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PARAMETRIC DEPENDENCE OF THE SOLUTION PROCESSES ON THE COEFFICIENTS IN McSHANE'S STOCHASTIC INTEGRAL EQUATION SYSTEMS

1. INTRODUCTION

We consider families of stochastic integral equation systems of the type:

$$(1)\qquad X_\lambda^i(t,\omega) = \alpha_\lambda^i(t,\omega) + \sum_{\rho=1}^{r}\int_a^t g_{\lambda,\rho}^i(s,X_\lambda(s,\omega),\omega)dz^\rho(s,\omega) + \sum_{\rho,\sigma=1}^{r}\int_a^t h_{\lambda,\rho\sigma}^i(s,X_\lambda(s,\omega),\omega)dz^\rho(s,\omega)dz^\sigma(s,\omega) \qquad (i=1,\dots n)$$

($t\in[a,b]\subset\mathbb{R}$; $\omega\in\Omega$, being (Ω,A,P) some probability space) in which the coefficients depend on some parameter λ, and understanding the integrals as McShane's stochastic belated integrals.

The main aim of this paper is to establish convergence conditions relative to the coefficients α_λ^i, $g_{\lambda,\rho}^i$ and $h_{\lambda,\rho\sigma}^i$, that allow to assure the convergence of the corresponding solution processes (we prove in a previous paper the existence and uniqueness of the solution processes). In that sense, we state two results that show the regularity of the solution processes, by means of two differents kinds of stochastic convergence.

Our study on the considered problem, previously treated for other types of stochastic integrals, is completely original in that referred to McShane's stochastic belated integrals.

In this work we consider as starting point the fundamental theorem on the existence and uniqueness of the solution processes in McShane's stochastic integral equation systems that we set out in the above mentioned previous paper that we present together with this one (see Angulo-Gutiérrez (1987)).

2. DEFINITIONS AND NOTATION

In relation to the previous elements of the McShane's stochastic calculus theory, we basically refer to McShane's (1974) and Elworthy's (1982) works. Also, in that referred to the previous definitions (classes of processes, norms,...) and basic notation, see our work (Angulo-Gutiérrez (1987)).

With regard to the stochastic convergence types that we will use here, in order to specify the stochastic convergence notion, we will consider the followings:

. **"L_2-convergence"**: it is the process convergence based on the L_2-norm ($|||\cdot|||$).

. **"Uniform-sample probability convergence (u.s.P.-convergence)"**: Given a family $\{x_\lambda\}_{\lambda\epsilon\Lambda}$ (Λ some parametric space) of processes, we will say that x_λ converge to x_{λ_0} (for a fixed λ_0) sample-uniformly in probability iff for each $\varepsilon>0$,

$$\lim_{\lambda\to\lambda_0} P[\sup_{t\epsilon[a,b]} |x_\lambda(t,\omega)-x_{\lambda_0}(t,\omega)|>\varepsilon] = 0$$

Finally, by means of the notation $\{E[\alpha_\lambda,g_\lambda,h_\lambda];\lambda\epsilon\Lambda\}$ we will refer to the family of systems (1), and by means of $I[g_\lambda,h_\lambda;x_\lambda]$ to the sum of the integrals in the rigth member of (1) (for each λ).

3. LIPSCHITZ CHARACTER OF THE SOLUTIONS WITH RESPECT TO THE INITIAL CONDITION

Let us suppose a family $\{E[\alpha,g,h];\alpha\epsilon H^2_{F,n}\}$ of systems where the functions g^i_ρ, $h^i_{\rho\sigma}$ and the integrators z^ρ are common, and such that the hypotheses [HE] are true with a common constant L.

Consider the function

$$F:\ \tilde{H}^2_{F,n} \longrightarrow \tilde{H}^2_{F,n}$$

that associates to each element $\tilde{\alpha}\epsilon\tilde{H}^2_{F,n}$ the element $F(\tilde{\alpha})=\tilde{X}^*$, defined as the equivalence class of the solutions in $H^2_{F,n}$ relative to the system

$E[\alpha,g,h]$ (see Angulo-Gutiérrez (1987)).

