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APPLICATIONS OF DECOMPOSITIONS OF HOLOMORPHIC FUNCTIONS TO  
PARTIAL DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

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NOTATIONS.

We consider  $\mathbb{R}^n$  endowed with the usual scalar product defined by  $\langle y, \xi \rangle = \sum y_j \xi_j$  and the euclidean norm  $|y| = \sqrt{\langle y, y \rangle}$  as a closed submanifold of  $\mathbb{C}^n$ . We shall denote by  $S^{n-1}$  the unit sphere of  $\mathbb{R}^n$  and for any cone  $\Gamma \subset \mathbb{R}^n$  we define the polar of  $\Gamma$  by  $\Gamma^\Delta = \{\xi \in \mathbb{R}^n \setminus \{0\} : \langle y, \xi \rangle \geq 0, \forall y \in \Gamma\}$ .

By a salient cone, we mean a cone that does not contain any straight line. Given an open subset  $\Omega$  of  $\mathbb{R}^n$  and an open convex cone  $\Gamma \subset \mathbb{R}^n$ , a subset  $A$  of  $\mathbb{C}^n$  will be called of profile  $\Omega + i\Gamma$  if for every compact sets  $K \subset \Omega$  and  $\mathcal{K} \subset \Gamma \cap S^{n-1}$ , there exists  $\rho_0 > 0$  such that the wedge

$$\{x + i\rho y : x \in K, y \in \mathcal{K}, \rho \in ]0, \rho_0]\}$$

is contained in  $A$ . We are going to represent by  $\mathcal{D}$  the ring of linear partial differential operators with constant complex coefficients. It is well known that  $\mathcal{D}$  is unitary and noetherian. If  $P = \sum_{|\alpha| \leq m} a_\alpha D^\alpha$  belongs to  $\mathcal{D}$ , we shall write  $\overset{\circ}{P}$  the principal symbol of  $P$  and  $\text{car}(P)$  the characteristic variety of  $P$ , i.e. the set  $\{\xi + i\eta \in \mathbb{C}^n : |\xi|^2 + |\eta|^2 = 1, P(\xi + i\eta) = 0\}$ . Finally, let us denote by  $\mathcal{O}$  the sheaf of holomorphic functions on  $\mathbb{C}^n$  and by  $\mathcal{A}$  the linear space of  $\mathbb{C}$ -valued analytic functions on  $\mathbb{R}^n$ .

Let us first recall two decomposition theorems proved in [5] and [6].

Theorem 1: For any  $F \in \mathcal{B}$  and any finite family of open convex salient cones  $\Gamma_j$  of  $\mathbb{R}^n$  whose polars cover  $S^{n-1}$ , there exist domains of holomorphy  $V_j$  containing  $\mathbb{R}^n$  and an open convex tube  $\mathbb{R}^n + i\Omega_j$  of profile  $\mathbb{R}^n + i\Gamma_j$  and  $F_j \in \mathcal{O}(V_j)$  such that  $F = \sum F_j$  holds on a neighborhood of  $\mathbb{R}^n$ . Moreover, if the interiors of the polars of the  $\Gamma_j$ 's cover  $S^{n-1}$ , given  $r \in ]0, +\infty[$ , one can assume that the  $V_j$ 's are open pseudoconvex neighborhoods of the closed tubes

$$\mathbb{R}^n + i\{y \in \bar{\Gamma}_j : |y| \leq r\}.$$

Theorem 2: Let  $\Gamma$  be an open convex cone of  $\mathbb{R}^n$ ,  $\Omega$  an open subset of  $\mathbb{R}^n$  and  $V$  an open subset of  $\mathbb{C}^n$  of profile  $\Omega + i\Gamma$ . For any  $r \in ]0, +\infty[$ , any  $F \in \mathcal{O}(V)$  and any open subcone  $\Gamma'$  of  $\Gamma$  whose intersection with  $S^{n-1}$  is relatively compact in  $\Gamma$ , there exist an open convex neighborhood  $\Omega'$  of  $\{y \in \bar{\Gamma}' : 0 < |y| \leq r\}$  in  $\mathbb{R}^n$ , an open pseudoconvex neighborhood  $W$  of  $\Omega$  contained in  $\Omega + i\mathbb{R}^n$ ,  $A \in \mathcal{O}(W)$  and  $G \in \mathcal{O}(\mathbb{R}^n + i\Omega')$  such that  $W \cap (\mathbb{R}^n + i\Omega') \subset V$  and  $F = G + A$  on  $W \cap (\mathbb{R}^n + i\Omega')$ .

