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Correction to “Cohomology of line bundles on $G/B$”

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Correction to

COHOMOLOGY OF LINE BUNDLES ON G/B

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G. Kempf has pointed out that the computation of the line bundle $K_r$ on $X(w_n)$, \[\text{cf. § 3, B, type } B_n, \text{ 6, 7 (b) and 8; p. 115 to 121}\] is incorrect and that in fact it turns out to be the trivial line bundle. However this does not affect the proof of the main theorem of paragraph 3, Type $B_n$ (Theorem B. 11), in fact the proof of the essential step I on p. 121 now becomes immediate after writing the exact cohomology sequence. Further as we shall now see, the proof that $K_r$ is trivial also turns out to be simpler than the considerations of the paper for computing $K_r$.

Thus one has to make the following correction: In place of Proposition B. 9 (p. 119) one has

**Proposition.** — $K_r$ is isomorphic to the trivial line bundle and in particular, we have the exact sequence.

$$0 \to \mathcal{O}_{X(w_n)} \to \mathcal{O}_{Z_r} \to \mathcal{O}_{X(w_n)_r} \to 0.$$ 

**Proof.** — Let $P = P_9$, $T$, $B$ be the subgroups of $G = \text{SO}(2n+1) \subset \text{GL}(2n+1)$ and identify $P \setminus G$ with the quadric $Q \equiv x_1 y_n + \ldots + x_n y_1 + z^2 = 0$ in $\mathbb{P}^{2n} = \{ (x_1, \ldots, x_n, z, y_1, \ldots, y_n) \}$ as in the paper. The coordinate functions $x_1, \ldots, x_n, z, y_1, \ldots, y_n$ can be canonically identified with functions on $G$, namely the entries of the last row. We have the ideals $I = (x_1, \ldots, x_n, z)$ and $J = (x_1, \ldots, x_n)$ in $A = k[G]$. Take the action of $G$ on $A$ induced by right translation. Recall that $I$ and $J$ are $B$-stable ideals. Further notice that the element $z$ is $B$-invariant modulo $J$ (not merely $B$-stable modulo $J$, we see that $B$ acts on $z$ mod $J$ through the trivial character).

Let $K = I/J$ as in the paper. Let $R = A/I$; then $R = k[X(w_n)]$. Since $I^2 \subset J$, $I/J$ acquires a $B$-action consistent with the canonical $B$-action on $R$ ($B$-actions induced by right multiplication). To prove that $K_r$ is the trivial line bundle on $X(w_n)_r$, we have to show that (as $R$-module) $I/J$ is $B$-isomorphic to $R$, $R$ being considered as a module over itself. Since $K_r$ is a line bundle, we know that $I/J$ is a projective $R$-module of rank 1. Hence it suffices to show that there exists $m \in I/J$ such that: $1^m$ generates $I/J$ over $R$ and $2^m$ is $B$-invariant. For $m$ we take the image in $I/J$ of $z \in I$. Since $z^2 \in J$ it follows that $z$ generates $I/J$ over $R$ and we have seen that $z$ mod $J$ is a $B$-invariant element.

Q. E. D.