We establish the following result:

THEOREM: "Under the previous conditions, the function F is lipschitziain on $\tilde{H}^2_{F,n}$: it exists a positive constant M such that for each pair of elements $\tilde{\alpha}_1$, $\tilde{\alpha}_2 \in \tilde{H}^2_{F,n}$,

$$|||F(\tilde{\alpha}_1)-F(\tilde{\alpha}_2)||| \leq M|||\tilde{\alpha}_1-\tilde{\alpha}_2||| \text{ ."}$$

Proof: Let α_1 and α_2 be two elements in $H^2_{F,n}$, and let x_1 and x_2 be versions of the respective solutions of the systems $E[\alpha_1,g,h]$ and $E[\alpha_2,g,h]$.

We can state that

$$\|x_1^{*i}(t,\omega)-x_2^{*i}(t,\omega)\|^2 \leq (1+r+r^2)[\|\alpha_1^i(t,\omega)-\alpha_2^i(t,\omega)\|^2 +$$

$$+\sum_{\rho=1}^{r}\|\int_a^t[g_\rho^i(s,x_1^*(s,\omega),\omega)-g_\rho^i(s,x_2^*(s,\omega),\omega)]dz^\rho(s,\omega)\|^2 +$$

$$+\sum_{\rho,\sigma=1}^{r}\|\int_a^t[h_{\rho\sigma}^i(s,x_1^*(s,\omega),\omega)-h_{\rho\sigma}^i(s,x_2^*(s,\omega),\omega)]dz^\rho(s,\omega)dz^\sigma(s,\omega)\|^2] \leq$$

$$\leq (1+r+r^2)[\|\alpha_1^i(t,\omega)-\alpha_2^i(t,\omega)\|^2 +$$

$$+(r+r^2)C^2L^2\int_a^t\|x_1^*(s,\omega)-x_2^*(s,\omega)\|^2ds]$$

Adding for i=1,...n and taking supremes on [a,b] to the initial condition term, we obtain that

$$\|x_1^*(t,\omega)-x_2^*(t,\omega)\|^2 \leq$$

$$\leq (1+r+r^2)[|||\alpha_1-\alpha_2|||^2 + n(r+r^2)C^2L^2\int_a^t\|x_1^*(s,\omega)-x_2^*(s,\omega)\|^2ds]$$

The Gronwall's lemma yields

$$\|x_1^*(t,\omega)-x_2^*(t,\omega)\|^2 \leq (1+r+r^2)e^{n(1+r+r^2)C^2L^2(t-a)}|||\alpha_1-\alpha_2|||^2$$

Taking now supremes on [a,b] in both members, and noting

$$M = (1+r+r^2)e^{n(1+r+r^2)C^2L^2(b-a)}$$

(M is independent of x_1 and x_2) we have

$$|||x_1^*-x_2^*|||^2 \leq M^2|||\alpha_1-\alpha_2|||^2$$

as we would like to prove.

4. PARAMETRIC DEPENDENCE OF THE SOLUTION PROCESSES ON THE COEFFICIENTS

In this paragraph we state and prove two results on the parametric dependence of the solution processes on the coefficients: the first one in terms of "L_2convergence", and the second one in terms of "L_2-" and "u.s.P.-convergence". Previously, we establish two lemmas that will be used in their proofs.

Let us consider a family $\{E[\alpha_\lambda,g_\lambda,h_\lambda];\lambda\in\Lambda\}$ of systems with coefficients depending on a parameter λ (being the integrators z^ρ common to all the elements of the family). We will say that the mentioned family satisfies the hypotheses [HE(λ)] if for every $\lambda\in\Lambda$ the system $E[\alpha_\lambda,g_\lambda,h_\lambda]$ satisfies the hypotheses [HE], being L independent of λ.

By means of the following lemma, we propose two equivalent ways to state the convergence conditions we later require to the coefficients $\{g_\lambda,h_\lambda:(i,\rho,\sigma)\}$ in order to prove the regularity of the corresponding solutions:

<u>LEMMA I</u>: "Let $\{f_\lambda\}_{\lambda\in\Lambda}$ be a family of funtions satisfying the hypotheses [HE(λ)] (in relation to [HE(λ)-4]). Let x be a process belonging to $H^2_{F,n}$. Let λ_0 be a fixed point in Λ. Then, the following conditions are equivalent:

<u>condition a[x]</u>: for every $t\in[a,b]-N$ (being N a Lebesgue-null subset of [a,b]),

$$f_\lambda(t,x(t,\omega),\omega) \xrightarrow{P} f_{\lambda_0}(t,x(t,\omega),\omega)$$

(when $\lambda\to\lambda_0$). (P expresses the convergence in Probability of r. v.).

