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SEPARABILITY OF ANALYTIC IMAGES OF SOME BANACH SPACES

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Abstract

A Banach space contains a nonseparable analytic image of a ball in $c_0(\Gamma)$ iff it contains an isomorphic copy of $c_0(B)$, B uncountable.

Let Γ be an uncountable set. It is known that the (complex) space $c_0(\Gamma)$ has some interesting properties with respect to analytic maps. For instance, every scalar-valued analytic map on $c_0(\Gamma)$ factors through a separable subspace of $c_0(\Gamma)$ [8, 1]. All separable complex Banach spaces X and the spaces $X = 1^p(B)$ for any B, $1 \leq p < \infty$ have the property that every nonempty open connected subset of X can be filled densely with an analytic image of a ball in X [4, 5], while the space $c_0(\Gamma)$ does not have this property [9]. No space $1^p(B)$ $(1 \leq p < \infty)$ and no space with countable total set contains a nonseparable analytic image of a ball in $c_0(\Gamma)$ [8, 6]. In the present paper we sharpen the last result by proving that a Banach space contains a nonseparable analytic image of a ball in $c_0(\Gamma)$ iff it contains an isomorphic copy of $c_0(B)$, B uncountable. This is known in the linear case (see Remark 1 below).

Preliminaries

The scalar field (R or C) is the same for all Banach spaces considered. We denote by N the set of all positive integers. If A is a map we denote its image by $R(A)$. Let Γ be an infinite set. By $c_0(\Gamma)$ we denote the Banach space of all scalar-valued functions on Γ which

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are arbitrarily small outside finite subsets of Γ, with sup norm. If $x \in c_0(\Gamma)$ we write supp $x = \{\gamma \in \Gamma : x(\gamma) \neq 0\}$. We denote by $\{e(\gamma) : \gamma \in \Gamma\}$ the standard basis in $c_0(\Gamma) : e(\gamma)(\delta) = 1 (\gamma = \delta)$, $e(\gamma)(\delta) = 0$ $(\gamma \neq \delta)$. Given a metric space M we denote by dens M the density character of M, i.e. the smallest cardinal of a dense subset of M. Note that if M_1 is a subspace of M then dens $M_1 \leq$ dens M. Let X be a Banach space. By $B_1(X)$ we denote the open unit ball of X and by X' we denote the dual of X. If $S \subset X$ we write $\overline{sp}\ S$ for the closed linear span of S. Let Y be another Banach space and let $n \in N$. A map $P : X \to Y$ is called a bounded n-homogeneous polynomial if there is a bounded symmetric n-linear map $Q : X^n \to Y$ such that $P(x) = Q(x, x, \ldots, x)$ $(x \in X)$. We use the term 0-homogeneous polynomial for constant maps. A map $A : B_1(X) \to Y$ is called analytic if given any $x_0 \in B_1(X)$ there are an $r > 0$ and for each n a bounded n-homogeneous polynomial $P_n : X \to Y$ such that $A(x) = \sum_{n=0}^{\infty} P_n(x - x_0) (\|x - x_0\| < r)$, the series being uniformly convergent for $\|x - x_0\| < r$ [11]. When the scalar field is C then A is analytic iff for each $x \in B_1(X)$ the Fréchet derivative of A at x exists as a bounded complex-linear map from X to Y, or equivalently, if A is G-analytic and continuous on $B_1(X)$ [7].

Our main result is the following

THEOREM: *Let Y be a Banach space and let d be any infinite cardinal. Suppose that there exists an analytic map A from the open unit ball of some $c_0(\Gamma)$ to Y such that dens $R(A) > d$. Then Y contains an isomorphic copy of $c_0(B)$ where card $B > d$.*

REMARK 1: In the special case when A is bounded linear map the assumptions above imply that card $\{\gamma \in \Gamma ; A(e(\gamma)) \neq 0\} > d$ so for some $\delta > 0$ card $\{\gamma \in \Gamma : \|A(e(\gamma))\| \geq \delta\} > d$ and the assertion follows by [12 p. 30, Rem. 1]; see also [2, 3].

COROLLARY 1: *A Banach space contains a nonseparable analytic image of a ball in $c_0(\Gamma)$ iff it contains an isomorphic copy of $c_0(B)$ where B is uncountable.*

LEMMA 1: *Let X, Y be two Banach spaces and let d be any infinite cardinal. Suppose that there exists an analytic map $A : B_1(X) \to Y$ such that dens $R(A) > d$. Then there are an $n \in N$ and a bounded n-homogeneous polynomial $P : X \to Y$ such that dens $R(P) > d$.*

PROOF: There is some $r > 0$ such that

$$A(x) = \sum_{n=0}^{\infty} P_n(x) \quad (\|x\| < r) \tag{1}$$

where for each n, P_n is a bounded n-homogeneous polynomial. With no loss of generality assume that $P_0 = 0$.

