COMPOSITIO MATHEMATICA

WILLIAM CASSELMAN
M. SCOTT OSBORNE

The n-cohomology of representations with an infinitesimal character

Compositio Mathematica, tome 31, nº 2 (1975), p. 219-227
<http://www.numdam.org/item?id=CM_1975__31_2_219_0>

COMPOSITIO MATHEMATICA, Vol. 31, Fasc. 2, 1975, pag. 219–227
Noordhoff International Publishing
Printed in the Netherlands

THE $\mathfrak{n}$-COHOMOLOGY OF REPRESENTATIONS WITH AN INFINITESIMAL CHARACTER

William Casselman and M. Scott Osborne

1

Let $\mathbb{F}$ be a field of characteristic 0, $\mathfrak{g}$ a reductive Lie algebra over $\mathbb{F}$, $\mathfrak{p}$ a parabolic subalgebra with $\mathfrak{n}$ as nilpotent radical and $\mathfrak{m}$ a reductive complement in $\mathfrak{p}$ to $\mathfrak{n}$. If V is an irreducible finite-dimensional $\mathfrak{g}$-module, then the Lie algebra cohomology $H^*(\mathfrak{n}, V)$ has a canonical $\mathfrak{m}$-module structure (see Section 2). Over an algebraic closure of $\mathbb{F}$, this module decomposes completely into a direct sum of absolutely irreducible $\mathfrak{m}$-modules, and Kostant [5] has determined this decomposition completely. In this paper we shall prove a partial generalization of Kostant's result for the class of all $\mathfrak{g}$-modules with an infinitesimal character (Theorem 2.6 and its corollary). We include (in Section 4) an elementary derivation of the structure of $H^*(\mathfrak{n}, V)$ when V is finite-dimensional.

One of the applications Kostant made of his result was to give a new proof of the Weyl character formula. In order to give the $\mathfrak{n}$-cohomology groups for infinite-dimensional V some general interest, we would like to point out here that one might expect, even for them, a relationship between characters and $\mathfrak{n}$-cohomology (see [7]).

We have in mind other applications of these groups also, notably to the question of analytic continuation of interwining operators between principal series, and even further to a detailed analysis of the decomposition of principal series. This is what one would expect in light of the theory for p-adic groups, where a crucial role is played by the fact that the V_N-functor (see [3] for notation) is exact.

Notation: if $\mathfrak{h}$ is a Lie algebra, $U(\mathfrak{h})$ is its universal enveloping algebra and $Z(\mathfrak{h})$ the centre of $U(\mathfrak{h})$.

2

Let $\mathfrak{h}$ be an arbitrary Lie algebra over $\mathbb{F}$, and let U and V be $\mathfrak{h}$-modules. If

$$\cdots \to U_1 \to U_0 \to U \to 0$$

is any $U(\mathfrak{h})$-projective resolution of U, then $\mathrm{Ext}^*_{U(\mathfrak{h})}(U, V)$ is the cohomology of the complex $\mathrm{Hom}_{U(\mathfrak{h})}(U_*, V)$. Since one can show fairly easily that any two projective resolutions are homotopically equivalent, the Ext-groups are independent of the choice of resolution. If one has $U = \mathbb{F}$ with the trivial action of $\mathfrak{h}$, then one recovers the cohomology of $\mathfrak{h}$ with coefficients in V:

$$\mathrm{Ext}^*_{U(\mathfrak{h})}(\mathbb{F}, V) \cong H^*(\mathfrak{h}, V).$$

One also has

$$\mathrm{Ext}^*_{U(\mathfrak{h})}(U, V) \cong H^*(\mathfrak{h}, \mathrm{Hom}\ (U, V)).$$

There exists a standard resolution of the $\mathfrak{h}$-module $\mathbb{F}$, which is finite in length. This is obtained by letting U_m be $U(\mathfrak{h}) \otimes_{\mathbb{F}} \Lambda^m\mathfrak{h}$ (with $\mathfrak{h}$ acting on the first factor alone) and defining a differential by the formula

$$d(1 \otimes (h_1 \wedge \cdots \wedge h_m)) = \sum_{1 \leq i \leq m} (-1)^{i+1} h_i \otimes (h_1 \wedge \cdots \wedge \hat{h}_i \wedge \cdots \wedge h_m)$$
$$+ \sum_{1 \leq i < j \leq m} (-1)^{i+j} \otimes ([h_i, h_j] \wedge \cdots \wedge \hat{h}_i \wedge \cdots \wedge \hat{h}_j \wedge \cdots \wedge h_m).$$