<u>condition b[x]</u>:

$$\int_a^b \|f_\lambda(t,x(t,\omega),\omega)-f_{\lambda_0}(t,x(t,\omega),\omega)\|^2 dt \longrightarrow 0$$

(when $\lambda\to\lambda_0$)."

REMARK: We have proved this result considering, in relation to x and the condition b[x], any L_p-norm (not only for p=2) and any exponent r (not only for r=2). On the other hand in this case the previous hypotheses [HE(λ)] can be weakened (in fact, the hypotheses [HE(λ)-2,3,5] are superflous, and the hypothese

[HE(λ)-4] is excessive). But, to simplify, we have preferred to state here the previous lemma according to its later application in this work.

(Proof: see APPENDIX A).

From the fundamental existence and uniqueness theorem (see Angulo-Gutiérrez (1987)) and the previous lemma, we establish the following regularity theorem:

THEOREM I: "Let $\{E[\alpha_\lambda, g_\lambda, h_\lambda]; \lambda\in\Lambda\}$ a family of systems satisfying the hypotheses [HE(λ)], and being $\alpha_\lambda \in H^2_{F,n}$ (for every λ). Let $\tilde{X}^*_\lambda$ be the only equivalence class of solutions of the system $E[\alpha_\lambda, g_\lambda, h_\lambda]$. Let λ_0 be a fixed point in Λ. Let us suppose that the following condition are true:

a) For each $x \in H^2_{F,n}$, the families

$$\{g^i_{\lambda,\rho}\}_{\lambda\in\Lambda} \qquad \{h^i_{\lambda,\rho\sigma}\}_{\lambda\in\Lambda}$$

($i=1,\dots n$; $\rho,\sigma=1,\dots r$) satisfy the condition a[x].

b) $$\alpha_\lambda \xrightarrow{L_2} \alpha_{\lambda_0} \qquad \text{(when } \lambda\to\lambda_0).$$

Then,

$$\tilde{X}^*_\lambda \xrightarrow{L_2} \tilde{X}^*_{\lambda_0} \qquad \text{(when } \lambda\to\lambda_0)."$$

Proof: The conditions of the theorem allow to state that

$$\|x^{*i}_\lambda(t,\omega) - x^{*i}_{\lambda_0}(t,\omega)\|^2 \le$$

$$\le (1+r+r^2)\{\|\alpha^i_\lambda(t,\omega) - \alpha^i_{\lambda 0}(t,\omega)\|^2 +$$

$$+ C^2 \sum_{\rho=1}^{r} \int_a^t \|g^i_{\lambda,\rho}(s, x^*_\lambda(s,\omega),\omega) - g^i_{\lambda_0,\rho}(s, x^*_{\lambda_0}(s,\omega),\omega)\|^2 ds +$$

$$+ C^2 \sum_{\rho,\sigma=1}^{r} \int_a^t \|h^i_{\lambda,\rho\sigma}(s, x^*_\lambda(s,\omega),\omega) - h^i_{\lambda_0,\rho\sigma}(s, x^*_{\lambda_0}(s,\omega),\omega)\|^2 ds\} \le$$

$$\le (1+r+r^2)\{\|\alpha^i_\lambda(t,\omega) - \alpha^i_{\lambda_0}(t,\omega)\|^2 +$$

$$+ 2C^2 \sum_{\rho=1}^{r} \int_a^t \|g^i_{\lambda,\rho}(s, x^*_\lambda(s,\omega),\omega) - g^i_{\lambda,\rho}(s, x^*_{\lambda_0}(s,\omega),\omega)\|^2 ds +$$

$$+ \quad 2C^2 \sum_{\rho=1}^{r} \int_a^t \| g^i_{\lambda,\rho}(s,x^*_{\lambda_0}(s,\omega),\omega) - g^i_{\lambda,\rho}(s,x^*_{\lambda_0}(s,\omega),\omega) \|^2 ds \; +$$

