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Théorie de l'homotopie - Colloque CNRS-NSF-SMF au C.I.R.M. du 11 au 15 juillet 1988 organisé par Haynes R. Miller (M.I.T.), Jean-Michel Lemaire (Nice) Lionel Schwartz (Paris-sud), Michel Zisman (Paris VII) - Pages préliminaires

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THÉORIE DE L'HOMOTOPIE

Colloque CNRS-NSF-SMF au C.I.R.M.

du 11 au 15 juillet 1988

organisé par

Haynes R. MILLER (M.I.T.), Jean-Michel LEMAIRE (Nice)

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SOCIÉTÉ MATHÉMATIQUE DE FRANCE

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INTRODUCTION

On trouvera dans ce volume les articles issus du colloque international sur la théorie de l'homotopie qui s'est tenu au Centre International de Rencontres Mathématiques de Luminy du 11 au 15 juillet 1988.

La théorie de l'homotopie a connu dans les années 80 des développements spectaculaires; le but du colloque était de faire le point sur un certain nombre de ces développements et leurs perspectives: deux thèmes principaux ont été particulièrement évoqués pendant le colloque et se retrouvent dans la plupart des articles de ce volume:

- l'étude de l'homotopie des espaces fonctionnels, notamment lorsque la source est l'espace classifiant d'un groupe fini ou de Lie, dont la solution des conjectures de Segal et de Sullivan a montré la richesse. Au plan des méthodes, l'accent a été mis sur les suites spectrales d'Adams instables et sur les modules instables sur l'algèbre de Steenrod.

- l'utilisation et la construction de modèles algébriques de l'homotopie, particulièrement performants en homotopie rationnelle, mais aussi à présent avec des coefficients plus généraux, à savoir des corps quelconques ou des sous-anneaux appropriés de \mathbb{Q} .

Ce colloque a permis de réunir une bonne part des spécialistes américains et européens du sujet, et a bénéficié à ce titre d'un soutien important de la National Science Foundation et du Centre National de la Recherche Scientifique dans le cadre de leur accord de coopération; il a également bénéficié de l'aide financière des Universités de Paris VII, Paris-Sud et Nice, et bien entendu de celle de la SMF sous la forme de la subvention accordée par le Conseil Scientifique du CIRM et de la publication du présent volume.

Les organisateurs tiennent à exprimer leur reconnaissance à toutes ces institutions, ainsi qu'au personnel du C.I.R.M., grâce auquel cette semaine a été - comme toujours - aussi riche en échanges scientifiques et humains qu'agréable sur le plan matériel.

INTRODUCTION

This volume contains papers submitted at the international conference on Homotopy Theory held at the C.I.R.M. in Marseille-Luminy from July 11 to 15, 1988.

Striking progress has been achieved in homotopy theory during the eighties. The purpose of this conference was to survey some of these achievements and their prospects: two main subjects were especially discussed during the conference and can be found in most contributions to this volume:

-The study of the homotopy type of function spaces, in particular when the domain is the classifying space of a finite group or a Lie group, whose richness has been revealed by the proofs of Segal's and Sullivan's conjectures. Among the methods used in this field, unstable Adams spectral sequences and unstable modules over the Steenrod algebra were emphasized.

-The use and construction of algebraic models of homotopy types, which have proven themselves especially fruitful in rational homotopy theory, but also now with more general coefficient rings, namely arbitrary fields and suitable subrings of \mathbf{Q} .

A fair number of the experts in the field from America and Europe had the opportunity to meet at this conference, which was indeed largely supported by the NSF-CNRS cooperation agreement. Financial support was also provided by the Universities of Paris VII, Paris-Sud and Nice, and by the Mathematical Society of France through the grant distributed by the Scientific Committee of the C.I.R.M. and through the publication of this volume of Astérisque.

The organizers of this conference are pleased to express their warmest thanks to all sponsoring institutions, and to the C.I.R.M. staff who did their usual best to make that week a most profitable and pleasant one.

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RÉSUMÉS DES ARTICLES

ANICK David J. & DROR-FARJOUN Emmanuel - *On the space of maps between R-local CW complexes.*

Over a subring R of the rationals, we construct a simplicial skeleton for the space of pointed maps between two R -local simply-connected CW complexes. The construction makes use of an R -local DG Lie algebra model for spaces.

AVRAMOV Luchezar & FÉLIX Yves - *Espaces de Golod.*

Nous considérons des fibrations nilpotentes $F \rightarrow E \rightarrow B$, où E et B sont des CW complexes finis simplement connexes. Un espace X est dit *de Golod* s'il existe un entier n tel que le revêtement n -connexe de X ait le type d'homotopie rationnelle d'un bouquet de sphères. Cette notion topologique correspond à celle des anneaux de Golod en algèbre locale. Nous montrons que si la base B de la fibration est un espace de Golod, la série de Poincaré de la fibre F est rationnelle.

BAKER Andrew - *Exotic multiplications on Morava K-theories and their liftings.*

For each prime p and (finite) integer $n > 0$, there is a ring spectrum $K(n)$ called the n -th Morava K-theory at p . We discuss exotic multiplications upon $K(n)$ and their liftings to certain characteristic zero spectra $E\hat{(n)}$.

BROWN Edward H. & SZCZARBA R.H. - *Continuous homology and real homotopy type II.*

In our earlier paper "Continuous homology and real homotopy type", we studied localization of simplicial spaces at the reals and established an equivalence between the category of free nilpotent differential graded commutative algebras of finite type over the reals, and nilpotent simplicial spaces of finite type localized at the reals. In this paper, we extend these results by eliminating the nilpotent condition on the algebraic side, thus proving a conjecture of Sullivan. The main technical work consists in introducing local coefficients into continuous cohomology, continuous de Rham cohomology, the Serre spectral sequence and the constructions involved in real homotopy type.