Remark 3: This last statement constitutes in fact a slight improvement of the result obtained in [6]. To establish it, one only needs (besides evident modifications) to remark that lemma 6 of [5] can be precised as follows: if the  $U_j$ 's are strictly pseudoconvex tubes with  $C_2$ -boundaries  $\partial U_j \supset \mathbb{R}^n$ , for any complex neighborhood  $W$  of an open set  $\Omega$  of  $\mathbb{R}^n$ , there exists an open pseudoconvex neighborhood  $V_W$  of  $\Omega \cup (\cap U_j)$  such that  $V_W \setminus (\cap U_j) \subset W$ .

Lemma 4: Let  $\Gamma$  be an open convex non void cone of  $\mathbb{R}^n$ . The polar of  $\mathbb{R}^n \times \Gamma$  coincides with  $0 \times \Gamma^\perp$  and is a convex salient cone closed in  $\mathbb{R}^{n+n'} \setminus \{0\}$ . Moreover, for any open cone  $\gamma \supset 0 \times \Gamma^\perp$ , there exists  $\rho > 0$  such that  $\gamma^\perp$  is contained in

$$\tilde{\Gamma}_\rho = \{(x, y) : |x| \leq \frac{|y|}{\rho}, y \neq 0, d(\frac{y}{|y|}, S^{n-1} \setminus \Gamma) \geq \rho\},$$

where  $d$  denotes the euclidean distance.

Proof: One obtains immediately the equality  $(\mathbb{R}^n \times \Gamma)^\perp = 0 \times \Gamma^\perp$  and as  $\Gamma$  is open and non void, this is a salient cone closed in  $\mathbb{R}^{n+n'} \setminus \{0\}$ .

For the second part of the statement, let us first choose  $\varepsilon > 0$  such that

$$\omega_\varepsilon = \{(\xi, \eta) \in S^{n+n'-1} : |\xi| < \varepsilon, d\left(\frac{\eta}{|\eta|}, \Gamma^\perp \cap S^{n'-1}\right) < \varepsilon\} \subset \gamma,$$

hence such that  $\gamma^\perp$  is contained in

$$\Gamma' = \bigcap_{(\xi, \eta) \in \omega_\varepsilon} \{(x, y) : \langle x, \xi \rangle + \langle y, \eta \rangle \geq 0\}.$$

As it is clear by definition of  $\omega_\varepsilon$  that  $\Gamma' \setminus \{0\}$  is disjoint from

$\{(x, 0) : x \in \mathbb{R}^n\}$ , we are going to show that the existence of sequences  $(x_m, y_m)$  or  $(x'_m, y'_m) \in \Gamma' \setminus \{0\}$  such that  $|x_m| > m|y_m|$  or such that  $|y'_m| = 1$  and

$d(y'_m, S^{n'-1} \setminus \Gamma) < \frac{1}{m}$  leads to a contradiction in each case. As a matter of fact,

for any  $\eta_0 \in \Gamma^\perp \cap S^{n'-1}$ , the points  $(\xi_m, \eta_m) = \frac{1}{\sqrt{1+m^2}} \left(-\frac{x_m}{|x_m|}, m\eta_0\right)$  belong to

$\omega_\varepsilon$  for  $m$  sufficiently large and one obtains

$$0 \leq \langle x_m, \xi_m \rangle + \langle y_m, \eta_m \rangle < -\frac{m|y_m|}{\sqrt{1+m^2}} + \frac{m|y_m|}{\sqrt{1+m^2}} = 0,$$

hence a first contradiction. In the second case, we can find  $y''_m \in S^{n'-1} \setminus \bar{\Gamma}$  such

that  $|y'_m - y''_m| < \frac{1}{m}$  and by convexity of  $\bar{\Gamma}$ , there are points  $\eta'_m \in \Gamma^\perp \cap S^{n'-1}$

verifying  $\langle y''_m, \eta'_m \rangle < 0$ .

