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CERTAIN UNIPOTENT REPRESENTATIONS OF FINITE CHEVALLEY GROUPS AND PICARD–LEFSCHETZ MONODROMY

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ABSTRACT. – The purpose of this note is to give a strange relation between the dimension of certain unipotent representations of finite Chevalley groups of type G_2, F_4, and E_8 on the one hand, and the minimal polynomials of the Picard–Lefschetz monodromy on the other hand.

RÉSUMÉ. – Le but de cette note est d'établir une relation étrange entre la dimension de certaines représentations unipotentes de groupes de Chevalley finis de type G_2, F_4, et E_8 d'une part, et le polynôme minimal de la monodromie de Picard–Lefschetz d'autre part.

Introduction. The purpose of this note is to report a strange relation between the dimensions of certain unipotent representations of finite Chevalley groups of type G_2, F_4, and E_8 on the one hand, and the minimal polynomials of the Picard–Lefschetz monodromy on the other hand. We shall give this relation in §10 after explaining the concept and the notation which are necessary to state it.

1. Prehomogeneous vector spaces. First we recall some basic results in the theory of prehomogeneous vector spaces to fix notation. See [4, §1] for the detail.

Let G be a complex reductive group. A G-module V is called a prehomogeneous vector space if V has an open G-orbit. Let f be a polynomial function on V. We say f is a relative invariant with the character $\phi \in \mathrm{Hom}(G, \mathbb{C}^\times)$ if

$$f(gx) = \phi(g)f(x) \quad \text{for all } g \in G \text{ and } x \in V.$$

Let $V^\vee$ be the dual G-module of V. Then $V^\vee$ is again a prehomogeneous vector space and there exists a relative invariant $f^\vee \in \mathbb{C}[V^\vee]$ with the character ϕ^{-1}.

Now regard $\mathbb{C}[V^\vee] = \mathbb{C}[y_1, \ldots, y_n]$ as the ring of differential operators $\mathbb{C}[\partial] = \mathbb{C}[\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}]$ acting on $\mathbb{C}[V] = \mathbb{C}[x_1, \ldots, x_n]$.

2. b-Functions. Let f and $f^\vee$ be as in §1. Then the functional equation

$$f^\vee(\partial)f(x)^{s+1} = b_f(s)f(x)^s$$

holds with some polynomial $b_f(s) \in \mathbb{C}[s]$, which is called the b-function. This polynomial $b_f(s)$ is known [4, §2.5] to be the same as the b-function in the sense of I.N. Bernstein [1] up to scalar multiple. Hence by a result of M. Kashiwara [9], the b-function is of the form

$$b_f(s) = a_f \prod_{j=1}^{d} (s + \alpha_j)$$

with

$$\begin{aligned} &a_f \in \mathbb{C}^\times, \\ &d = \deg f = \deg f^\vee, \\ &\alpha_j \in \mathbb{Q}_{>0}. \end{aligned}$$

3. Exponential b-functions and Picard–Lefschetz monodromy. Using the above expression for the b-function, we consider the following modification, which we shall call *the exponential b-function*:

$$b_f^{\exp}(t) := \prod_{j=1}^{d} \left(t - \mathrm{e}^{2\pi\sqrt{-1}\alpha_j}\right).$$

The exponential b-functions are related to the Picard–Lefschetz monodromy as follows.

LEMMA. – *The exponential b-function $b_f^{\exp}(t)$ is the minimal polynomial of the Picard–Lefschetz monodromy of the nearby cycle sheaf $R\psi_f(\mathbb{C}) = R\psi_f(\mathbb{Q}) \otimes \mathbb{C}$, and hence a product of cyclotomic polynomials.*

See P. Deligne [2] for $R\psi_f(\mathbb{C})$. See B. Malgrange [15] and M. Kashiwara [10] for the D-module theoretic description of $R\psi_f(\mathbb{C})$, from which the above lemma follows; 'the minimal polynomial of the monodromy of $R\psi_f(\mathbb{C})$' is defined in terms of the D-module which corresponds to $R\psi_f(\mathbb{C})$ via the Riemann–Hilbert correspondence.