CRABB Michael C. - *The Fuller index and T-equivariant stable homotopy theory.*

In 1967, F.B. Fuller introduced a remarkable index for counting periodic orbits of smooth flows. It has become apparent in recent work of J. Ize and E.N. Dancer that the natural setting for Fuller's index is

\mathbb{T} -equivariant homotopy theory, where \mathbb{T} is the circle group. This paper describes their work in the conventional framework of equivariant stable homotopy theory over a base and index theory for fixed-points of maps and zeroes of vector fields.

DROR-FARJOUN Emmanuel & SMITH Jeffrey - *A geometric interpretation of Lannes' functor T.*

In this paper we prove a version of a conjecture of Lannes concerning the mod. p cohomology of the space of maps from $B\mathbb{Z}/p\mathbb{Z}$ to a rather general space X . This gives a topological meaning to an algebraic functor for modules over the Steenrod algebra, defined by Lannes. That functor has proven very useful in understanding spaces of maps from classifying spaces. As a corollary we get new proofs of several results of Lannes.

DWYER William G. & WILKERSON Clarence - *Spaces of null homotopic maps.*

We study the null component of the space of pointed maps from $B\pi$ to X when π is a locally finite group, and other components of the mapping space when π is elementary abelian. Results about the null component are used to give a general criterion for the existence of torsion in arbitrary high dimensions in the homotopy of X .

GOERSS Paul G.- *André-Quillen cohomology and the Bousfield-Kan spectral sequence.*

This paper undertakes to exploit the observation that the non-abelian homological algebra of Quillen and, in particular, the commutative algebra cohomology of André and Quillen provides a framework for discussing the unstable Adams spectral sequence of Bousfield and Kan. We take this observation in a variety of directions; for instance, we show that the long exact "transitivity sequence" in André-Quillen cohomology is related to the homotopy long exact sequence of a fibration, and we show that a product in André-Quillen cohomology can be used to compute the Whitehead product in homotopy.

HENN Hans-Werner. - *Cohomological p -nilpotence criteria for compact Lie groups.*

We introduce the concept of a p -nilpotent compact Lie group and discuss various group theoretical characterisations of such groups. These characterizations are then used to generalize cohomological p -nilpotence criteria for finite groups due to Atiyah and Quillen to the case of compact Lie groups.

MARKL Martin - *The rigidity of Poincaré duality algebras and classification of homotopy types of manifolds.*

We prove that Poincaré duality algebras are characterized by a certain rigidity property. As a consequence of this fact, we show that the k -isomorphism class of a Poincaré duality algebra H^* of top dimension n is uniquely determined by the factor H^*/H^n , provided k is algebraically closed. Using this and usual methods of descent theory, we obtain a description of the set of k -isomorphism classes of Poincaré duality algebras with the same given isomorphism class of H^*/H^n , for any field k of characteristic zero. These results are then applied to the study of homotopy types of simply connected Poincaré duality spaces.

MAY J. Peter - *Some remarks on equivariant bundles and classifying spaces.*

A number of results are given on the relationship between equivariant and non-equivariant bundles and their classification. The bundles dealt with are the projections to orbits $E \rightarrow E/\Pi$, where Π is a normal subgroup of a compact Lie group Γ and E is a Π -free Γ -space. The base space has an action by $G = \Gamma/\Pi$, and such bundles are classified by a G -space $B(\Pi, \Gamma)$. Information about the homotopy type of this G -space gives information about the set of equivalence classes of such bundles with base a given G -space X . The bundle theory considerably simplifies when G acts freely on X , and the main theme is the study of the transformation on bundle theories induced by the natural projection $EG \times X \rightarrow X$.

VIGUÉ-POIRRIER Micheline - *Homologie de Hochschild et homologie cyclique des algèbres différentielles graduées.*

Pour toute algèbre différentielle graduée libre $(T(V), d)$ sur un corps commutatif quelconque, nous donnons une description explicite de deux complexes: l'homologie du premier est l'homologie de Hochschild de $(T(V), d)$ et celle du second est l'homologie cyclique de $(T(V), d)$. Ces complexes servent aussi de modèles pour calculer l'homologie (resp. l'homologie équivariante) de l'espace des lacets libres sur un espace simplement connexe.

WOJTKOWIAK Zdzisław - *Maps between p -completions of the Clark-Ewing spaces.*

Let \mathbf{Z}_p denote the ring of p -adic integers. Let $W \subset GL(n, \mathbf{Z}_p)$ be a finite group such that p does not divide the order of W . The group W acts on $K((\mathbf{Z}_p)^n, 2)$. Let $X(W, p, n)_p$ be the p -completion of the space $K((\mathbf{Z}_p)^n, 2) \times_W EW$. We classify homotopy classes of maps between spaces $X(W, p, n)_p$.

ZARATI Saïd - *Derived functors of the destabilization and the Adams spectral sequence.*

In this note we prove the following

Theorem - Let X and Y be two pointed CW complexes such that

(i) $\bar{H}^*(X; \mathbb{F}_2) \cong \Sigma^2 I$, where I is an injective unstable module

(ii) $\bar{H}^*(Y; \mathbb{F}_2)$ is gradually finite and nil-closed.

Then the Adams spectral sequence for the group $[S^\infty X, S^\infty Y]$ degenerates at the E_2 term.

This theorem is deduced from the theory of the higher Hopf invariant introduced by Lannes and the author, and from the relationship between the Ext groups and the derived functors of the destabilization.

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