By the analyticity of A given any $x \in B_1(X)$ and any $u \in Y'$ the scalar-valued map $t \mapsto F(t) = \langle A(tx)|u\rangle$ defined on $I = \{t: 0 \leq t \leq 1\}$ has an analytic extension to an open subset of C containing I so by the identity theorem $F(t) = 0$ $(0 < t < r)$ implies that $F(1) = 0$. By the Hahn-Banach theorem it follows that $A(x) \in \overline{sp}\{A(tx); 0 < t < r\}$ so

$$R(A) \subset \overline{sp}\{Ax; \|x\| < r\}. \tag{2}$$

Assume that dens $R(P_n) \leq d$ for all n and for each n let B_n be a dense subset of $R(P_n)$ satisfying card $B_n \leq d$. The set B of all vectors $y \in Y$ of the form $Y = \Sigma_{i=1}^{n} y_i$ where $y_i \in B_i$ $(1 \leq i \leq n)$ and $n \in N$ satisfies card $B \leq d$ so dens $\overline{sp}$ $B \leq D$. On the other hand, by (1) and (2) $R(A) \subset \overline{sp} B$ so dens $R(A) \leq d$, a contradiction which proves that for some $n \in N$ dens $R(P_n) > d$. Q.E.D.

PROOF OF THE THEOREM: Let Γ be an infinite set, put $X = c_0(\Gamma)$ and let $A: B_1(X) \to Y$ be an analytic map satisfying dens $R(A) > d$. By Lemma 1 there are an $n \in N$ and a bounded n-homogeneous polynomial $P: X \to Y$ such that dens $R(P) < d$. Let $Q: X^n \to Y$ be a bounded symmetric m-linear map such that $P(x) = Q(x, x, \ldots, x)$ $(x \in X)$. Let $\mathscr{A} \subset \Gamma^n$ be the set of all those $a = (a_1, a_2, \ldots, a_n)$ for which $Q(e(a_1), e(a_2), \ldots, e(a_n)) \neq 0$. We prove that card $\mathscr{A} > d$. To see this, assume that card $\mathscr{A} \leq d$. For i, $1 \leq i \leq n$ write $\mathscr{A}_i = \{\beta \in \Gamma: \beta = a_i$ for some $a \in \mathscr{A}\}$. Clearly card $\mathscr{A}_i \leq$ card $\mathscr{A} \leq d$ $(1 \leq i \leq n)$ so writing $\mathscr{U} = \bigcup_{i=1}^{n} \mathscr{A}_i$ we have card $\mathscr{U} \leq d$. By the boundedness of Q it follows that $Q(e(\gamma), x_2, x_3, \ldots, x_n) = 0$ for any $\gamma \in \Gamma - \mathscr{U}$ and any $x_i \in X$ $(2 \leq i \leq n)$ so $Q(y, x_2, x_3, \ldots, x_n) = 0$ for any $x_i \in X$ $(2 \leq i \leq n)$ and any $y \in X$, supp $y \cap \mathscr{U} = \emptyset$. Since Q is symmetric it follows that $P(x + y) = Q(x + y, x + y, \ldots, x + y) = Q(x, x, \ldots, x) = P(x)$ for any x, $y \in X$, supp $y \cap \mathscr{U} = \emptyset$. Consequently $P = P \circ L$ where L is the projection from X onto $c_0(\mathscr{U})$ defined by

$$L(x)(\gamma) = \begin{cases} x(\gamma) & \gamma \in \mathscr{U} \\ 0 & \gamma \in \Gamma - \mathscr{U} \end{cases}$$

Now, card $\mathcal{U} \leq d$ implies that dens $c_0(\mathcal{U}) \leq d$ and it follows that dens $R(P) \leq d$, a contradiction which proves that card $\mathcal{A} > d$.

By Remark 1 the proof will be complete once we have proved the following

LEMMA 2: *Let Γ be an infinite set and put $X = c_0(\Gamma)$. Let Y be a Banach space, let $m \in N$ and let d be any infinite cardinal. Suppose that $P: X^m \to Y$ is a bounded m-linear map such that the set*

$$\mathcal{A} = \{a = (a_1, a_2, \ldots, a_m) \in \Gamma^m : P(e(a_1), e(a_2), \ldots, e(a_m)) \neq 0\}$$

satisfies card $\mathcal{A} > d$.