Since for any $\mathbb{F}$-space X and $\mathfrak{h}$-module Y one has

$$\mathrm{Hom}_{U(\mathfrak{h})}(U(\mathfrak{h}) \otimes X, Y) = \mathrm{Hom}\ (X, Y),$$

this gives rise to the common definition of $H^*(\mathfrak{h}, V)$ as the cohomology of the complex $C^*(\mathfrak{h}, V) = \mathrm{Hom}\ (\Lambda^*\mathfrak{h}, V)$ with the differential

$$df(h_1, \cdots, h_{n+1}) = \sum (-1)^{i+1} h_i \cdot f(h_1, \cdots, \hat{h}_i, \cdots, h_{n+1})$$
$$+ \sum_{1 \leq i < j \leq n+1} (-1)^{i+j} f([h_i, h_j], \cdots, \hat{h}_i, \cdots, \hat{h}_j, \cdots, h_{n+1}).$$

If $\mathfrak{h}$ is an ideal in another Lie algebra $\mathfrak{g}$ and V a $\mathfrak{g}$-module, then one has the adjoint action of $\mathfrak{g}$ on $\mathfrak{h}$, hence on $\Lambda^*\mathfrak{h}$, hence on $C^*(\mathfrak{h}, V)$. The differentials above are $\mathfrak{g}$-morphisms ([8], Lemma 2.5.1.1) and one obtains thus a representation of $\mathfrak{g}$ on $H^*(\mathfrak{h}, V)$.

We now describe a second way to obtain a representation of $\mathfrak{g}$ on $H^*(\mathfrak{h}, V)$.

2.1. LEMMA: *If $\mathfrak{g}$ is any Lie algebra and $\mathfrak{h}$ a subalgebra, then $U(\mathfrak{g})$ is a free $U(\mathfrak{h})$-module. Any injective $U(\mathfrak{g})$-module is also $U(\mathfrak{h})$-injective.*

PROOF: The first claim follows from Poincare-Birkhoff-Witt. The second is an elementary (and well known) exercise.

Now again assume $\mathfrak{h}$ to be an ideal in $\mathfrak{g}$, V a $\mathfrak{g}$-module. If

$$\cdots \to U_1 \to U_0 \to \mathbb{F} \to 0$$

is any resolution of the trivial $\mathfrak{g}$-module by free $U(\mathfrak{g})$-modules, then by Lemma 2.1 one has that $H^*(\mathfrak{h}, V)$ is the cohomology of the complex $\mathrm{Hom}_{U(\mathfrak{h})}(U_*, V)$. Now clearly $U(\mathfrak{g})$ acts on this complex and the differentials are $U(\mathfrak{g})$-morphisms, and this gives the second representation of on $H^*(\mathfrak{h}, V)$ that we want to consider. This one is for the usual reasons independent of the resolution.

2.2 PROPOSITION: *Assume $\mathfrak{h}$ to be an ideal in $\mathfrak{g}$, V a $\mathfrak{g}$-module. The two representations of $\mathfrak{g}$ on $H^*(\mathfrak{h}, V)$ just defined are the same.*

PROOF: It will suffice to choose, in the definition of the second representation, the standard $U(\mathfrak{g})$-resolution of $\mathbb{F}$. There are two representations of $\mathfrak{g}$ on each term $U(\mathfrak{g}) \otimes \Lambda^m\mathfrak{g}$ in the complex: $g \circ (u \otimes \lambda) = (gu \otimes \lambda)$ and $g \times (u \otimes \lambda) = (u \otimes g \cdot \lambda)$. *These operations are homotopic.* More precisely, for any $g \in \mathfrak{g}$ let K_g be the linear map:

$$U(\mathfrak{g}) \otimes \Lambda^m\mathfrak{g} \to U(\mathfrak{g}) \otimes \Lambda^{m+1}\mathfrak{g}, \qquad u \otimes \lambda \to u \otimes g \wedge \lambda.$$

Then as one may check easily,

$$(dK_g + K_g d)(u \otimes \lambda) = g \circ (u \otimes \lambda) - g \times (u \otimes \lambda).$$

This implies, for example, that the two induced representations of $\mathfrak{g}$ on $H^*(\mathfrak{g}, V)$ are the same – hence trivial, since the first one clearly is. From this point, one can prove Proposition 2.2 by considering the restriction map from $C^*(\mathfrak{g}, V)$ to $C^*(\mathfrak{h}, V)$.

