \leqslant X(c)$ on the set where $X(c) \geqslant 0$; therefore, the event (3.9) implies

$$\max_T [X(t) - E(X(t) \mid X(c))] > u - X(c), \qquad 0 \leqslant X(c) \leqslant u - \gamma Q(h).$$

Since $X(t) - E(X(t) \mid X(c))$ and $X(c)$ are independent, the probability of the event above is, by the total probability formula, representable as

$$\int_0^{u - \gamma Q(h)} P(\max_T [X(t) - E(X(t) \mid X(c))] > u - y) \frac{1}{\sigma_T} \phi\left(\frac{y}{\sigma_T}\right) dy.$$

By Lemma 2.2, the integral above is at most equal to

$$\lambda \int_0^u \Psi\left(\frac{u-y}{Q(h)}\right) \frac{1}{\sigma_T} \phi\left(\frac{y}{\sigma_T}\right) dy.$$

By the change of variable $z = u(u - y)$, the latter is at most equal to

$$\lambda \int_0^\infty \Psi\left(\frac{z}{uQ(h)}\right) \frac{1}{u\sigma_T} \phi\left(\frac{u - z/u}{\sigma_T}\right) dz.$$

By (3.1), this is at most equal to

$$\frac{\lambda}{u\sigma_T} \phi\left(\frac{u}{\sigma_T}\right) \int_0^\infty \Psi\left(\frac{z}{uQ(h)}\right) e^{z/\sigma_T^2} dz,$$

which, by a change of variable from z to z/σ_T^2, is equal to

$$\frac{\lambda\sigma_T}{u} \phi\left(\frac{u}{\sigma_T}\right) \int_0^\infty \Psi\left(\frac{z\sigma_T^2}{uQ(h)}\right) e^z dz. \tag{3.10}$$

By integration by parts, the integral in (3.10) is equal to

$$\frac{\sigma_T^2}{uQ(h)} \int_0^\infty \phi\left(\frac{z\sigma_T^2}{uQ(h)}\right) e^z dz - \frac{1}{2}$$

which, by the moment generating function formula, is at most equal to

$$\exp\left\{\frac{1}{2} \frac{u^2 Q^2(h)}{\sigma_T^4}\right\}.$$

Therefore (3.10) is at most equal to the last member of (3.6); this completes the proof.

In the following corollary we show that if T is a set of diameter at most h, and $Q(h) \leqslant \frac{1}{u}$, then, for $u \to \infty$ the probability $P(\max_T X(t) > u)$ is of the same order as $P(X(c) > u)$, where c is defined by (2.9).

COROLLARY 3.1. — *Let* S *be a subset of* $[0, 1]^n$ *such that*

$$\sigma^2 = \inf_S EX^2(t) > 0 ; \tag{3.11}$$

then

$$\limsup_{\substack{u \to \infty h \to 0 \\ uQ(h) \leqslant 1}} \sup_{\substack{T \subset S \\ \operatorname{diam}(T) \leqslant h}} \frac{P(\max_T X(t) > u)}{\Psi(u/\sigma_T)} \leqslant e^{\gamma/\sigma^2} + \lambda e^{1/2\sigma^4} \tag{3.12}$$

Proof. — (3.3) implies that

$$\limsup_{\substack{u \to \infty, h \to 0 \\ uQ(h) \leqslant 1}} \frac{\Psi\left(\dfrac{u - \gamma Q(h)}{\sigma_T}\right)}{\Psi(u/\sigma_T)} \leqslant e^{\gamma/\sigma^2} .$$

Thus we obtain the first term on the right hand side of (3.12) when we consider the first term in the bound (3.6).

Since $Q(h) \to 0$, the second term on the right hand side of (3.6) is $o(\Psi(u/\sigma_T))$ for $u \to \infty$.

The last term in (3.12) is obtained by dividing the corresponding term in (3.6) by $\Psi(u/\sigma_T)$, applying (3.2), and then taking the lim sup.

