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Multiple orbits for Hamiltonian systems on starshaped surfaces with symmetries

by

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ABSTRACT. — This paper deals with the existence of periodic solutions of Hamiltonian system with N degrees of freedom, on a given energy surface. The surface is supposed to be symmetric and starshaped with respect to the origin. We show that any such surface carries at least one symmetric periodic solution and we give a sufficient condition on the surface for the existence of N such solutions.

RÉSUMÉ. — Dans ce travail nous considérons l'existence de solutions périodiques de systèmes hamiltoniens à N degrés de liberté sur une surface d'énergie donnée que nous supposons symétrique et étoilée par rapport à l'origine. Nous montrons que toute surface de ce type admet au moins une solution périodique et symétrique, et nous donnons une condition suffisante pour l'existence de N telles solutions.

§ 1. INTRODUCTION

The existence of periodic solutions of Hamiltonian systems with N degrees of freedom on a given energy surface has been investigated by several authors (see [*10*], [*14*] for local results and [*8*], [*1*] for global ones).

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In particular I. Ekeland and J. M. Lasry have proved a remarkable theorem (see also [*1*]) concerning the existence of N distinct periodic orbits in the case of convex Hamiltonian surfaces. These results have recently been extended by Berestycki, Lasry, Mancini and Ruf (see [*6*] [*7*]) to the case of a starshaped surface.

The purpose of this note is to investigate the case of a starshaped surface $V = \{ z \in \mathbb{R}^{2N} : H(x, y) = \text{const} \}$ which is symmetric w. r. t. the origin, i. e. $H(x, y) = H(-x, -y)$. We are able to weaken the assumption of [*7*] for this class of Hamiltonian surfaces.

Our results have to be compared with a paper by Van Groesen [*13*], which was also a motivation for our investigation. In [*13*] the Author deals with Hamiltonian surfaces which are *convex* and such that

$$H(-x, y) = H(x, y) = H(x, -y),$$

while in the present paper $H(-x, -y) = H(x, y)$ and no convexity is required; of course the solutions found in [*13*] and here have the corresponding symmetry properties. The proof here relies in a variational principle in a suitable function space which characterizes symmetric periodic orbits and, in contrast to [*13*] is more in the spirit of [*7*].

§ 2. THE RESULT

Let V be a regular C^2-manifold of $\mathbb{R}^{2n}$. If $H, \overline{H} \in C^2(\mathbb{R}^{2n}, \mathbb{R})$ are such that $V = H^{-1}(a) = \overline{H}^{-1}(b)$, $a, b \in \mathbb{R}^+/\{0\}$, and $H'(z) \neq 0$, $\overline{H}'(z) \neq 0$ $\forall z \in V$, then it is well known (see [*11*]) that the Hamiltonian systems

$$J\dot{z} = H'(z) \qquad \text{and} \qquad J\dot{z} = \overline{H}'(z)$$

where $z = (x, y) \in \mathbb{R}^{2N}$, $Jz = (y, -x)$ $\forall z \in \mathbb{R}^{2N}$ have the same trajectories on V, called « Hamiltonian trajectories on V ».

For $\rho \in \mathbb{R}^+$ let B_ρ denote the ball of radius ρ in $\mathbb{R}^{2N}$. For $z_1, z_2 \in \mathbb{R}^{2N}$ let (z_1, z_2) denote the scalar product in $\mathbb{R}^{2N}$. For $z \in V$ let $n(z) = \dfrac{H'(z)}{|H'(z)|}$ be the unitary exterior normal vector to V at z and let $d(z) := (n(z), z)$, i. e. the distance between the origin of $\mathbb{R}^{2N}$ and the tangent hyperplane to V at z. If V is starshaped, i. e. $d(z) > 0$ $\forall z \in V$, let $d = \min \{ d(z) : z \in V \}$. Let $R = \max_V |z|$ and $r = \min_V |z|$.

We shall assume

(H_1) $V = \partial\Omega$, where Ω is an open bounded subset of $\mathbb{R}^{2N}$, $O \in \Omega$, starshaped w. r. t. the origin.

(H_2) V is symmetric w. r. t. the origin

(H_3) $R^2 < 3(rd)$.