Incidentally, since – with assumptions as in Proposition 2.2 – one can see easily from 2.2 that $\mathfrak{h}$ acts trivially, one obtains in fact a canonical representation of $\mathfrak{g}/\mathfrak{h}$ on $H^*(\mathfrak{h}, V)$.

2.3 Proposition: (a) *If U and V are two $\mathfrak{g}$-modules and $f: U \to V$ is a $\mathfrak{g}$-morphism, then the induced* map: $H^*(\mathfrak{h}, U) \to H^*(\mathfrak{h}, V)$ *is a $\mathfrak{g}$-morphism.*

(b) *If*

$$0 \to V_1 \to V_2 \to V_3 \to 0$$

is a short exact sequence of $\mathfrak{g}$-modules, then the connecting morphisms: $H^m(\mathfrak{h}, V_3) \to H^{m+1}(\mathfrak{h}, V_1)$ are $\mathfrak{g}$-morphisms.

Proof: Straightforward.

Now assume $\mathfrak{h}$ to be a subalgebra of $\mathfrak{g}$. Let $N_{\mathfrak{g}}(\mathfrak{h})$ be the subspace of $X \in U(\mathfrak{g})$ such that $[X, \mathfrak{h}] \subseteq \mathfrak{h}$. If V is any $\mathfrak{g}$-module, $N_{\mathfrak{g}}(\mathfrak{h})$ takes $V^{\mathfrak{h}}$ to itself. If one applies this to the spaces Hom (U_*, V), where V is a $\mathfrak{g}$-module and U_m a free $U(\mathfrak{g})$-resolution of $\mathbb{F}$, one obtains a representation of $N_{\mathfrak{g}}(\mathfrak{h})$ on $H^*(\mathfrak{h}, V)$. This representation is of course independent of the resolution.

Now assume, as in Section 1, $\mathfrak{g}$ to be a reductive Lie algebra over $\mathbb{F}$, $\mathfrak{p}$ a parabolic subalgebra of $\mathfrak{g}$ with Levi decomposition $\mathfrak{p} = \mathfrak{m} \oplus \mathfrak{n}$. Let $N(\mathfrak{n}) = N_{\mathfrak{g}}(\mathfrak{n})$. This space contains $U(\mathfrak{n})$, $U(\mathfrak{m})$, and $Z(\mathfrak{g})$ (identifying these with their canonical copies in $U(\mathfrak{g})$ if necessary).

2.4 Lemma: *There exists a unique homomorphism $\sigma: Z(\mathfrak{g}) \to Z(\mathfrak{m})$ such that for any $X \in Z(\mathfrak{g})$ one has $X - \sigma(X) \in U(\mathfrak{g})\mathfrak{n}$. It is injective.*

Proof: Let $\mathfrak{a}$ be the centre of $\mathfrak{m}$, $\mathfrak{g} = \oplus\, \mathfrak{g}_\alpha$ the eigenspace decomposition of $\mathfrak{g}$ with respect to the adjoint action of $\mathfrak{a}$. Let Σ be the set of eigencharacters other than 0. Then one knows that $\mathfrak{g}_0 = \mathfrak{m}$ and that there exists a subset $\Sigma^+ \subseteq \Sigma$ such that (1) $\mathfrak{n} = \oplus_{\alpha\in\Sigma^+} \mathfrak{g}_\alpha$; (2) $\mathfrak{n}^- = \oplus_{\alpha\in\Sigma-\Sigma^+} \mathfrak{g}_\alpha$ is a Lie subalgebra of $\mathfrak{g}$. From this point the argument is exactly as for Lemmas 2.3.3.4 and 2.3.3.5 of [8].

If V is a $\mathfrak{g}$-module, then one has the representation defined earlier of $N(\mathfrak{n})$ – hence of $U(\mathfrak{n})$, $U(\mathfrak{m})$, and $Z(\mathfrak{g})$ – on $H^*(\mathfrak{n}, V)$. The algebra $\mathfrak{n}$ acts trivially, so that according to Lemma 2.4 one has $X\gamma = \sigma(X)\gamma$ for all $X \in Z(\mathfrak{g})$, $\gamma \in H^*(\mathfrak{n}, V)$.