4. Contraction. Let notation be as in the first section. Put $V_f := V \setminus \{f = 0\}$. Then there exists a unique G-orbit O_f which is closed in V_f. Consider the isotropy group $G_{v_f} := \{g \in G \mid gv_f = v_f\}$ at a point $v_f \in O_f$. Then G_{v_f} is reductive. Take a maximal torus T_f of G_{v_f}. Put

$$G^{(f)} := Z_G(T_f)/T_f, \quad \text{and} \quad V^{(f)} := V^{T_f},$$

where $Z_G(T_f)$ denotes the centralizer, and V^{T_f} the space of T_f-fixed points.

LEMMA ([7, Theorem A], [8, §11]). –
(1) *The $G^{(f)}$-module $V^{(f)}$ is a prehomogeneous vector space. More precisely, v_f belongs to the open $G^{(f)}$-orbit.*
(2) *The exponential b-functions of f and its restriction to $V^{(f)}$ are the same*:

$$b^{\exp}_{f|_{V^{(f)}}}(t) = b_f^{\exp}(t).$$

We call $(G^{(f)}, V^{(f)})$ the *contraction* of the prehomogeneous vector space (G, V) with respect to the relative invariant f.

5. Exact prehomogeneous vector space. Let notation be as in the previous section. Here we assume that

$$\dim G = \dim V.$$

This condition is equivalent to saying that the generic isotropy group, i.e., the isotropy group G_{v_0} at a point v_0 belonging to the open G-orbit, is finite. A prehomogeneous vector space satisfying these equivalent conditions is called *exact*.

Remark. – The result of a contraction is always exact. On the other hand the contraction of an exact prehomogeneous vector space with respect to a generic f is necessarily trivial. In short, the exact prehomogeneous vector spaces are the 'dead end' resulting from the naive application of the contraction.

6. Singular contraction. Let (G,V) be an exact prehomogeneous vector space, and v_0 a point of the open G-orbit in V. Take an element $h \in G_{v_0}$. Put

$$G^{((h))} := Z_G(h) \quad (= \text{the centralizer of } h \text{ in } G), \quad \text{and}$$
$$V^{((h))} := V^h \quad (= \text{the space of the } h\text{-fixed points}).$$

LEMMA ([7, Theorem B]). – *The $G^{((h))}$-module $V^{((h))}$ is a prehomogeneous vector space.*

We call $(G^{((h))}, V^{((h))})$ the *singular contraction* of the prehomogeneous vector space (G,V) with respect to h. The singular contraction does not preserve the exponential b-function.

7. Prehomogeneous vector spaces arising from nilpotent orbits. In this section, we review the Dynkin–Kostant theory concerning the nilpotent orbits. See [16] for the detail.

Let $\tilde{G}$ be a complex semisimple Lie group and $\tilde{\mathfrak{g}}$ its Lie algebra. Take an sl_2-triple $\{N,H,N'\} \subset \tilde{\mathfrak{g}}$, i.e., a triple of elements such that $[N,N'] = H$, $[H,N] = 2N$, and $[H,N'] = -2N'$. Then N is a nilpotent element of $\tilde{\mathfrak{g}}$ and all the nilpotent orbits are obtained as $\mathrm{ad}\,\tilde{G}\cdot N$ from sl_2-triples in a unique way up to $\tilde{G}$-conjugacy. Put $\tilde{\mathfrak{g}}_j := \{x \in \tilde{\mathfrak{g}} \mid [H,x] = jx\}$. Then $\tilde{\mathfrak{g}}_0$ is a reductive Lie subalgebra of $\tilde{\mathfrak{g}}$. Let $G = \tilde{G}_0$ be the connected (reductive) subgroup of $\tilde{G}$ whose Lie algebra is $\tilde{\mathfrak{g}}_0$. Put $V := \tilde{\mathfrak{g}}_2$. Then V has a natural G-module structure, and is a prehomogeneous vector space. More precisely, the above nilpotent element N belongs to V and its G-orbit, say O, is open in V. Moreover the complement $V \setminus O$ is a hypersurface of V, i.e., purely of codimension 1 in V. Let f be the reduced defining polynomial of $V \setminus O$. Then f is a relative invariant on the G-module V. We shall be interested in the exponential b-functions of such relative invariants f.