THEOREM 3.2. — *Define*

$$Q^{-1}(x) = \sup(y : Q(y) \leqslant x) . \tag{3.13}$$

Let S *be a closed cube in* $[0, 1]^n$ *of edge* δ *such that* (3.11) *holds. Then, for* $p \geqslant 2$,

$$\limsup_{u \to \infty} \left(\frac{Q^{-1}(1/u)}{\delta}\right)^n \frac{P(\max_S X(t) > u)}{\Psi(u/\sigma_S)} \leqslant e^{\gamma/\sigma^2} + \lambda e^{1/2\sigma^4} . \tag{3.14}$$

Proof. — For every $h > 0$, S is representable as the union of at most

$$\left(\left[\frac{\delta}{h}\right] + 1\right)^n$$

closed cubes T, each of edge of length h. For these cubes we have

$$\operatorname{diam}(T) = h, \qquad \max_{T \subset S} \sigma_T = \sigma_S ;$$

therefore, by Boole's inequality,

$$P(\max_S X(t) > u) \leqslant \left(\frac{\delta}{h} + 1\right)^n \max_{T \subset S} P(\max_T X(t) > u). \qquad (3.15)$$

Put $h = Q^{-1}(1/u)$, and apply Corollary 3.1 to the right hand member of (3.15): The latter is asymptotically at most equal to $\Psi(u/\sigma_T) \leqslant \Psi(u/\sigma_S)$ times the right hand member of (3.14). This completes the proof.

We have the following version of Theorem 3.2 without the assumption (3.11).

THEOREM 3.3. — *In Theorem* 3.2, *the assumption* (3.11) *may be dropped, and then* σ *is replaced by* σ_S *on the right hand side of* (3.14).

Proof. — For every $\varepsilon > 0$, let S_ε be the union of the cubes T in the proof of Theorem 3.2 such that $\sigma_T \leqslant \sigma_S - \varepsilon$. (The dependence of S_ε on u is understood but not explicitly noted.) According to the original Fernique inequality (2.2), we have

$$\max_{T \subset S_\varepsilon} P(\max_T X(t) > u) \leqslant \lambda \Psi\left(\frac{u}{\sigma_S - \varepsilon + Q(h)}\right);$$

thus, by the reasoning leading to (3.15),

$$P(\max_{S_\varepsilon} X(t) > u) \leqslant \left(\frac{\delta}{h} + 1\right)^n \lambda \Psi\left(\frac{u}{\sigma_S - \varepsilon + Q(h)}\right). \qquad (3.16)$$

Put $h = Q^{-1}(1/u)$ and divide each side of (3.16) by $(Q^{-1}(1/u))^{-n}\Psi(u/\sigma_S)$: It follows from (3.2) that the quotient on the right hand side of (3.16) converges to 0.

As a consequence of the result above, it follows that, in the estimation of the left hand member of (3.14), we may omit the subset S_ε and consider the maximum only over $S - S_\varepsilon$. Then the constant σ in the right hand member of (3.14), originally defined by (3.11), may be taken as $\sigma = \min_{T \subset S - S_\varepsilon} \sigma_T$, so that

$$\liminf_{u \to \infty} \sigma \geqslant \sigma_S - \varepsilon.$$

Since $\varepsilon > 0$ is arbitrary, the proof is complete.

4. COMPARISON WITH THE ORIGINAL BOUND OF FERNIQUE

Theorem 3.2 usually provides a sharper asymptotic bound on the tail of the distribution of the maximum than that furnished by Fernique's original result.

First we note the obvious fact that $P(\max X(t) > u) \leqslant P(\max |X(t)| > u)$. Then we note that if $X(t)$ is a Gaussian process with mean 0, then it is equivalent in distribution to the process $-X(t)$; therefore,

$$P(\max |X(t)| > u) \leqslant P(\max X(t) > u) + P(\min X(t) < -u) = 2P(\max X(t) > u).$$

Therefore a bound for $P(\max X(t) > u)$ furnishes a bound for

$$P(\max |X(t)| > u)$$

of the same order, and conversely.

The main idea of Theorem 3.2 is to cover the set T with cubes of diameter at most $h = Q^{-1}(1/u)$, and then estimate the tail of the distribution of the maximum over each cube. As a result we find that $P(\max_T X(t) > u)$ is of the order

$$\Psi(u/\sigma_T)/(Q^{-1}(1/u))^n \tag{4.1}$$

for $u \to \infty$.