Then there exist a set D, card $D > d$ and a bounded linear map $L: c_0(D) \to Y$ such that $L(e(\delta)) \neq 0$ $(\delta \in D)$.

PROOF: We prove the lemma by induction on m.

If $m = 1$ put $D = \mathcal{A}$ and $L = P|c_0(D)$.

Assume that we have proved the lemma for $m = n - 1$ and let $P: X^n \to Y$ be a bounded n-linear map such that card $\mathcal{A} > d$ where $\mathcal{A} = \{a = (a_1, a_2, \ldots, a_n) \in \Gamma^n : P(e(a_1), e(a_2), \ldots, e(a_n)) \neq 0\}$.

Assume first that there is some k, $1 \leq k \leq n$ and some $\gamma \in \Gamma$ such that card$\{a \in \mathcal{A} : a_k = \gamma\} > d$. Consider the bounded $(n-1)$-linear map $Q: X^{n-1} \to Y$ defined by

$$Q(x_1, \ldots, x_{k-1}, x_{k+1}, \ldots, x_n) = P(x_1, \ldots, x_{k-1}, e(\gamma), x_{k+1}, \ldots, x_n).$$

Now

$$\text{card}\{a = (a_1, a_2, \ldots, a_{n-1}) \in \Gamma^{n-1} : Q(e(a_1), e(a_2), \ldots, e(a_{n-1})) \neq 0\} > d,$$

and the assertion of the lemma for $m = n$ follows by the induction hypothesis.

In the sequel we assume that

$$\left.\begin{array}{l}\text{for every } \gamma \in \Gamma \text{ and for each } k: 1 \leq k \leq n \\ \text{card}\{a \in \mathcal{A} : a_k = \gamma\} \leq d.\end{array}\right\} \tag{3}$$

Consider the class $\mathcal{C}$ of all nonempty subsets $Q \subset \mathcal{A}$ having the following property

Let $\{a_1, a_2, \ldots, a_k\}$ be any finite subset (of distinct elements) of Q. Write $a_j = (a_{j1}, a_{j2}, \ldots, a_{jn})$ $(1 \leq j \leq k)$. If $1 \leq j_i \leq k$ $(1 \leq i \leq n)$ then $(a_{j_1 1}, a_{j_2 2}, \ldots, a_{j_n n}) \in \mathcal{A}$ implies that $j_1 = j_2 = \ldots = j_n$. (4)

Observe that $\mathscr{C}$ is not empty since every set Q consisting of one element belongs to $\mathscr{C}$. Partially order $\mathscr{C}$ by inclusion. Let $\{Q(i), i \in I\}$ be a chain in $\mathscr{C}$. Put $Q = \cup_{i \in I} Q(i)$, let $k \in N$ and let $a_j (1 \le j \le k)$ be distinct elements of Q. There are $i_j \in I$ $(1 \le j \le k)$ such that $a_j \in Q(i_j)$ $(1 \le j \le k)$. Since $\{Q(i), i \in I\}$ is a chain there is some $j_0 : 1 \le j_0 \le k$ such that $\cup_{j=1}^{k} Q(i_j) = Q(i_{j_0})$. Since $Q(i_{j_0}) \in \mathscr{C}$ and since $a_j \in Q(i_{j_0})$ $(1 \le j \le k)$ it follows that a_j satisfy (4) and consequently $Q \in \mathscr{C}$. By Zorn lemma there exists a maximal element Q in $\mathscr{C}$.

Assume first that card $Q \le d$. Write $\mathscr{B} = \mathscr{A} - Q$. Clearly card $\mathscr{B} > d$. Given i, $1 \le i \le n$ denote $Q_i = \{\beta \in \Gamma : \beta = a_i$ for some $a = (a_1, a_2, \dots, a_n) \in Q\}$. Let $b = (b_1, b_2, \dots, b_n) \in \beta$. Assume that for every decomposition $\{1, 2, \dots, n\} = A \cup B$ where $A, B \ne \emptyset$; $A \cap B = \emptyset$,