8. Special cases. Let $\tilde{G}$ be of type G_2, F_4 or E_8 and N be the nilpotent element of $\tilde{\mathfrak{g}}$ such that the component group $\pi_0(Z_{\tilde{G}}(N))$ of the centralizer $Z_{\tilde{G}}(N)$ is isomorphic to $\mathcal{G} := \mathfrak{S}_3$, $\mathfrak{S}_4$ or $\mathfrak{S}_5$, respectively. Such N is unique up to $\tilde{G}$-conjugacy, and its weighted Dynkin diagram (cf. [16]) is

$$2 \Rrightarrow 0, \quad 02 \Rightarrow 00, \quad \underset{0}{0002000},$$

respectively. The unipotent orbits corresponding to these N's are the supports of certain cuspidal character sheaves on $\tilde{G}$; such unipotent orbit is uniquely determined for each exceptional simple algebraic group over $\mathbb{C}$, and plays an important role in the representation theory. See [13,14].

Now, as in §7, we can consider the prehomogeneous vector space (G,V) and the relative invariant f associated to such $\tilde{G}$ and N. Then (G,V) is an exact prehomogeneous vector space whose generic isotropy is isomorphic to the above symmetric group $\mathcal{G}$. Hence we can consider the singular contraction

$$\left(G^{((h))}, V^{((h))}\right) \quad \text{for each } h \in \mathcal{G},$$

the relative invariant

$$f^{((h))} := f|_{V^{((h))}},$$

and its exponential b-function

$$b^{\exp}_{f^{((h))}}(t).$$

9. Unipotent representations. In this section, we review the representation theory due to G. Lusztig [12].

Let $\tilde{G}(\mathbb{F}_q)$ be the group of $\mathbb{F}_q$-rational points of the simple algebraic group of the same type as in §8; $\tilde{G}$ is of type G_2, F_4 or E_8. Assume the characteristic of $\mathbb{F}_q$ is large. In Lusztig's theory on the complex irreducible representations of $\tilde{G}(\mathbb{F}_q)$, the unipotent representations are of basic importance. These are classified into several families. The set of families is parametrized by a certain class of nilpotent orbits in $\tilde{\mathfrak{g}}(\mathbb{C})$, called *special nilpotent orbits*. The adjoint orbits of N's which were considered in §8 are special. So there associates a family of unipotent representations of $\tilde{G}(\mathbb{F}_q)$. The representations in this family are parametrized by the set

$$\mathcal{M}(\mathcal{G}) := \left\{(h,\rho) \mid h \in \mathcal{G},\ \rho \in \operatorname{Irr}\left(Z_{\mathcal{G}}(h)\right)\right\}/\mathcal{G}\text{-conjugacy},$$

where $\mathcal{G}$ is the symmetric group given in §8, and $\operatorname{Irr}(Z_{\mathcal{G}}(h))$ denotes the set of the isomorphism classes of the irreducible representations of the centralizer $Z_{\mathcal{G}}(h)$ of h in $\mathcal{G}$. Let $R(h,\rho)$ be the unipotent representation of $\tilde{G}(\mathbb{F}_q)$ associated to $(h,\rho) \in \mathcal{M}(\mathcal{G})$. (See [12, Appendix] for the explicit correspondence $(h,\rho) \leftrightarrow R(h,\rho)$.) Then the dimension $\dim R(h,\rho)$ can be expressed as

$$\dim R(h,\rho) = \frac{\dim \rho}{|Z_{\mathcal{G}}(h)|} \cdot \frac{q^a \prod_{i=1}^{l}(q^{m_i+1}-1)}{d_{h,\rho}(q)},$$

where $m_1, \ldots, m_l$ are the exponents of $\tilde{G}$; a is 1, 4 or 16 according as $\tilde{G}$ is of type G_2, F_4 or E_8; and $d_{h,\rho}(t)$ is a polynomial in $\mathbb{Z}[t]$ which is independent of $\mathbb{F}_q$ and is not divisible by t. Now we are ready to explain our observation.

