A special class of two dimensional exponential-Bessel series of Fox’s $H$-function

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A SPECIAL CLASS OF TWO DIMENSIONAL EXPONENTIAL-BESSEL SERIES
OF FOX'S H-FUNCTION

Abstract: In this paper, we present a special class of two-dimensional Exponential-Bessel series of Fox's H-function [2], and present one two-dimensional series of this class.

The following formulae are required in the proof:

The integral [1, p.46, (1)]:

\[ \int_0^\pi e^{-2iu \sin x} \left( \frac{1}{w_1 - 1} H_{m,n}^{p,q} \right) \left( z(\sin x)^{-2h} \left( \frac{a_p e_p}{(b_q f_q)} \right) \right) dx \]

\[ = \frac{e^{-iu \frac{w_1 + 1}{2}}}{2^{w_1 - 1}} H_{m+1,n}^{p+2,q+2} \left( z 2^{-2h} \left( \frac{a_p e_p}{(w_1 2h) b_q f_q} \right) \pm u, h \right) \]

where

\[ h > 0, \sum_{j=1}^p e_j - \sum_{j=1}^q f_j = A \leq 0, \sum_{j=1}^n e_j - \sum_{j=n+1}^p e_j + \sum_{j=1}^m f_j - \sum_{j=m+1}^q f_j = B > 0, \]

\[ |\arg z| < \frac{1}{2} B, \text{Re} w_1 - 2h \max_{1 \leq j \leq n} \left( \text{Re} (a_j - 1) / e_j \right) > 0. \]

The integral [4, p.94, (2.2)]:

\[ \ldots \]
The orthogonality property of the Bessel functions \[3, p. 291, (6)\]:

2. TWO-DIMENSIONAL EXPONENTIAL-BESSEL SERIES. The two-dimensional Exponential-Bessel series to be established is

\[
(1.2) \int_0^\infty y^{w_2-1} \sin y \ J_v(y) \ H_p^{m,n}_{p,q} \left[ zy^{2k} \left( \frac{a_{p,e_p}}{b_q^{f_q}} \right) \right] dy
\]

\[
= 2^{w_2-1} \sqrt{n} \ H_{p+4,q+1}^{m+1,n+1} \left[ 2^{2k} \left( \frac{1-w_2-v}{2}, k \right), \left( 1 + \frac{v-w_2}{2}, k \right), \left( 1 - \frac{v+w_2}{2}, k \right), \left( \frac{v-w_2}{2}, k \right), \left( \frac{1+v-w_2}{2}, k \right), \left( \frac{1-v-w_2}{2}, k \right), \left( 1-\frac{v+w_2}{2}, k \right), \left( \frac{1-v-w_2}{2}, k \right) \right), \left( \frac{v-w_2}{2}, k \right), \left( \frac{1-v-w_2}{2}, k \right) \right]
\]

where \( k > 0, A \leq 0, B > 0, |\arg z| < \frac{1}{2} B, \|Re \left( w_2 + v \right) + 2k \min_{1 \leq j \leq m} \left( Re b_j / f_j \right) > 0. \)

The orthogonality property of the Bessel functions \(3, p. 291, (6)\):

\[
(1.3) \int_0^\infty x^{-1} J_{a+2n+1}(x) J_{a+2m+1}(x) \ dx
\]

\[
= \begin{cases} 
0, \text{ if } m \neq n; \\
(4n+2a+2)^{-1}, \text{ if } m = n, \text{ Re } a + m + n > -1.
\end{cases}
\]

2. TWO-DIMENSIONAL EXPONENTIAL-BESSEL SERIES. The two-dimensional Exponential-Bessel series to be established is

\[
(2.1) \ (\sin x)^{w_1-1} \ y^{w_2} \sin y \ H_{p,q}^{m,n} \left[ z(\sin x)^{-2h} y^{2k} \left( \frac{a_{p,e_p}}{b_q^{f_q}} \right) \right]
\]
valid under the conditions of (1.1), (1.2) and (1.3).

Proof. Let

\[ \sum_{r = -\infty}^{\infty} \sum_{s = 0}^{\infty} \left( a + 2s + 1 \right) e^{2i\pi (x - \Pi/2)} J_{a + 2s + 1} (y) \]

Equation (2.2) is valid, since \( f(x, y) \) is continuous and bounded variation in the region \( 0 < x < \Pi, 0 < y < \infty \).

Multiplying both sides of (2.2) by \( y^{-1} J_{a + 2\nu + 1} (y) \) and integrating with respect to \( y \) from 0 to \( \infty \), then using (1.2) and (1.3). Now multiplying both sides of the resulting expression by \( e^{-2i\pi x} \) and integrating with respect to \( x \) from 0 to \( \Pi \), then using (1.1) and the orthogonality property of exponential functions, we obtain the value of \( C_{r,s} \).

Substituting this value of \( C_{r,s} \) in (2.2), the expansion (2.1) is obtained.
Note: On applying the same procedure as above, we can establish three other forms of two-dimensional expansions of this class with the help of alternative forms of (1.1) and (1.2).

Since on specializing the parameters Fox's H-function yields almost all special functions appearing in applied mathematics and physical sciences. Therefore, the result presented in this paper is of a general character and hence may encompass several cases of interest.

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S. D. BAJPAI
UNIVERSITY OF BAHRAIN
P.O. BOX 32038
ISA TOWN.

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