$$+ \; 2C^2 \sum_{\rho,\sigma=1}^{r} \int_a^t \| h^i_{\lambda,\rho\sigma}(s,x^*_\lambda(s,\omega),\omega) - h^i_{\lambda,\rho\sigma}(s,x^*_{\lambda_0}(s,\omega),\omega) \|^2 ds \; +$$

$$+ \; 2C^2 \sum_{\rho,\sigma=1}^{r} \int_a^t \| h^i_{\lambda,\rho\sigma}(s,x^*_{\lambda_0}(s,\omega),\omega) - h^i_{\lambda_0,\rho\sigma}(s,x^*_{\lambda_0}(s,\omega),\omega) \|^2 ds \}$$

By means of $J^i_1(t)$, $J^i_2(t)$, $J^i_3(t)$ and $J^i_4(t)$ we will respectively note the last four terms in the last member of the previous inequalities.

In relation to $J^i_1(t)$ and $J^i_3(t)$ the hypothese [HE(λ)-4] allows to state that

$$\| x^{*i}_\lambda(t,\omega) - x^{*i}_{\lambda_0}(t,\omega) \|^2 \leq$$

$$\leq (1+r+r^2) \{ \| \alpha^i_\lambda(t,\omega) - \alpha^i_{\lambda_0}(t,\omega) \|^2 + J^i_2(t) + J^i_4(t) \; +$$

$$+ \; 2(r+r^2)C^2L^2 \int_a^t \| x^*_\lambda(s,\omega) - x^*_{\lambda_0}(s,\omega) \|^2 ds$$

Adding for i=1,...n and noting

$$C_1 = (||| \alpha_\lambda - \alpha_{\lambda_0} |||^2 + J_2 + J_4)(1+r+r^2)n$$

where $J_k = \sum_{i=1}^{n} J^i_k(b)$ (k=2,4), and

$$C_2 = 2(1+r+r^2)(r+r^2)C^2L^2n$$

we obtain, for every $t\varepsilon[a,b]$, that

$$\| x^*_\lambda(t,\omega) - x^*_{\lambda_0}(t,\omega) \|^2 \leq C_1 + C_2 \int_a^t \| x^*_\lambda(s,\omega) - x^*_{\lambda_0}(s,\omega) \|^2 ds$$

The Gronwall's lemma yields

$$\| x^*_\lambda(t,\omega) - x^*_{\lambda_0}(t,\omega) \|^2 \leq C_1 e^{C_2(t-a)}$$

and, taking supremes on [a,b],

$$||| x^*_\lambda - x^*_{\lambda_0} |||^2 \leq C_1 e^{C_2(b-a)}$$

On the other hand, from the hypothese (a) and the Lemma I, it is verified that

$$J_2+J_4 \longrightarrow 0$$

when $\lambda \to \lambda_0$. This statement together with the hypothese (b) allows to assure that

$$C_1 \longrightarrow 0$$

when $\lambda \to \lambda_0$. Consequently,

$$|||x^*_\lambda - x^*_{\lambda_0}||| \longrightarrow 0$$

when $\lambda \to \lambda_0$, and so the proof is concluded.

To state a similar result, introducing a u.s.P.-convergence condition respecting to the initial condition α_λ, we establish a previous lemma where we relate the condition a[x] to this last type of convergence:

<u>LEMMA II</u>: "Let $\{f_\lambda\}_{\lambda\in\Lambda}$ be a family of function satisfying the hypotheses [HE(λ)] (in relation to [HE(λ)-4]). Let λ_0 be a fixed point in Λ, and let $\{x_\lambda\}_{\lambda\in\Lambda}$ a family of processes belonging to $H^2_{F,n}$ and such that

$$x_\lambda \xrightarrow{L_2} x_{\lambda_0} \qquad \text{(when } \lambda\to\lambda_0\text{)}.$$

Suppose that the family $\{f_\lambda\}_{\lambda\in\Lambda}$ satisfies the condition $a[x_{\lambda_0}]$. Let z, z^1 and z^2 be processes satifying the hypotheses [HE(λ)] (in relation to [HE(λ)-3]), and let $J_\lambda(t,\omega)$ and $H_\lambda(t,\omega)$ (for each λ) be separable versions of the indefinite integrals

$$\int_a^t f_\lambda(s,x_\lambda(s,\omega),\omega)dz(s,\omega)$$

$$\int_a^t f_\lambda(s,x_\lambda(s,\omega),\omega)dz^1(s,\omega)dz^2(s,\omega)$$

respectively.