An estimate of $P(\max_T |X(t)| > u)$ for each cube T is obtained directly from Fernique's inequality by a similar covering operation. Put

$$q(h) = Q(h) - \varphi(h) = (2 + \sqrt{2}) \int_1^\infty \varphi(hp^{-y^2})dy;$$

then (2.2) is equivalent to

$$P(\max_T |X(s)| \geqslant u) \leqslant \lambda \Psi\left(\frac{u}{\sigma_T + q(h)}\right). \tag{4.2}$$

for $u \geqslant \gamma(\sigma_T + q(h))$. Write, for $u \to \infty$, $h \to 0$,

$$\frac{u}{\sigma_T + q(h)} = \frac{u/\sigma_T}{1 + q(h)/\sigma_T} = \frac{u}{\sigma_T}\left\{1 - \frac{q(h)}{\sigma_T}(1 + o(1))\right\}. \tag{4.3}$$

If we cover a fixed cube T with cubes of diameter h, and let $h \to 0$ and $u \to \infty$, as we did to obtain (4.1), then we find, by applying (4.2) and (4.3) to each of the latter cubes, that $P(\max_T |X(s)| > u)$ is of the order

$$h^{-n}\Psi\left(\frac{u}{\sigma_T} - \frac{uq(h)}{\sigma_T^2}(1 + o(1))\right). \tag{4.4}$$

The bound (4.1) is compared to the bound (4.4) in the following theorem.

THEOREM 4.1. — *If*

$$\liminf_{h\to 0} \varphi(ch)/\varphi(h) > 0, \qquad \text{for} \quad c \geqslant 0, \tag{4.5}$$

and

$$\lim_{h,h'\to 0} (h/h')e^{-cQ(h)/Q(h')} = 0, \qquad \text{for} \quad c > 0, \tag{4.6}$$

then the bound (4.1) *is of smaller order than* (4.4) *for* $u \to \infty$ *and* $h \to 0$.

Proof. — It follows from (3.2) and the fact that $q(h) \to 0$ that (4.4) is asymptotically equal to

$$h^{-n} \frac{\sigma_T}{u} \phi\left(\frac{u}{\sigma_T} - \frac{uq(h)}{\sigma_T^2}(1 + o(1))\right).$$

By the form of ϕ, and another application of (3.2), the expression above is asymptotically equal to

$$h^{-n}\Psi(u/\sigma_T) \exp\left[\frac{u^2 q(h)}{\sigma_T^3}(1 + o(1))\right].$$

Hence, the ratio of (4.1) to (4.4) is asymptotically equal to

$$\left[\frac{h}{Q^{-1}(1/u)}\right]^n \exp\left[-\frac{u^2 q(h)}{\sigma_T^3}(1 + o(1))\right]. \tag{4.7}$$

Under the assumption (4.5), we have, by Fatou's lemma,

$$\liminf_{h\to 0} q(h)/\varphi(h) \geqslant (2 + \sqrt{2}) \int_1^\infty \liminf_{h\to 0} \frac{\varphi(hp^{-y^2})}{\varphi(h)}\, dy > 0\,;$$

hence,

$$\liminf_{h\to 0} q(h)/Q(h) = \liminf_{h\to 0} \frac{q(h)}{q(h) + \varphi(h)} > 0\,;$$

hence, (4.7) is at most equal to

$$\left(\frac{h}{Q^{-1}(1/u)}\right)^n \exp\,[-\,u^2 Q(h)k]$$

for some $k > 0$. Now define $h' = Q^{-1}(1/u)$; then the expression above is equal to

$$(h/h')^n \exp\,[-\,kQ(h)/Q^2(h')]\,,$$

which, by the assumption (4.6), has the limit 0.

EXAMPLE 4.1. — Suppose that $\varphi(t) \sim C\,|\,t\,|^{\alpha/2}$, $t \to 0$, for some $C > 0$ and $0 < \alpha \leqslant 2$; then, it follows that $Q(t) \sim C_1\,|\,t\,|^{\alpha/2}$, for some $C_1 > 0$, and thus (4.5) and (4.6) hold. In this case $Q^{-1}(1/u)$ is of the order $u^{-2/\alpha}$, so that the asymptotic form of the bound in Theorem 3.2 is of the same order as the exact asymptotic value [*5*], p. 232.

EXAMPLE 4.2. — Suppose that $\varphi(t) \sim C\,|\log t|^{-(1/2)(1+\varepsilon)}$, $t \to 0$, for some $C > 0$ and $\varepsilon > 0$; then, $Q(t) \sim C_1\,|\log t|^{-\varepsilon/2}$, for $C_1 > 0$, and conditions (4.5) and (4.6) hold.

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