10. Observation. *The 'denominators' $d_{h,1}(t)$ of the dimensions of the unipotent representations $R(h,1)$ can be expressed in terms of the exponential b-functions as*

$$b^{\exp}_{f^{((h))}}(t) = d_{h,1}(t) \quad \textit{for all } h \in \mathcal{G}.$$

The author obtained this equality by a case study. At present it seems very strange to him. The left member is purely of topological nature; it is the minimal polynomial of the monodromy of the nearby cycle sheaf $R\psi_{f^{((h))}}(\mathbb{C})$ on the singular contraction $(\tilde{G}_0^{((h))}, \operatorname{ad}, \tilde{\mathfrak{g}}_2^{((h))})$. On the other hand the right member comes from an irreducible representation of the finite Chevalley group $\tilde{G}(\mathbb{F}_q)$. A naive generalization of the above equality is not true as the example in §11 shows; the author does not know how it sits in a more general picture.

In the following tables, we give the data which the above observation is based on, following the notation of G. Lusztig [12, §4.8 and Appendix]. In particular, ϕ_n denotes the nth cyclotomic polynomial, e.g., $\phi_3 = t^2 + t + 1$.

In the fourth column we use the weighted Dynkin diagram to describe prehomogeneous vector spaces. For example $2 \Rightarrow 02 \quad 2$ denotes the prehomogeneous vector space obtained as in §7 from the nilpotent orbit of $\mathfrak{g}(C_3) \times \mathfrak{g}(A_1)$ which is the product of the ones represented by $2 \Rightarrow 02$ and 2 of $\mathfrak{g}(C_3)$ and $\mathfrak{g}(A_1)$, respectively.

At the top of each table, we include the values of the exponents.

Type G_2 $\quad 1, 5$

h	$R(h,1)$	$\lvert Z_{\mathcal{G}}(h)\rvert$	$(G^{((h))}, V^{((h))})$	$b^{\exp}_{f^{((h))}}(t) = d_{h,1}(t)$
1^3	$[V]$	6	$2 \Rrightarrow 0$	$\phi_1^2\phi_6$
21	$[V']$	2	$2 \quad 2$	$\phi_1^2\phi_3$
3	$[\varepsilon_2]$	3	$2 \quad 2$	$\phi_1^2\phi_2^2$

Type F_4 $\quad 1, 5, 7, 11$

h	$R(h,1)$	$\lvert Z_{\mathcal{G}}(h)\rvert$	$(G^{((h))}, V^{((h))})$	$b^{\exp}_{f^{((h))}}(t) = d_{h,1}(t)$
1^4	$[12_1]$	24	$02 \Rightarrow 00$	$\phi_1^4\phi_4^2\phi_6^2$
21^2	$[16_1]$	4	$2 \Rightarrow 02 \quad 2$	$\phi_1^4\phi_3^2\phi_8$
31	$[6_1]$	3	$2 \quad 2 \quad 2 \quad 2$	$\phi_1^4\phi_2^4\phi_4^2$
4	$[4_3]$	4	$2 \quad 2 \quad 2 \quad 2$	$\phi_1^4\phi_2^2\phi_3^2\phi_4$
2^2	$[9_2]$	8	$202 \Rightarrow 0$	$\phi_1^4\phi_2^4\phi_6^2$