As the points  $(\xi_m, \eta_m) = \frac{1}{\sqrt{1+m^2}} (0, m\eta'_m - y'_m)$  belong to  $\omega_\varepsilon$  for  $m$

sufficiently large, we obtain another contradiction:

$$0 \leq \langle x'_m, \xi_m \rangle + \langle y'_m, \eta_m \rangle \leq \frac{1}{\sqrt{1+m^2}} [m \langle y'_m - y''_m, \eta'_m \rangle + m \langle y''_m, \eta'_m \rangle - 1]$$

$$< \frac{1}{\sqrt{1+m^2}} [m |y'_m - y''_m| - 1] \leq 0.$$

Definition 4: Adapting a definition due to Bony and Schapira (cf. [1]) to the particular situation we have in mind, we shall say that a point  $z_0$  of the boundary of an open subset  $V$  of  $\mathbb{C}^n$  verifies the condition  $c(z_0, \gamma, V)$  with respect to a convex salient cone  $\gamma$  closed in  $\mathbb{R}^n \setminus \{0\}$  if for any  $\varepsilon > 0$ , there exists  $\eta > 0$  such that

$$\{z \in V : |z - z_0| < \eta\} - i\{y \in (\gamma_\varepsilon)^\perp : |y| < \eta\}$$

is contained in  $V$ , where  $\gamma_\varepsilon$  denotes the conic hull of the set of points of  $S^{n-1}$  whose distance to  $\gamma \cap S^{n-1}$  is less than  $\varepsilon$ .

Lemma 5: a) If  $\Gamma \subset \mathbb{R}^n$  is an open convex non void cone and  $V = \mathbb{R}^n + i\Omega$  an open tube of  $\mathbb{C}^n$ , then for any  $z_0 \in \partial V$ , the condition  $c(z_0, -\Gamma^\perp, V)$  is equivalent to the cone condition  $C(z_0, (0 \times \Gamma^\perp) \cap S^{2n-1})$  stated in 4.1 of [1].

b) Let  $r$  belong to  $]0, +\infty[$ ,  $\Gamma$  be an open convex non void cone of  $\mathbb{R}^n$  and  $V$  denote the open tube  $\{z \in \mathbb{R}^n + i\Gamma : |y| < r\}$ . Every  $x \in \mathbb{R}^n \subset \partial V$  verifies then the condition  $c(x, -\Gamma^\perp, V)$ . Moreover, if  $\gamma$  is an open conic neighborhood of  $(-\Gamma^\perp) \setminus \{0\}$  and if  $V' = \mathbb{R}^n + i\Omega$  is an open tube of profile  $\mathbb{R}^n + i\Gamma'$ , there exists an open convex subcone  $\Gamma'$  of  $\Gamma$  such that  $-\Gamma'^\perp \subset \gamma$  and any  $x \in \mathbb{R}^n \subset \partial[V' \cap (\mathbb{R}^n + i\Gamma')]$  verifies the condition  $c[x, -\Gamma'^\perp, V' \cap (\mathbb{R}^n + i\Gamma')]$

Proof: a) We first show that  $c$  implies  $C$ . Let  $I'$  be an open neighborhood of  $(0 \times \Gamma^\perp) \cap S^{2n-1}$  in  $S^{2n-1}$ . By Lemma 4, the polar of  $I'$  in the sense of Bony and Schapira (which is the opposite of ours) is contained in  $-\tilde{\Gamma}'_\rho$  for some  $\rho > 0$ . There exists  $\varepsilon > 0$  such that  $(\Gamma^\perp_\varepsilon)^\perp$  contains  $\{y \in S^{n-1} : d(y, S^{n-1} \setminus \Gamma) \geq \rho\}$ . Now let  $\eta$  be the number which corresponds to  $\varepsilon$  by application of  $c(z_0, -\Gamma^\perp, V)$ . For  $V' = \{z : |z - z_0| < \frac{\eta}{2}\}$  and the set  $A$  of the points of the polar of  $I'$  (in the sense of [1]) whose module is less than  $\frac{\eta}{2}$ , the inclusion  $(V' \cap V) + A \subset V$  is easy to obtain because  $V$  is a tube.