THEOREM 1. — *Assume* (H_1) *and* (H_2). *Then there exists at least one symmetric periodic Hamiltonian trajectory on* V.

THEOREM 2. — *Assume* (H_1), (H_2) *and* (H_3). *Then there exist at least* N *distinct, symmetric periodic Hamiltonian trajectories on* V.

REMARK 1. — If we replace (H_3) by
$(\exists_2')$ $k \in \mathbb{N}$, $1 \leq k \leq N$ such that

$$R^2 < (2k + 1)(rd).$$

We can obtain [*1*] that there exist at least $\left[\frac{N}{k}\right]$ distinct symmetric periodic trajectories on V, where for $a \in \mathbb{R}^+$, $[a] := \min [n \in \mathbb{N} : a \leq n]$.

REMARK 2. — If V is convex ; (H_3) becomes

(H_3') $$R^2 < 3r^2$$

and theorem 2 is an improvement, for symmetric trajectories, of the result of Ekeland and Lasry [*8*].

REMARK 3. — The theorems and proofs are given in the special case of surfaces lying between two spheres ; the general result, i. e. for surfaces which are close to an ellipsoid, can be obtained as in [*7*]. It follows that also an analogous of Weinstein theorem [*14*] can be stated for the existence of N symmetric periodic solutions.

REMARK 4. — The variational principle introduced here for symmetric Hamiltonian systems can be easily stated for the unconstrained problem, i. e. the problem of finding periodic solutions with a given period. Existence results of the type of [*11*] [*12*] [*5*] still hold in this case, and the solutions so found will also be symmetric.

REMARK 5. — In [*13*] the Author considers the Hamiltonian surfaces $V = \{ (x, y) \in \mathbb{R}^{2N} : H(x, y) = \text{const} \}$ where H is *convex* and

(H_2') $H(x, y) = H(-x, y) = H(x, -y) = H(-x, -y) \qquad \forall (x, y) \in \mathbb{R}^{2N}$

The existence of N periodic orbits with the same symmetry properties is proved under the assumptions (H_1), (H_2') and (H_3').

The same result could be obtained for a starshaped case, following the same argument as in [*13*], but using an appropriate $\mathbb{Z}_2$ pseudo index theory constructed as in § 4.

§ 3. THE PROOF

We shall prove theorem 2. Theorem 1 will be an easy consequence. It is well known (see [*11*]) that there exists

$$H \in C^2(\mathbb{R}^{2N} / \{ 0 \}, \mathbb{R}) \cap C^{1, \text{lip}}(\mathbb{R}^{2N}, \mathbb{R}),$$

homogeneous of degree two, such that $V = H^{-1}(1)$ and $H(z) = H(-z)$ $\forall z \in \mathbb{R}^{2N}$.

We seek distinct periodic solutions of the Hamiltonian system

$$J\dot{z} = H'(z) \tag{1}$$

lying on V, symmetric with respect to the origin, (i. e. a solution $z(t)$ such that there exists τ with $z(t+\tau) = -z(t)\ \forall t$). The proof will be carried out in the following steps:

STEP 1. — Use a variational principle to find periodic symmetric Hamiltonian trajectories as critical points of the action integral $f(u) = \frac{1}{2}\int_0^{2\pi} uJ\dot{u}$ on $S = \left\{ u \in E_1 : \frac{1}{2\pi}\int_0^{2\pi} H(u) = 1 \right\}$ where E_1 is a suitable space.

STEP 2. — Prove that the positive critical value of the action integral have a lower bound on S, which can be easily estimated from below in terms of d and r.

STEP 3. — Use a pseudo-index (see [4] [5]) to find N positive critical levels of f on S, $c_1, \dots, c_N$ say, and estimate c_N through (H_3), to ensure that the corresponding critical points give rise to different trajectories.

STEP 1. — Variational principle.

Let $E = H^{1/2}(S^1, \mathbb{R}^{2N})$. For any smooth $u \in E$, we consider the action integral $f(u) = \frac{1}{2}\int_0^{2\pi} uJ\dot{u}$ and, by extension, we define f as continuous quadratic form on E. One has $f \in C^1(E, \mathbb{R})$. Denoting by $\langle \cdot, \cdot \rangle$ the scalar product in E, we define the bounded selfadjoint linear operator $L : E \to E$ by

$$\langle Lu, v \rangle := f'(u)v \qquad \forall u, v \in E$$

so that $\frac{1}{2}\langle Lu, u \rangle = f(u)$.