2.5 Lemma: *If V is a $\mathfrak{g}$-module annihilated by the $Z(\mathfrak{g})$-ideal I, then $\sigma(I)$ annihilates $H^0(\mathfrak{n}, V) \simeq V^{\mathfrak{n}}$.*

PROOF: Clear from the preceding remarks.

The main result of this paper is the analogous assertion for higher-dimensional $\mathfrak{n}$-cohomology:

2.6 THEOREM: *If V is a $\mathfrak{g}$-module annihilated by the $Z(\mathfrak{g})$-ideal I, then $\sigma(I)$ annihilates $H^*(\mathfrak{n}, V)$.*

The proof will occupy the next Section.

If V is a $\mathfrak{g}$-module, one says that V is $Z(\mathfrak{g})$-*scalar* and has *infinitesimal character* $\theta : Z(\mathfrak{g}) \to \mathbb{F}$ if θ is a ring homomorphism such that for all $X \in Z(\mathfrak{g})$, $v \in V$, one has $Xv = \theta(X)v$. This means that V is annihilated by the $Z(\mathfrak{g})$-ideal generated by $\{X - \theta(X) | X \in Z(\mathfrak{g})\}$, hence Theorem 2.6 immediately implies:

2.7 COROLLARY: *If V is a $\mathfrak{g}$-module with infinitesimal character θ, then for all $X \in Z(\mathfrak{g})$ and $\gamma \in H^*(\mathfrak{n}, V)$ one has $\sigma(X)\gamma = \theta(X)\gamma$.*

These results impose a severe restriction on the structure of $H^*(\mathfrak{n}, V)$ as an $\mathfrak{m}$-module. If V is finite-dimensional, it is equivalent to a crucial lemma ([1], Proposition 6, or [8], Lemma 2.5.2.3) in Aribaud's derivation of Kostant's result. Our proof is new even in that case.

In light of the remarks made just before Lemma 2.5, one can also phrase Theorem 2.6 as this: if V is annihilated by I, then so is $H^*(\mathfrak{n}, V)$. However, this seems only curious rather than useful; we only require in this paper (through Lemma 2.5) the representation of $Z(\mathfrak{g})$ on $H^0(\mathfrak{n}, V)$, which is trivial to obtain.

We remark also that an analogous result is true for the homology spaces $H_*(\mathfrak{n}, V)$.

3

Continue to assume $\mathfrak{g}, \mathfrak{p}, \mathfrak{m}, \mathfrak{n}$ as in Section 1, and let $\mathfrak{p}^-$ be the parabolic subalgebra *opposite* to $\mathfrak{p}$ (in the terminology of the proof of 2.4, $\mathfrak{p}^- = \mathfrak{m} + \mathfrak{n}^-$). Then $\mathfrak{g} = \mathfrak{p}^- \oplus \mathfrak{n}$, so that by Poincare-Birkhoff-Witt one has $U(\mathfrak{g}) = U(\mathfrak{p}^-) \oplus U(\mathfrak{g})\mathfrak{n}$.

3.1 LEMMA: *If X is any element of $Z(\mathfrak{g})$, then $U(\mathfrak{p}^-)X \cap U(\mathfrak{g})\mathfrak{n} = 0$.*

PROOF: The case $X = 0$ is trivial, so assume $X \neq 0$. Let R be an element of $U(\mathfrak{p}^-)$ such that $RX \in U(\mathfrak{g})\mathfrak{n}$. Since $X - \sigma(X) \in U(\mathfrak{g})\mathfrak{n}$, one also has

$R\sigma(X) \in U(\mathfrak{g})\mathfrak{n}$. Hence $R\sigma(X) = 0$. Since $X \neq 0$ and σ is injective (Lemma 2.4, $\sigma(X) \neq 0$ as well. Since $U(\mathfrak{g})$ has no 0-divisors, $R = 0$.

3.2 COROLLARY: *If X is any element of $Z(\mathfrak{g})$, then*

$$U(\mathfrak{g})X \cap U(\mathfrak{g})\mathfrak{n} = U(\mathfrak{g})X\mathfrak{n}.$$

PROOF: Since $X\mathfrak{n} = \mathfrak{n}X$, one has

$$U(\mathfrak{g})X = U(\mathfrak{p}^-)X \oplus U(\mathfrak{g})X\mathfrak{n}.$$

Since $U(\mathfrak{g})X\mathfrak{n} \subseteq U(\mathfrak{g})\mathfrak{n}$,

$$\begin{aligned} U(\mathfrak{g})X \cap U(\mathfrak{g})\mathfrak{n} &= (U(\mathfrak{p}^-)X \cap U(\mathfrak{g})\mathfrak{n}) \oplus \mathrm{U}(\mathfrak{g})\mathrm{X}\mathfrak{n} \\ &= U(\mathfrak{g})X\mathfrak{n}, \end{aligned}$$

by 3.1.

