AST

Pages préliminaires

Astérisque, tome 228 (1995), p. 1-4

<http://www.numdam.org/item?id=AST_1995__228__1_0>

228 ASTÉRISQUE

1995

COLUMBIA UNIVERSITY NUMBER THEORY SEMINAR

New-York, 1992

SOCIÉTÉ MATHÉMATIQUE DE FRANCE
Publié avec le concours du CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

AMS Subject classification :
11, 14, 11F66, 11G18, 11G40, 14K15, 11L05, 11G25, 11L07, 14F20, 14G15, 14G25, 14G05, 11E, 11H, 11D72, 14F30, 11G35, 11D85, 11F07, 11F12, 05C50, 11R04.

In 1992, Professor Dorian Goldfeld organized a special year on Number Theory at Columbia University. On this occasion, he collected the papers appearing in the present volume. Professor Goldfeld, for personal reasons, does not want to appear as an editor of this volume. This we regret sincerely. We want to express our heartfelt thanks to him for the work which he has accomplished.

The Editorial Board of Astérisque

CONTENTS

ABSTRACTS

D. ABRAMOVICH: *Formal finiteness and the torsion conjecture on elliptic curves*

Suppose N is a huge prime number. We show that if E is an elliptic curve and P an N torsion point on E, defined over a number field K, then the degree of K over the field of rational numbers is greater than 12. In fact, a computational procedure is given for proving the same for degree higher than 12.

E. BOMBIERI, J. MUELLER: *Trinomial equations in function fields*

Let $a, b, c, h \in k(t)$ be rational functions in one variable over the algebraically closed field k ($\mathrm{char}(k) = 0$). Up to scaling by elements of k, the equation $ax^r + bx^s y^{r-s} + cy^s = h$ has at most three solutions, provided r, s and $|r - 2s|$ are bigger than some absolute constant. The proof uses the *abc* theorem of Voloch and Brownawell-Masser.

A. BRUMER: *The rank of $J_0(N)$*

Under suitable Riemann Hypotheses, we show that the rank of the Mordell-Weil group $A(\mathbb{Q})$ is bounded by $(\frac{3}{2} + \varepsilon) \dim A$ for any Atkin-Lehner component A of the Jacobian $J_0(N)$ of the modular curve $X_0(N)$.

We also show using new conductor bounds that the field of endomorphisms of an abelian variety with real multiplications must contain the real subfield of a suitable cyclotomic field when its conductor is divisible by a high power of prime.

A table gives the splitting and rank of the "even part" $J_0^-(N)$ for N prime at most 10000.

B. FISHER: *Equidistribution theorems (d'après P. Deligne et N. Katz)*

The purpose of this paper is to explain Deligne's equidistribution theorem to those unfamiliar with the language of étale cohomology. Starting with a global field, we define a "fundamental group"; consider representations of this group and give a few examples; state Deligne's theorem; and summarize Katz's work to show that Deligne's theorem applies to Kloosterman sums. We also state explicitly the consequences for elliptic curves.

S. KAMIENNY, B. MAZUR (with an appendix by A. GRANVILLE): *Rational torsion of prime order in elliptic curves over number fields*

In this paper we investigate the set $S(d)$ of prime numbers that can occur as rational torsion in elliptic curves over number fields of degree less than or equal to d. We show that $S(d)$ is finite for $d < 9$, and that it is of density zero for any d.

W.-C. W. LI: *Number theoretic constructions of Ramanujan graphs*

A Ramanujan graph is a k-regular graph with nontrivial eigenvalues bounded by $2\sqrt{k-1}$. In this article we review three explicit constructions of Ramanujan graphs using results from automorphic forms, representation theory of GL_2 over finite fields, and character sum estimates derived from the Riemann Hypothesis for curves over finite fields.

Y. MANIN: *Lectures on zeta functions and motives (according to Deninger and Kurokawa)*

The lectures describe some recent ideas of Deninger and Kurokawa suggesting the existence of a category of "absolute motives", containing an absolute point, in which the tensoring corresponds to the adding zeroes of zetas. Various classical constructions in combinatorics and special functions theory are interpreted from this perspective.

B. MAZUR: *Speculations about the topology of rational points: an up-date*

This paper is devoted to a discussion concerning the conjecture that if V is a smooth variety over the rational numbers, and if its set of rational points is Zariski-dense in V, then the topological closure of the set of rational points of V is an open subset of the real locus of V.

J. PILA: *Density of integral and rational points on varieties*

Let V be an irreducible variety defined over $\mathbb{R}$. We give an upper bound for the number of integral points of V of height $\leq H$. Our bound is uniform for all varieties of given dimension, degree and ambient space, and is best possible in some aspects. We discuss related results in the literature.

W. M. SCHMIDT: *Number fields of given degree and bounded discriminant*

How many number fields are there, of degree d and discriminant of modulus $\leq X$? We will show that there are $\ll X^{(d+2)/4}$ such fields.

E. DE SHALIT: *The explicit reciprocity law of Bloch-Kato*

We give a detailed account of the "fundamental exact sequence", which is basic to understanding the ring B_{cris} (both B_{cris} and the fundamental exact sequence are due to Fontaine). We present a proof of the explicit reciprocity law of Bloch and Kato, which is somewhat different than the original one. We then discuss the possibility of extending there results to Lubin-Tate groups of height > 1.

J. H. SILVERMAN: *Counting integer and rational points on varieties*

Let V/K be an algebraic variety defined over a number field, and let H be a height function on V. We study the asymptotic behavior of the counting function $N(V(R), B) = \#\{P \in V(R)\colon H(P) \leq B\}$ as $B \to \infty$. We describe some known and conjectural formulas for the behaviour of $N(V(R), B)$ which are given in terms of elementary geometric invariants of the variety V and elementary arithmetic invariants of the ring R.