Let $\phi_{Jm} = \frac{1}{2i}[e^{imt}\xi_J - e^{-imt}\bar{\xi}_J]$ $J = 1, \dots, N, m \in \mathbb{N}$; let

$$E_1 = \operatorname{span}\{\phi_{Jm}, m \text{ odd}\}$$
$$E_2 = \operatorname{span}\{\phi_{Jm} : m \text{ even}\}$$
$$E_0 = \ker L = \mathbb{R}^{2n}$$

Then $E = E_0 \oplus E_1 \oplus E_2$ is an orthogonal decomposition of E.

One has $f \in C^1(E_1, \mathbb{R})$. Let $S = \left\{ u \in E_1 : \frac{1}{2\pi}\int_0^{2\pi} H(u) = 1 \right\}$. S is a regular C^1-manifold of E_1, of codimension 1, radially diffeomorphic to the unit sphere of E_1.

LEMMA 1. — *If* $u \in S$ *is a critical point of* $f|_S$, *with* $f(u) > 0$, *then there exists* $\lambda > 0$ *such that*

$$J\dot{u} = \lambda H'(u). \tag{2}$$

Proof. — Denote $\mathscr{H} \in C^1(E, \mathbb{R})$ the weakly continuous functional $u \to \frac{1}{2\pi}\int_0^{2\pi} H(u)$, $u \in E$. If u is a critical point of $f|_S$ we have

$$\langle Lu, v \rangle = \mathscr{H}'(u)v \qquad \forall v \in E_1 .$$

To prove the lemma we have to show that

$$\langle Lu, w \rangle = \mathscr{H}'(u)w \qquad \forall w \in E .$$

From this it follows, by a simple regularity argument, [*5*] that $u \in C^1$ and verifies (2) for some $\lambda \in \mathbb{R}$; the positivity of λ easy follows (see (6)).

Let $w \in E$ and $w = w_0 + w_1 + w_2$ where $w_i \in E_i$, $i = 0, 1, 2$. Clearly $\langle Lu, w_0 \rangle = 0$. Let $u_n \to u$ where $u_n \in C^\infty(S^1 ; \mathbb{R}^{2N}) \cap E_1$; then, by definition of L, one has $\langle Lu, w_2 \rangle = \lim \int_0^{2\pi} (J\dot{u}_n, w_2) = 0$ since $J\dot{u}_n \in E_1$.

Recall that $|H'(z)| \leq C|z|\ \forall z \in \mathbb{R}^{2N}$, where $C = \max_{|z|=1} |H'(z)|$; H' is a continuous mapping from $L^2(S^1, \mathbb{R}^{2N})$ to $L^2(S^1, \mathbb{R}^{2N})$. From $u_n \underset{E}{\to} u$ it follows $u_n \underset{L^2}{\to} u$ and $H'(u_n) \underset{L^2}{\to} H'(u)$.

As $u_n \in E_1$, $H'(u_n(t + \pi)) = H'(-u_n(t)) = -H'(u_n)$ by the symmetry of V ; then $H'(u_n) \in \underset{L^2}{\mathrm{span}} \{ \phi_{Jm} : m \text{ odd} \}$. It follows

$$\int_0^{2\pi} (H'(u_n), w_0) = \int_0^{2\pi} (H'(u_n), w_2) = 0$$

and

$$\mathscr{H}'(u)w_0 = \mathscr{H}'(u)w_2 = 0 .$$

The lemma is proved. □

In view of lemma 1, if u satisfies (2), $u(\lambda^{-1}t)$ is a solution of (1), lying on V, of period $T = 2\pi\lambda$, and if u has minimal period 2π, then $u(\lambda^{-1}t)$ has minimal period T.

Moreover, since $u \in E_1$, $u(t + \pi) = -u(t)\ \forall t \in S^1$, hence $u\left(\frac{t}{\lambda}\right)$ is symmetric w. r. t. the origin.