3.3 PROPOSITION: *If X is any element of $Z(\mathfrak{g})$ and V an injective $U(\mathfrak{g})/U(\mathfrak{g})X$-module, then $H^q(\mathfrak{n}, V) = 0$ for $q > 0$.*

PROOF: Let U_X be the ring $U(\mathfrak{g})/U(\mathfrak{g})X$. If E is a vector space over $\mathbb{F}$, define $I_X(E)$ to be the U_X-module $\mathrm{Hom}_{\mathbb{F}}(U_X, E)$. This is an injective U_X-module, and if E is a U_X-module there exists a canonical injection of $E \cong \mathrm{Hom}_{U_X}(U_X, E)$ into it. Thus every injective U_X-module is a summand of a suitable $I_X(E)$, and it suffices to prove the proposition for $V = I_X(E)$.

One has the exact sequence of $\mathfrak{g}$-modules

$$0 \to I_X(E) \to \mathrm{Hom}_{\mathbb{F}}(U(\mathfrak{g}), E) \to \mathrm{Hom}\ (U(\mathfrak{g})X, E) \to 0.$$

Since $U(\mathfrak{g})$ and $U(\mathfrak{g})X$ are free over $U(\mathfrak{g})$, the last two modules in this sequence are $U(\mathfrak{g})$-injective, hence $U(\mathfrak{n})$-injective (Lemma 2.1). The long sequence of cohomology groups gives us that $H^q(\mathfrak{n}, I_X(E)) = 0$ for $q > 1$, and reduces proving that $H^1(\mathfrak{n}, I_X(E)) = 0$ to proving that the canonical map

$$H^0(\mathfrak{n}, \mathrm{Hom}_{\mathbb{F}}(U(\mathfrak{g}), E)) \to H^0(\mathfrak{n}, \mathrm{Hom}_{\mathbb{F}}(U(\mathfrak{g})X, E))$$

is surjective. This map is equivalent to:

$$\mathrm{Hom}_{\mathbb{F}}(U(\mathfrak{g})/U(\mathfrak{g})\mathfrak{n}, E) \to \mathrm{Hom}_{\mathbb{F}}(U(\mathfrak{g})X/U(\mathfrak{g})X\mathfrak{n}, E)$$

which is surjective by Corollary 3.2.

PROOF OF THEOREM 2.6: We must show that if V is a $\mathfrak{g}$-module and $X \in Z(\mathfrak{g})$ $^{+}$nnihilates V, then $\sigma(X)$ annihilates $H^*(\mathfrak{n}, V)$. As above, let U_X be the ring $U(\mathfrak{g})/U(\mathfrak{g})X$. Then V is a U_X-module, and there exists an injective U_X-module A into which V embeds. If B is the quotient, then we have this sequence of U_X-modules:

$$0 \to V \to A \to B \to 0.$$

Applying 3.3, one sees that the long exact sequence of $\mathfrak{n}$-cohomology decomposes:

$$0 \to H^0(\mathfrak{n}, V) \to H^0(\mathfrak{n}, A) \to H^0(\mathfrak{n}, B) \to H^1(\mathfrak{n}, V) \to 0$$

$$0 \to H^{m-1}(\mathfrak{n}, B) \to H^m(\mathfrak{n}, V) \to 0 \qquad (m > 1).$$

From Proposition 2.3 and Lemma 2.5, one concludes Theorem 2.6 for $H^1(\mathfrak{n}, V)$; proceed by induction.

3.4 REMARK: By an argument a great deal more complicated, using among other things the fact that $U(\mathfrak{g})$ is free over $Z(\mathfrak{g})$ (see [6]), one can prove the following generalization of Proposition 3.3: *if I is any ideal of $Z(\mathfrak{g})$ and V an injective $U(\mathfrak{g})/U(\mathfrak{g})I$-module, then $H^q(\mathfrak{n}, V) = 0$ for $q > 0$.*

4

Assume $\mathbb{F}$ to be algebraically closed, $\mathfrak{g}$ a reductive Lie algebra over $\mathbb{F}$, $\mathfrak{p}$ a maximal solvable subalgebra of $\mathfrak{g}$ with Levi decomposition $\mathfrak{p} = \mathfrak{m} \oplus \mathfrak{n}$ (so that $\mathfrak{m}$ is a Cartan algebra of $\mathfrak{g}$). Let Σ be the set of roots of $\mathfrak{g}$ with respect to $\mathfrak{m}$, and give to Σ the ordering associated to $\mathfrak{p}$. Let W be the Weyl group associated to Σ, and for each $w \in W$ let $l(w)$ be its length.