Type E_8 $\quad 1, 7, 11, 13, 17, 19, 23, 29$

h	$R(h,1)$	$\lvert Z_{\mathcal{G}}(h)\rvert$	$(G^{((h))}, V^{((h))})$	$b^{\exp}_{f^{((h))}}(t) = d_{h,1}(t)$
1^5	$[4480_y]$	120	0002000 0	$\phi_1^8\phi_3^4\phi_4^4\phi_6^4\phi_{10}^2$
21^3	$[7168_w]$	12	002002 2 0	$\phi_1^8\phi_3^4\phi_5^2\phi_6^2\phi_8^2\phi_{12}$
31^2	$[3150_y]$	6	2 2 20202 0	$\phi_1^8\phi_2^8\phi_3\phi_4^4\phi_9\phi_{10}^2$
41	$[1344_w]$	4	2 2 2 2 202 0	$\phi_1^8\phi_2^4\phi_3^4\phi_4^2\phi_5^2\phi_6^2\phi_{12}$
5	$[420_y]$	5	2 2 2 2 2 2 2 2	$\phi_1^8\phi_2^8\phi_3^4\phi_4^4\phi_6^4$
2^21	$[4200_y]$	8	2020020 0	$\phi_1^8\phi_2^8\phi_3^4\phi_6^4\phi_{10}^2$
32	$[2016_w]$	6	2 2 2 2 2 2 2 2	$\phi_1^8\phi_2^4\phi_3\phi_4^2\phi_5^2\phi_8^2\phi_9$

11. Example. We shall give the similar table as above for the nilpotent orbit of $N = \underset{0}{200200}$, which is a support of a cuspidal character sheaf. In this case, the component group $\pi_0(Z_{\tilde{G}}(N))$ as in §8 (for $\tilde{G}$ of adjoint type) is isomorphic to the group $\mathcal{G}$ whose $\mathcal{M}(\mathcal{G})$ parametrizes the family. Moreover both are isomorphic to the symmetric group $\mathfrak{S}_3$.

Type E_7 $\quad 1,5,7,9,11,13,17$

h	$R(h,1)$	$\lvert Z_{\mathcal{G}}(h)\rvert$	$(G^{((h))},V^{((h))})$	$b^{\exp}_{f^{((h))}}(t)$	$d_{h,1}(t)$
1^3	$315'_a$	6	$\underset{0}{200200}$	$\phi_1^7\phi_3^3\phi_4^2\phi_6^2$	$\phi_1^7\phi_2^7\phi_4^2\phi_6^3\phi_9$
21	280_b	2	$\underset{0}{2\,20202}$	$\phi_1^7\phi_2^4\phi_3^3\phi_6^2$	$\phi_1^7\phi_2^3\phi_3^3\phi_4^2\phi_8\phi_9$
3	$70'_a$	3	2 2 2 2 2 2 2	$\phi_1^7\phi_2^4\phi_3^3\phi_4^2$	$\phi_1^7\phi_2^7\phi_3^3\phi_4^2\phi_6^3$

12. Remark. The unipotent representation $R(h,1)$ ($h \in \mathcal{G}$) of $\tilde{G}(\mathbb{F}_q)$ appears as an irreducible constituent of the induced representation $\operatorname{ind}_{\tilde{B}(\mathbb{F}_q)}^{\tilde{G}(\mathbb{F}_q)} 1$ where 1 is the trivial representation of the Borel subgroup $\tilde{B}(\mathbb{F}_q)$. Hence it is associated to an irreducible representation, say $R_q(h,1)$, of the Iwahori–Hecke algebra $H_q(W)$, where W is the Weyl group. We know [11,3] (see also the references cited there) that the $\mathbb{C}$-algebra $H_q(W)$ and its representation $R_q(h,1)$ can be naturally deformed into a family $(H_\alpha(W), R_\alpha(h,1))_{\alpha\in\mathbb{C}^\times}$ of $\mathbb{C}$-algebras and their representations, and [18,6] that, for $1 \neq \alpha \in \mathbb{C}^\times$, the representation $R_\alpha(h,1)$ is irreducible and projective if and only if $d_{h,1}(\alpha) \neq 0$. Using our observation, we can state this also as follows: *for $1 \neq \alpha \in \mathbb{C}^\times$, the representation $R_\alpha(h,1)$ of $H_\alpha(W)$ is irreducible and projective if and only if $b^{\exp}_{f^{((h))}}(\alpha) \neq 0$.*

