AST

Summary

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Summary

Let G be a real Lie group acting on a manifold M. H. Cartan introduced the equivariant de Rham complex and its cohomology $H_G^*(M)$. For a free action, $H_G^*(M)$ is the cohomology $H^*(G\backslash M)$ of the orbit space and if M is the point $\bullet$, $H_G^*(\bullet)$ is the algebra of invariant polynomial functions on the Lie algebra $\mathfrak{g}$ of G. To G-equivariant vector bundles on M, equipped with a G-invariant connection, are associated equivariant Chern classes. It became necessary to consider more general cohomological objects, like the algebra $H_G^\infty(M)$ of equivariant cohomology with C^∞-coefficients, which is an algebra over $H_G^\infty(\bullet) = C^\infty(\mathfrak{g})^G$, and also the space $H_G^{-\infty}(M)$ of equivariant cohomology with $C^{-\infty}$-coefficients, which is a module for $H_G^\infty(M)$, and for which $H_G^{-\infty}(\bullet)$ is the space of invariant generalized functions $C^{-\infty}(\mathfrak{g})^G$.

The first of the two articles of this volume, *Cohomologie équivariante et descente, by Michel Duflo and Michèle Vergne*, studies a generalization, denoted by $\mathcal{K}_G(M)$, of the cohomology $H_G^\infty(M)$. It is an algebra over $\mathcal{K}_G(\bullet) = C^\infty(G)^G$. It can be considered as a global analog of $H_G^\infty(M)$, and also as a de Rham version of the equivariant K-theory of M. The construction of $\mathcal{K}_G(M)$ uses the fixed point sets in M of the elements of G which are contained in compact subgroups of G. At least when G is compact, and under certain orientation assumptions, the "integral" over M of an element of $\mathcal{K}_G(M)$ is a G-invariant function on G.

The second article, *Equivariant cohomology with generalized coefficients, by Shrawan Kumar and Michèle Vergne*, undertakes a systematic study of the spaces $H_G^{-\infty}(M)$. Remarkable classes, with no analogs in the C^∞-theory, are discovered. In particular, for a free action, the integral over M of an element of $H_G^{-\infty}(M)$ is a generalized function on $\mathfrak{g}$ with support 0. When G is compact, a spectral sequence allows a comparison of $H_G^{-\infty}(M)$ with the equivariant cohomology $H_G^*(M)$.

The two articles have common motivations, but they can be read independently.