Let V be an irreducible finite-dimensional $\mathfrak{g}$-module, θ its infinitesimal character, Λ its highest weight.

Let δ be the character $\frac{1}{2}\sum_{\alpha>0} \alpha$.

4.1 LEMMA: *The characters $\{w(\Lambda+\delta)-\delta \mid w \in W\}$ are precisely those one-dimensional representations λ of $\mathfrak{m}$ (hence of $U(\mathfrak{m})$) such that $\lambda(\sigma(X)) = \theta(X)$ for all $X \in Z(\mathfrak{g})$.*

PROOF: See § 23 of [4].

We remark that these characters are all distinct.

4.2 LEMMA: *For a given* $w \in W$, *the character* $w(\Lambda+\delta)-\delta$ *occurs exactly once in* Hom $(\Lambda^q\mathfrak{n}, V)$ *if* $l(w)$ *is* q, *and not at all if the length of* w *is not* q.

PROOF: See [2].

(Note that the proof which both [5] and [8] give of this fact relies on the Weyl character formula for the $\mathfrak{g}$-representation with highest weight δ, while Cartier avoids this.)

4.3 THEOREM: (*Kostant*) *If* V *is an irreducible finite-dimensional representation of* $\mathfrak{g}$ *with highest weight* Λ, *then as an* $\mathfrak{m}$-*module one has* $H^q(\mathfrak{n}, V) \cong \oplus_{l(w)=q}\, w(\Lambda+\delta)-\delta$.

PROOF: Lemmas 4.1 and 4.2 and Theorem 2.6 imply that if λ is a character of $\mathfrak{m}$ occurring in $H^q(\mathfrak{n}, V)$, then $\lambda = w(\Lambda+\delta)-\delta$ for some $w \in W$ with $l(w) = q$. It remains to show that if $l(w) = q$, then $\mathrm{Hom}_{\mathfrak{m}}\,(w(\Lambda+\delta)-\delta, H^q(\mathfrak{n}, V)) \neq 0$. Let ϕ be an $\mathfrak{m}$-eigenvector $\neq 0$ in $C^q(\mathfrak{n}, V)$ with eigencharacter $w(\Lambda+\delta)-\delta$, which exists by Lemma 4.2. Because (i) the differential of the complex $C^*(\mathfrak{n}, V)$ is an $\mathfrak{m}$-morphism, and (ii) the character $w(\Lambda+\delta)-\delta$ does not occur in either $C^{q-1}(\mathfrak{n}, V)$ or $C^{q+1}(\mathfrak{n}, V)$ (4.2), the cochain ϕ is a cocycle but not a coboundary.

4.4 REMARK: One can deal similarly with parabolic subalgebras other than the minimal one to obtain Kostant's result for these, too.

REFERENCES

[1] F. ARIBAUD: Une nouvelle demonstration d'un théorème de R. Bott et B. Kostant. *Bull. Math. Soc. France 95* (1967) 205–242.

[2] P. CARTIER: Remarks on 'Lie algebra cohomology and the generalized Borel-Weil theorem' by B. Kostant. *Ann. of Math. 74* (1961) 388–390.

[3] W. CASSELMAN: *Some general results on admissible representations of* $\mathfrak{p}$-*adic groups*. (to appear)

[4] J. HUMPHREYS: *Introduction to Lie algebras and representation theory*. New York: Springer, 1972.

[5] B. KOSTANT: Lie algebra cohomology and the generalized Borel-Weil theorem. *Ann. of Math. 74* (1961) 329–387.

[6] B. KOSTANT: Lie group representations on polynomial rings. *Amer. J. of Math.* 85 (1963) 327–404.

[7] M. S. OSBORNE: *Yale University Ph.D. thesis.* (1973).
[8] G. WARNER: *Harmonic analysis on semi-simple groups* I. New York, Springer, 1972.

(Oblatum 22–I–1975)

W. Casselman
University of British Columbia
Vancouver

M. S. Osborne
University of Washington
Seattle