Then,

$$J_\lambda \xrightarrow{u.s.P.} J_{\lambda_0}$$

$$H_\lambda \xrightarrow{u.s.P.} H_{\lambda_0} \qquad \text{(when } \lambda\to\lambda_0\text{)}."$$

REMARK: It can be proved that the choice of separable versions of the indefinite integrals is always possible, under convenient conditions, in particular under the conditions of the lemma.

(Proof: see APPENDIX B).

Taking into account the Lemma II and the Theorem I, we stablish the following regularity theorem:

THEOREM II: "Let the hypotheses of the Theorem I be satisfied. Suppose that for each $\lambda\in\Lambda$ the process α_λ is separable, and

$$\alpha_\lambda \xrightarrow{u.s.P.} \alpha_{\lambda_0} \qquad \text{(when } \lambda\to\lambda_0\text{)}.$$

Then, if each solution x^*_λ (for each $\lambda\in\Lambda$) is choiced separable,

$$x^*_\lambda \xrightarrow{u.s.P.} x^*_{\lambda_0} \qquad \text{(when } \lambda\to\lambda_0\text{)."}$$

Proof: For each $\varepsilon>0$,

$$P[\sup_{[a,b]} |x^*_\lambda(t,\omega)-x^*_{\lambda_0}(t,\omega)|>\varepsilon] \leq$$

$$\leq P[\sup_{[a,b]} |\alpha_\lambda(t,\omega)-\alpha_{\lambda_0}(t,\omega)|>\varepsilon/2] +$$

$$+ P[\sup_{[a,b]} |I[g_\lambda,h_\lambda;x^*_\lambda](t,\omega)-I[g_{\lambda_0},h_{\lambda_0};x^*_{\lambda_0}](t,\omega)|>\varepsilon/2]$$

Taking in account the hypotheses of the theorem, the first term in the last member of the previous inequalities converges to 0 (when $\lambda\to\lambda_0$). Also, from the hypotheses of the theorem and applying the Lemma II, we can assert that the second term in the mentioned member converges to 0 (when $\lambda\to\lambda_0$). Thus,the proof is concluded.

5. FINAL OBSERVATION

As we have just remarked at the end of our previous work (Angulo-Gutiérrez (1987)), we have study the problem of the existence and uniqueness of the solution processes in systems of the considered type under conditions wider respecting to the coefficients α^i, g^i_ρ and $h^i_{\rho\sigma}$. Using the results we have obtained in that sense, we have also study some extensions relative to the regularity problem we consider in this paper (see the indications we have made in (Angulo-Gutiérrez (1986)). From obvious reasons of space, we don't develop here all that additional results.

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APPENDIX A. Proof of the Lemma I:

We will prove the double implication separately in both ways:

a[x]⟹b[x]: From [HE(λ)-4], it can be asserted that a. s., for every $t\in[a,b]$ and for every $\lambda\in\Lambda$,

$$|f_\lambda(t,x(t,\omega),\omega)-f_{\lambda_0}(t,x(t,\omega),\omega)| \leq 2L(1+|x(t,\omega)|)$$

Being the process x L_2-bounded, and considering the condition a[x], from a consequence of the "L_p-convergence theorem" (see Loève (1976)) we can assert that for every $t\in[a,b]-N$,

$$\|f_\lambda(t,x(t,\omega),\omega)-f_{\lambda_0}(t,x(t,\omega),\omega)\|^2 \longrightarrow 0$$

when $\lambda\to\lambda_0$.