The existence of N distinct symmetric periodic Hamiltonian trajectories on V, is equivalent to the existence of N critical points of $f|_S$, $u_1, \ldots, u_n$ say, $f(u_i) > 0$, such that

$$u_i \text{ has minimal period } 2\pi \qquad i = 1, \ldots, N \tag{3}$$

$$u_i(t) \neq u_J(t + \theta),\ i \neq J,\ i, J \in \{1, \ldots, N\} \qquad \theta \in S^1 \tag{4}$$

(see [*7*] [*8*]).

STEP 2. — Let $Z = \{ u \in S : f'|_S(u) = 0,\ f(u) > 0 \}$; if we consider the functional f and the manifold S in the space $E = H^{1/2}(S^1 ; \mathbb{R}^{2N})$ we have:

LEMMA. — *If* $u \in Z$ *then* $f(u) \geq \pi d^2$.

Proof. — (see also [7]). Let $M = \max \{ | H'(z) | : z \in H^{-1}(1) \}$. One has

$$d(z) = (n(z), z) = \left(\frac{H'(z)}{| H'(z) |}, z \right) = \frac{2}{| H'(z) |}$$

then

$$d = \min_{z \in H^{-1}(1)} d(z) = \frac{2}{\max_{z \in H^{-1}(1)} \{ | H'(z) | \}} = \frac{2}{M} \tag{5}$$

Let u be a critical point of $f|_S$, $f(u) > 0$; there exists $\lambda > 0$ such that $J\dot{u} = \lambda H'(u)$. Then

$$f(u) = \frac{1}{2} \langle Lu, u \rangle = \frac{1}{2} \int_0^{2\pi} uJ\dot{u} = \frac{\lambda}{2} \int_0^{2\pi} (H'(u), u) = 2\pi\lambda . \tag{6}$$

Moreover, for every $v \in H^1(S^1, \mathbb{R}^{2N})$, setting $v = \tilde{v} + C$, $\int_0^{2\pi} \tilde{v} = 0$ and using the Wirtinger inequality for zero mean functions, we obtain

$$\int_0^{2\pi} vJ\dot{v} = \int_0^{2\pi} \tilde{v}J\dot{\tilde{v}} \leq \| \tilde{v} \|_{L^2} \cdot \| \dot{\tilde{v}} \|_{L^2} \leq \| \dot{\tilde{v}} \|_{L^2}^2 = \| \dot{v} \|_{L^2}^2$$

hence, from (6)

$$2\pi\lambda = \frac{1}{2} \int_0^{2\pi} uJ\dot{u} \leq \frac{1}{2} \| \dot{u} \|_{L^2}^2 = \frac{1}{2} \int_0^{2\pi} | \lambda H'(u) |^2 \leq \pi\lambda^2 M^2 . \tag{7}$$

By (7) we have $\lambda \geq \frac{2}{M^2}$; then by (6) and (5)

$$f(u) \geq \frac{4\pi}{M^2} = \pi d^2 . \tag{8}$$

□

The previous lower bound for the action on the critical points obviously still holds in the space E_1 ; in this case a slightly stronger result holds.

LEMMA 2. — *If* $u \in Z$, *then* $f(u) \geqslant \pi(dr)$.

Proof. — Let M and d as before ; one has

$$2\pi r^2 \leqslant \int_0^{2\pi} | u |^2 \leqslant \int_0^{2\pi} | \dot{u} |^2 \leqslant \int_0^{2\pi} | \lambda H'(u) |^2 \leqslant 2\pi\lambda^2 M^2 \tag{9}$$

using Wirtinger's inequality and the fact that

$$| u(t) | \geqslant r \qquad \forall t \in [0, 2\pi) .$$

Hence $\lambda \geqslant \frac{r}{M}$ and, by (6)

$$f(u) = 2\pi\lambda \geqslant \pi \frac{2}{M} r = \pi(dr) . \tag{10}$$

We have the following easy consequence of Lemma 2.

LEMMA 3. — *If $u \in Z$, $f(u) < 3\pi(dr)$, then u has minimal period* 2.

Proof. — First we remark that if u has not minimal period 2π, then its minimal period cannot ever be π; in fact this would imply $u(\pi) = u(0)$; but also by symmetry $u(\pi) = -u(0)$ and hence $u(0)=0$ which is absurd. If the minimal period of u is $\frac{2\pi}{m}$, $m \in \mathbb{N}$, $m \geq 3$, setting $u^*(t) = u\left(\frac{t}{m}\right)$ we find, by direct calculation, $f(u^*) = \frac{1}{m} f(u) < \frac{3\pi(rd)}{m} \leq \pi(rd)$ which is impossible, u^* being also a critical point of $f|_S$.