It would be worth recalling here a similar phenomenon [17,5]; *the irreducibility of generalized Verma modules, in place of $R_\alpha(h,1)$, would be conjecturally controlled by certain b-functions in a similar way as above.*

13. Remark. The phenomenon stated in §10 seems to be related to the following two properties of our orbit $O = \operatorname{ad}\tilde{G}\cdot N$.

(1) O is a distinguished nilpotent orbit, i.e., $Z_{\tilde{\mathfrak{g}}_0}(N) = 0$.

(2) $R(h,1)$ is self-dual (i.e., $R_1(h,1) \otimes \operatorname{sgn}_W \simeq R_1(h,1)$, where

$$R_1(h,1) := R_\alpha(h,1)|_{\alpha=1} \in \operatorname{Irr}(\mathbb{C}W)$$

in the notation of §12, and sgn_W is the sign character of W).

In fact from these two properties, we can deduce the identity

$$\deg b^{\exp}_{f^{((h))}}(t) = \deg d_{h,1}(t)$$

fairly intrinsically.

Proof. – On the one hand, we have

$$\deg b^{\exp}_{f^{((h))}}(t) = \deg f^{((h))} = \deg f = \dim \tilde{\mathfrak{g}}_0 \quad (= \dim \tilde{\mathfrak{g}}_2).$$

We can derive intrinsically the third equality from (1) using some results given in [8, §14]. On the other hand we can show that

$$\deg d_{h,1}(t) = \dim \tilde{\mathfrak{g}}_0$$

as follows. For any unipotent representation $R \in \mathrm{Irr}(\tilde{G}(\mathbb{F}_q))$ its dimension is known to be of the form

$$\dim R = \alpha(R)q^{a(R)} + \cdots + \beta(R)q^{b(R)},$$

where $\alpha(R), \beta(R) \in \mathbb{Q}$, $a(R), b(R) \in \mathbb{Z}$, and the omitted terms are of the form γq^c with some $\gamma \in \mathbb{Q}$ and $a(R) < c < b(R)$. From the condition (1), it follows that $\tilde{\mathfrak{g}}_i = 0$ for odd i and that O is special. Therefore a family of unipotent representations $\{R(h,\rho) \mid (h,\rho) \in \mathcal{M}(\mathcal{G})\}$ is associated to O, where $\mathcal{G}$ is a finite group determined by O. It is known that

$$a\bigl(R(h,\rho)\bigr) = \dim \tilde{\mathcal{B}}^N \quad \text{for all } (h,\rho) \in \mathcal{M}(\mathcal{G}),$$

where $\tilde{\mathcal{B}}^N$ is the variety of Borel subalgebras of $\tilde{\mathfrak{g}}$ containing N ($\in O$). Moreover from (2), it follows that

$$a\bigl(R(h,q)\bigr) + b\bigl(R(h,1)\bigr) = \sum_{i=1}^{l} m_i \quad (= \#\{\text{positive roots}\}).$$

Therefore we get

$$\begin{aligned}\dim d_{h,1}(t) &= a\bigl(R(h,1)\bigr) + \sum_{i=1}^{l}(m_i+1) - b\bigl(R(h,1)\bigr)\\ &= 2a\bigl(R(h,1)\bigr) + l\\ &= 2\dim \tilde{\mathcal{B}}^N + l = \dim Z_{\tilde{\mathfrak{g}}}(N)\\ &= \dim \tilde{\mathfrak{g}}_0 + \dim \tilde{\mathfrak{g}}_1 = \dim \tilde{\mathfrak{g}}_0. \qquad \square\end{aligned}$$

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