As the process x belongs to $H^2_{F,n}$, it can be affirmed that the function $\|2L(1+|x(t,\omega)|)\|^2$ is Riemann-integrable and, consequently, Lebesgue-integrable on [a,b]. The Lebesgue's dominated convergence theorem allows to assure that the Lebesgue-integrals

$$\int_a^b \|f_\lambda(t,x(t,\omega),\omega)-f_{\lambda_0}(t,x(t,\omega),\omega)\|^2 dt$$

converge to 0, when $\lambda\to\lambda_0$. Besides, such integrals also exist as Riemann-integrals. In fact, the process

$$2L(1+|x(t,\omega)|)$$

belongs to H^2_F, and then, all the processes in the form

$$f_\lambda(t,x(t,\omega),\omega)-f_{\lambda_0}(t,x(t,\omega),\omega)$$

are L_2-continuous a. e. on [a,b]and, taking in account the first inequality in this proof, L_2-bounded. The hypothese [HE(λ)-5] and our "L_p-continuity lemma" (see Angulo (1984)) yield the later integrals also exist as Riemann-integrals.

b[x]⟹ a[x]: Since the function in the integrand in b[x]is not-negative, it can be asserted that for every $t\in[a,b]$,

$$\|f_\lambda(t,x(t,\omega),\omega)-f_{\lambda_0}(t,x(t,\omega),\omega)\|^2 \longrightarrow 0$$

when $\lambda\to\lambda_0$, and this imply a[x].

APPENDIX B. Proof of the Lemma II:

Let us prove the result corresponding to J_λ (analogously we would proceed in relation to H_λ).

To simplify, we will note

$$G_{\lambda_1,\lambda_2}(x_1,x_2;t,\omega) = f_{\lambda_1}(t,x_1(t,\omega),\omega)-f_{\lambda_2}(t,x_2(t,\omega),\omega)$$

So, it is inmediate that

$$|f_\lambda(t,x_\lambda(t,\omega),\omega)-f_{\lambda_0}(t,x_{\lambda_0}(t,\omega),\omega)| \leq$$

$$\leq |G_{\lambda,\lambda}(x_\lambda,x_{\lambda_0};t,\omega)| + |G_{\lambda,\lambda_0}(x_{\lambda_0},x_{\lambda_0};t,\omega)|$$

It can be proved (see Remark at the end) that for each $\varepsilon>0$,

(2) $$P[\sup_{[a,b]} |\int_a^t G_{\lambda,\lambda}(x_\lambda,x_{\lambda_0};s,\omega)dz(s,\omega)|>\varepsilon] \leq$$

$$\leq \frac{K}{\varepsilon}\int_a^b \|G_{\lambda,\lambda}(x_\lambda,x_{\lambda_0};t,\omega)\|dt + \frac{K}{\varepsilon^2}\int_a^b \|G_{\lambda,\lambda}(x_\lambda,x_{\lambda_0};t,\omega)\|^2 dt \leq$$

$$\leq \frac{K}{\varepsilon}L(b-a)|||x_\lambda-x_{\lambda_0}||| + \frac{K}{\varepsilon^2}L^2(b-a)|||x_\lambda-x_{\lambda_0}|||^2$$

(this last inequality from [HE(λ)-4]).

As

$$x_\lambda \xrightarrow{L_2} x_{\lambda_0}$$

when $\lambda\to\lambda_0$, the last and so the first member of the previous inequalities converge to 0, when $\lambda\to\lambda_0$.

Analogously, it can be stated (see Remark at the end) that for each $\varepsilon>0$,

(3) $$P[\sup_{[a,b]} |\int_a^t G_{\lambda,\lambda_0}(x_{\lambda_0},x_{\lambda_0};t,\omega)dz(t,\omega)|>\varepsilon] \leq$$

$$\leq \frac{K}{\varepsilon}\int_a^b \|G_{\lambda,\lambda_0}(x_{\lambda_0},x_{\lambda_0};t,\omega)\|dt + \frac{K}{\varepsilon^2}\int_a^b \|G_{\lambda,\lambda_0}(x_{\lambda_0},x_{\lambda_0};t,\omega)\|^2 dt$$

From the condition $a[x_{\lambda_0}]$ and the Lemma I (see Remark at the end of the Lemma I), the last and so the first meber of the previous inequality converge to 0, when $\lambda \to \lambda_0$. Thus, the proof is completed.

REMARK: The inequalities (2) and (3) are consequence of the extensions we have made of the Lemma 5B, Proposition 5B and Theorem 5C (§5, Chapter IV) of the in the Elworthy's (1982) book. In fact, though in such results the author require the sample-continuity of the integrator processes, we have proved that this last condition can be substituted by the weaker condition of the separability of such processes. We have also proved that any integrator of the considered type always has a separable version.

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