STEEP 3. — In order to prove (4), define a pseudo-index (see [*4*] [*5*]) and make use of the invariance of f and S through the S^1-action $S^1 \times E_1 \to E_1$, $(\theta, u) \to u(t + \theta)$, $u \in E_1$, $\theta \in S^1$.

The following definitions and properties will be needed below (see [*4*] [*5*] [*6*] [*7*]).

Let ϕ_{Jm} be as before, $\sigma_{Jm} = f(\phi_{Jm})$, $J = 1, \ldots, N$, $m \in \mathbb{N}$: let

$$E_1^+ = \text{span} \{ \phi_{Jm} : \sigma_{Jm} > 0, m \text{ odd} \}$$
$$E_1^- = \text{span} \{ \phi_{Jm} : \sigma_{Jm} < 0, m \text{ odd} \} .$$

Then $E_1 = E_1^+ \oplus E_1^-$ (see [*12*]) is the orthogonal decomposition of E_1 w. r. t. the functional f. Let G be a C^1-manifold of E_1, radially diffeomorphic to the unit sphere of E_1, invariant under the S^1-action. Let $\mathcal{U}$ be the family of self-adjoint linear equivariant isomorphisms $U : E_1 \to E_1$ such that $U(E_1^+) \subset E_1^+$ and let

$$\Gamma_G = \{ h : G \to G \,|\, h \textit{ is an equivariant homeomorphism } \exists g : G \to \mathbb{R}^+ \text{ continuous and } U \in \mathcal{U} \text{ such that } h - gU|_G \text{ is compact} \}$$

Then Γ_G is a group. Let Σ be the family of closed, S^1-invariant subsets of E_1. For $A \subset G$, $A \in \Sigma$, the pseudo-index (in the sense of Benci [*5*]) is defined as

$$i^*(A) := \min_{h \in \Gamma_G} i(h(A) \cap E_1^+)$$

where i is the S^1-index introduced by Benci [*5*]. We recall the following results:

PROPOSITION 1. — *Let* G_1 *and* G_2 *be* C^1*-manifolds radial diffeomorphic to a sphere of* E_1 *and invariant under* S^1*-action, and let* $p : G_1 \to G_2$ *be the radial projection from* G_1 *to* G_2*. Then*

$$i^*(p(A)) = i^*(A) \qquad \forall A \subset G_1, A \in \Sigma .$$

PROPOSITION 2. — 1. *Let* $H_h \subset E_1$ *be a 2h-dimensional invariant subspace and let* $H_h \oplus W = E_1$*. Then for* $A \subset G, A \in \Sigma$

$$i^*_G(A) \geq h + 1 \qquad \textit{imply} \qquad A \cap W \neq \emptyset .$$

2. *Let* H_h *as above; then*

$$i^*_G(G \cap [H_h \oplus E_1^-]) = h .$$

By the standard argument of the Ljusternik-Schnirelman theory, if $f|_G$ verifies the P. S. condition, one obtains the following:

Minimax principle: Let $Z_a = \{ u \in G : f'|_G(u) = 0, f(u) = a \}$; for $k \in \mathbb{N}$ define:

$$a_k = \inf_{A \in A_k} \sup_{u \in A} f(u), \; A_k = \{ A \subset G, A \in \Sigma : i^*(A) \geq k \} .$$

Then a_k is a (positive) critical value of $f|_G$. Moreover if $a = a_{k+1} = \ldots = a_{k+p}$, then $i(Z_a) \geq p$.