The proof of  $C \rightarrow c$  is similar and as we shall not use this implication in what follows, we do not give further details.

b) The first assertion follows immediately from the inclusion

$[(\Gamma^{\perp})_{\varepsilon}^{\perp}]^{\perp} \subset \Gamma$ , which is easy to obtain. For the second one, let us denote by  $\gamma'$  an open convex cone of  $\mathbb{R}^n$  verifying

$$-\Gamma^{\perp} \subset \gamma' \subset \overline{\gamma'} \setminus \{0\} \subset \gamma$$

and set  $\Gamma' = -\gamma'^{\perp 0}$ . One has evidently  $\overline{\Gamma'} = -\gamma'^{\perp} = -\overline{\gamma'}^{\perp} \subset \Gamma^{\perp \perp 0} = \Gamma$  and  $-\Gamma'^{\perp} = \overline{\gamma'} \setminus \{0\} \subset \gamma$  and as  $\mathbb{R}^n + i\Omega$  is of profile  $\mathbb{R}^n + i\Gamma$ , there exists  $\rho_0 > 0$  such that  $\{y \in \Gamma' : |y| < \rho_0\} \subset \Omega$ . Hence the conclusion by application of the first part of this result.

Theorem 6: Let  $P \in \mathcal{D}$ ,  $r \in ]0, +\infty[$  and  $\Gamma_j$  be a finite family of open convex non void cones of  $\mathbb{R}^n$ . If  $\text{car}(P) \cap [-i \cup \Gamma_j^{\perp}]$  is empty, the equation  $Pu = f$  is solvable in the subspace of  $\mathcal{A}$ , whose elements can be written  $\Sigma F_j|_{\mathbb{R}^n}$  with  $F_j \in \mathcal{A} \cap \mathcal{O}[\mathbb{R}^n + i\{y \in \Gamma_j : |y| < r\}]$ .

If there moreover exists an open convex conic neighborhood  $\gamma$  of  $-U\Gamma_j^{\perp}$  such that  $\text{car}(P) \cap i\gamma$  is empty, the same equation is solvable in the subspace of  $\mathcal{A}$ , whose elements can be written  $\Sigma F_j|_{\mathbb{R}^n}$  with  $F_j \in \mathcal{A} \cap \mathcal{O}(V_j)$ , where  $V_j$  denotes an open convex tube of profile  $\mathbb{R}^n + i\Gamma_j$ .

In particular, when  $P$  is  $\xi_0$ -hyperbolic in the sense that it verifies the following two conditions

- a)  $\mathring{P}(\xi_0) \neq 0$
- b)  $P$  does not vanish on  $\mathbb{R}^n + i\{\lambda\xi_0 : \lambda > c\}$  for some  $c \geq 0$ ,

the second situation occurs if  $-U\Gamma_j^{\perp}$  is contained in the open convex cone

$$\gamma_P = \bigcap_{\lambda \geq 0} \{\xi \in \mathbb{R}^n : P(\xi + \lambda\xi_0) \neq 0\}$$

Proof: According to the Malgrange-Ehrenpreis theorem (cf. [4] or [7]), the equations  $PU_j = F_j$  are solvable in both cases in a convex tube of profile

$\mathbb{R}^n + i\Gamma_j$ . By the precedent lemma, we can apply theorem 4.1 of [1] and therefore suppose that the  $U_j$ 's are also holomorphic on a neighborhood of  $\mathbb{R}^n$ . Hence the conclusion by linearity of  $P$ .

The third assertion is a direct consequence of a slight modification of Gårding's well known result on hyperbolic polynomials (cf. [2]) which asserts under our hypotheses that  $\overset{\circ}{P}$  does not vanish on  $\mathbb{R}^n + i\gamma_P$ .

Remark 7: a) Our definition of  $\xi_0$ -hyperbolicity differs from Gårding's one because we interchange the roles of the real and imaginary parts of the complex directions.

b) When  $P$  is  $\xi_0$ -hyperbolic, it is easy to prove by Hurwitz's theorem that  $\gamma_P$  coincides with the connected component of

$$\{\xi \in \mathbb{R}^n : \overset{\circ}{P}(\xi) = 0\}$$

that contains  $\xi_0$ .

c) Combining theorems 1 and 6, one obtains immediately the following well known result:

Corollary 8: If  $P \in \mathcal{D}$  is elliptic, one has  $P(D)\mathcal{A} = \mathcal{A}$ .