It is known that $f|_S$ satisfies the Palais-Smale condition (see [7]); however we sketch the proof for reader's convenience. Let $\{ u_n \} \subset S$, with $f(u_n)$ bounded, and $w_n = f'|_S(u_n) = Lu_n - \lambda_n \mathscr{H}'(u_n) \to 0$. Since $\langle \mathscr{H}'(u_n), u_n \rangle = 2$ and $\langle Lu_n, u_n \rangle$ is bounded, it follows that $|\lambda_n| \leq c + d\|u_n\|$. Also since $u_n \in S$, u_n is bounded in L^2 and hence

$$\| \mathscr{H}'(u_n) \| = \sup_{\|v\|=1} \left| \int (H'(u_n), v) \right| \leq \text{const} .$$

If we write $u = u_n^+ + u_n^- \in E_1^+ \oplus E_1^-$, we get

$$c \| u_n^+ \|^2 \leq \langle Lu_n^+, u_n^+ \rangle \leq \| w_n \| \| u_n^+ \| + c \| u_n \| + d$$
$$- c \| u_n^- \|^2 \geq \langle Lu_n^-, u_n^- \rangle \geq - \| w_n \| \| u_n^- \| - c \| u_n \| - d$$

and therefore

$$\| u_n \|^2 = \| u_n^+ \|^2 + \| u_n^- \|^2 \leq c \| u_n \| + d$$

which implies $\| u_n \|$ bounded. Finally, from $Lu_n = \lambda_n \mathscr{H}'(u_n) + w_n$ and the compactness of $\mathscr{H}'(u_n)$ we see that $L(u_n^+ + u_n^-)$ has a convergent subsequence. This complete the proof.

We have now what is needed to prove theorems 1 and 2. Let us consider $S_R = \left\{ u \in E_1 : \dfrac{1}{2\pi} \displaystyle\int_0^{2\pi} |u|^2 = R^2 \right\}$ which is a C^1-manifold, radially diffeomorphic to a sphere of E_1, invariant under the S^1-action. Obviously $f|_{S_R}$ verifies the P. S. condition. We apply the minimax principle to S and S_R.

Let us denote by $b_1, \ldots, b_N$ the first N critical value of $f|_{S_R}$ of minimax type.
The following simple lemma (see also [*1*]) holds.

LEMMA 4. — *For* $f|_{S_R}$ *one has* $b_1 = \ldots = b_N = \pi R^2$.

Proof. — It is obvious, as in lemma 2, that if u is a critical (positive) point of $f|_{S_R}$, then $f(u) \geq \pi R^2$. By the P. S. conditions, there exists a minimal positive critical value of $f|_S$, $b_{\min}$ say. If u is a critical point at level $b_{\min}$, u has minimal period 2π. On the other hand

$$u = (\xi \cos \gamma t + \eta \operatorname{sen} \gamma t, \xi \operatorname{sen} \gamma t - \eta \cos \gamma t) \qquad \xi, \eta \in \mathbb{R}^N$$

and the minimality of period 2π implies $\gamma = 1$. Hence

$$\{ u \in S_R : f'|_{S_R}(u) = 0, f(u) = b_{\min} \} = \{ u = (\xi \cos t + \eta \operatorname{sen} t, \xi \operatorname{sen} t - \eta \cos t) : |\xi|^2 + |\eta|^2 = R^2 \} = S^{(N)}.$$

Let $H_N = \{ \xi \cos t + \eta \operatorname{sen} t, \xi \operatorname{sen} t - \eta \cos t : \xi, \eta \in \mathbb{R}^N \}$ and let

$$A_N = (H_N \oplus H_1^-) \cap S_R.$$

By proposition 2, one has $i^*(A_N) = N$. One sees immediately that

$$\pi R^2 = \sup_{A_N} f(u) \geq b_N \geq \ldots \geq b_1 \geq b_{\min} = \pi R^2 \qquad \square$$

Let $$c_J = \inf_{i_S^*(A) \geq J} \sup_A f \qquad J = 1, \ldots, N.$$

It is known that c_J's are positive ($c_J \geq \pi r \cdot d$) and that they are critical values of $f|_S$. Moreover for every $u \in S_R$ there exists a unique λ such that $\lambda u \in S$ and $\lambda \leq 1$. For $A \in S \cap \Sigma$ one has

$$\sup_{w \in A} f(w) = \sup_{u \in p(A)} f(\lambda u) = \sup_{u \in p(A)} \lambda^2 f(u) \leq \sup_{u \in p(A)} f(u)$$

where p is the radial projection from S to S_R.

By proposition 1, $c_J \leq b_J = \pi R^2$: by (H_3) $c_N \leq \pi R^2 \leq 3\pi(rd)$. Theorem 2 follows now from lemma 3.

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