Proof: One only needs to point out that  $\text{car}(P)$  does not meet  $i\mathbb{R}^n$ .

Proposition 9: Let  $\Gamma \subset \mathbb{R}^n$  be a closed convex cone with non void interior and  $\mathcal{M}$  be a finitely generated  $\mathcal{D}$ -module. For any  $r \in ]0, +\infty[$ , one has  $\text{Ext}_{\mathcal{D}}^j[\mathcal{M}, \mathcal{O}_{(F_{\Gamma,r})}] = 0$  for all  $j \geq 2$ , where  $F_{\Gamma,r}$  denotes

$$\{x + iy \in \mathbb{R}^n + i\Gamma : |y| \leq r\}.$$

Proof: The  $K_m = \{x + iy \in \mathbb{R}^n + i\Gamma : |x| \leq m, |y| \leq r\}$  ( $m \in \mathbb{N}$ ) form a sequence of compact convex sets which increases towards  $F_{\Gamma,r}$  in such a way that one has

$$\begin{aligned} \mathcal{O}_{(F_{\Gamma,r})} &= \varprojlim_m \mathcal{O}_{(K_m)} \text{ and consequently } \varprojlim_m \text{Hom}_{\mathcal{D}}[\mathcal{D}^N, \mathcal{O}_{(K_m)}] = \\ &= \text{Hom}_{\mathcal{D}}[\mathcal{D}^N, \mathcal{O}_{(F_{\Gamma,r})}] \text{ for any } N \in \mathbb{N}. \text{ Let} \end{aligned}$$

$$0 \leftarrow \mathcal{M} \leftarrow \mathcal{D}^{r_0} \xleftarrow{t_{\psi_0}} \mathcal{D}^{r_1} \xleftarrow{t_{\psi_1}} \dots \xleftarrow{t_{\psi_{q-1}}} \mathcal{D}^{r_q} \leftarrow 0 \quad (q \leq n)$$

be a free projective resolution of  $\mathcal{M}$  and consider the commutative diagram

$$\begin{array}{ccccccc}
 0 \rightarrow \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_0}, \mathcal{O}_{(\Gamma, r)}] & \rightarrow & \dots & \rightarrow & \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_q}, \mathcal{O}_{(\Gamma, r)}] & \rightarrow & 0 \\
 \downarrow \vdots & & & & \downarrow \vdots & & \\
 0 \rightarrow \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_0}, \mathcal{O}_{(K_{m+1})}] & \rightarrow & \dots & \rightarrow & \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_q}, \mathcal{O}_{(K_{m+1})}] & \rightarrow & 0 \\
 \downarrow & & & & \downarrow & & \\
 0 \rightarrow \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_0}, \mathcal{O}_{(K_m)}] & \rightarrow & \dots & \rightarrow & \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_q}, \mathcal{O}_{(K_m)}] & \rightarrow & 0 \\
 \downarrow \vdots & & & & \downarrow \vdots & & 
 \end{array}$$

A well known result (cf. [3], p. 410, for example) asserts that the canonical maps

$$\phi^j : \text{Ext}_{\mathcal{D}}^j[\mathcal{M}, \mathcal{O}_{(\Gamma, r)}] \rightarrow \varprojlim_m \text{Ext}_{\mathcal{D}}^j[\mathcal{M}, \mathcal{O}_{(K_m)}] , \quad \forall j \geq 2$$

are isomorphisms because Mittag-Leffler's condition is satisfied since we have for every  $j \geq 1$  and  $m \in \mathbb{N}$

$$H^j(0 \rightarrow \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_0}, \mathcal{O}_{(K_m)}] \rightarrow \dots \rightarrow \text{Hom}_{\mathcal{D}}[\mathcal{D}^{r_q}, \mathcal{O}_{(K_m)}] \rightarrow 0) = \text{Ext}_{\mathcal{D}}^j[\mathcal{M}, \mathcal{O}_{(K_m)}] = 0$$

by virtue of the Malgrange-Ehrenpreis theorem. Hence the conclusion.

Notation 10: If  $\Gamma$  is an open convex non void cone of  $\mathbb{R}^n$ , we set

$$\tilde{\mathcal{A}}_{\Gamma} = \varinjlim_{V} \mathcal{O}_{(V)} ,$$

where  $V$  runs over the open subsets of  $\mathbb{C}^n$  of profile  $\mathbb{R}^n + i\Gamma$ . We shall also denote by  $\mathcal{F}_{\Gamma}$  the family of open convex non void subcones  $\Gamma'$  of  $\Gamma$  such that  $\Gamma' \cap S^{n-1}$  is relatively compact in  $\Gamma$ .



Theorem 11: For any open convex non void cone  $\Gamma$  of  $\mathbb{R}^n$  and any finitely generated  $\mathcal{D}$ -module  $\mathcal{M}$ , one has

$$\varprojlim_{\Gamma' \in \mathcal{F}_\Gamma} \text{Ext}_{\mathcal{D}}^j[\mathcal{M}, \tilde{\mathcal{A}}_{\Gamma', \mathcal{A}}] = 0, \quad \forall j \geq 1.$$

Proof: Given any  $\Gamma' \in \mathcal{F}_\Gamma$ , we are going to establish that for every  $j \geq 1$  and every  $\Gamma'' \in \mathcal{F}_\Gamma$  such that  $\Gamma' \in \mathcal{F}_{\Gamma''}$ , the canonical operator

$$\text{Ext}_{\mathcal{D}}^j[\mathcal{M}, \tilde{\mathcal{A}}_{\Gamma'', \mathcal{A}}] \rightarrow \text{Ext}_{\mathcal{D}}^j[\mathcal{M}, \tilde{\mathcal{A}}_{\Gamma', \mathcal{A}}]$$

vanishes. Using the same notations as in the precedent proof to denote the free projective resolution of  $\mathcal{M}$ , we are lead to prove that the image of

$$(\tilde{\mathcal{A}}_{\Gamma'', \mathcal{A}})^{r_{j-1}} \xrightarrow{\psi_{j-1}} (\tilde{\mathcal{A}}_{\Gamma'', \mathcal{A}})^{r_j} \xrightarrow{\psi_j} (\tilde{\mathcal{A}}_{\Gamma'', \mathcal{A}})^{r_{j+1}}$$

in the complex

$$(\tilde{\mathcal{A}}_{\Gamma', \mathcal{A}})^{r_{j-1}} \xrightarrow{\psi_{j-1}} (\tilde{\mathcal{A}}_{\Gamma', \mathcal{A}})^{r_j} \xrightarrow{\psi_j} (\tilde{\mathcal{A}}_{\Gamma', \mathcal{A}})^{r_{j+1}}$$

is exact. In other words, we have to prove that given any  $F \in \tilde{\mathcal{A}}_{\Gamma''}^{r_j}$  verifying

$$\psi_j F = 0 \text{ mod } \mathcal{A}, \quad (1)$$

there exists  $U \in \tilde{\mathcal{A}}_{\Gamma'}^{r_{j-1}}$  such that

$$\psi_{j-1} U = F \text{ mod } \mathcal{A}. \quad (2)$$

By theorem 2, we can decompose  $F$  in  $G + A$  with

$G \in \mathcal{O}[\mathbb{R}^n + i\{y \in \bar{\Gamma}' : 0 < |y| \leq r\}]$  and  $A \in \mathcal{A}$ . Therefore (1) and (2)

become respectively

$$\psi_j G \in \mathcal{O}_{(\bar{\Gamma}', r)}^{r_{j+1}} \quad (3)$$

$$\psi_{j-1} U = G \text{ mod } \mathcal{A}. \quad (4)$$

As we have trivially  $\psi_{j+1} \psi_j G = 0$  the precedent lemma assures the existence of  $H \in \mathcal{O}_{(F_{\Gamma', r})}^{r^j}$  such that  $\psi_j H = \psi_j G$  and we can replace (3) and (4) respectively by

$$\begin{aligned} \psi_j(G - H) &= 0 \text{ on } F_{\Gamma', r} \\ \psi_{j-1}U &= G - H \text{ mod } \mathcal{A}. \end{aligned}$$

Since  $\mathbb{R}^n + i\{y \in \Gamma' : |y| < r\}$  is an open convex subset of  $F_{\Gamma', r}$ , another application of the Malgrange-Ehrenpreis theorem allows to conclude.

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R E F E R E N C E S

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