$$g_i = b_i \quad (i \in A)$$
$$g_i \in Q_i \quad (i \in B).$$

implies that $g = (g_1, g_2, \dots, g_n) \notin \mathscr{A}$. This means that $Q \cup \{b\} \in \mathscr{C}$ which contradicts the maximality of Q. This proves that given any $b \in \mathscr{B}$ there is a decomposition $\{1, 2, \dots, n\} = A \cup B$, A, $B \ne \emptyset$; $A \cap B = \emptyset$, such that there is some $g \in \mathscr{A}$ satisfying $g_i = b_i$ $(i \in A)$ and $g_i \in Q_i$ $(i \in B)$. Since the set of all possible decompositions is finite and since card $\mathscr{B} > d$ there is some fixed decomposition $\{1, 2, \dots, n\} = A \cup B$, A, $B \ne \emptyset$; $A \cap B = \emptyset$ and some set $\mathscr{B}_1 \subset \mathscr{B}$, card $\mathscr{B}_1 > d$ such that for every $b \in \mathscr{B}_1$ there is some $g \in \mathscr{A}$ satisfying $g_j = b_j$ $(j \in A)$ and $g_j \in Q_j$ $(j \in B)$. Write each $b \in \mathscr{B}_1$ in the form $b = P_A(b) \oplus P_B(b)$ where $P_A(b) \in \Pi_{j \in A} \Gamma$ and $P_B(b) \in \Pi_{j \in B} \Gamma$ are defined by $(P_A(b))_j = b_j$ $(j \in A)$ and $(P_B(b))_j = b_j$ $(j \in B)$. We show that card $P_A(\mathscr{B}_1) > d$. To see this, assume that card $P_A(\mathscr{B}_1) \le d$. Since card $\mathscr{B}_1 > d$ it follows that there is some $\mathscr{B}_2 \subset \mathscr{B}_1$, card $\mathscr{B}_2 > d$ and some $u \in \Pi_{j \in A} \Gamma$ such that $u = P_A(b)$ for all $b \in \mathscr{B}_2$. In particular, there are an $i \in A$ and a $\gamma \in \Gamma$ such that $\gamma = b_i$ for all $b \in \mathscr{B}_2$ which contradicts (3) since card $\mathscr{B}_2 > d$. This proves that card $P_A(\mathscr{B}_1) > d$. For each $u \in P_A(\mathscr{B}_1)$ choose an element from $P_A^{-1}(u) \cap \mathscr{B}_1$ and denote the set of all these elements by $\mathscr{B}_2$. Clearly card $\mathscr{B}_2 > d$ and

$$P_A(a) \ne P_A(b) \quad (a, b \in \mathscr{B}_2; a \ne b). \tag{5}$$

Recall that for every $b \in \mathscr{B}_2$ there is some $g \in \mathscr{A}$ such that $P_A(b) = P_A(g)$ and such that $g_j \in Q_j$ $(j \in B)$. Since card $Q_j \le$ card $Q \le d$ $(1 \le j \le n)$ it follows that card $\Pi_{j \in \mathscr{B}} Q_j \le d$. Since card $\mathscr{B}_2 > d$ it follows that there is some $\mathscr{B}_3 \subset \mathscr{B}_2$, card $\mathscr{B}_3 > d$ and some $v \in \Pi_{j \in B} \Gamma$

such that for every $b \in \mathscr{B}_3$ there is some $g \in \mathscr{A}$ satisfying $P_A(b) = P_A(g)$ and $P_B(g) = v$. By (5) it follows that there are an $i \in B$ and a $\gamma \in \Gamma$ such that $\operatorname{card}\{a \in \mathscr{A} : a_i = \gamma\} > d$ which contradicts (3). Thus we have proved that card $Q > d$.

Since $Q \in \mathscr{C}$ it follows that

$$a, b \in Q,\ a \neq b \text{ implies that } a_i \neq b_i\ (1 \leq i \leq n). \tag{6}$$

Let $k \in N$ and let a_j $(1 \leq j \leq k)$ be distinct elements of Q where $a_j = (a_{j1}, a_{j2}, \ldots, a_{jn})$ $(1 \leq j \leq k)$. Recall that $Q \in \mathscr{C}$. So if $1 \leq j_i \leq k$ $(1 \leq i \leq n)$ then $(a_{j_1 1}, a_{j_2 2}, \ldots, a_{j_n n}) \in \mathscr{A}$ implies that $j_1 = j_2 = \ldots = j_n$ i.e. if $j_1 = j_2 = \ldots = j_n$ is not satisfied then $P(e(a_{j_1 1}), e(a_{j_2 2}), \ldots, e(a_{j_n n})) = 0$. It follows that

$$\left.\begin{aligned} &P\Big(\sum_{i_1=1}^{k} \zeta_{i_1} e(a_{i_1 1}), \sum_{i_2=1}^{k} e(a_{i_2 2}), \ldots, \sum_{i_n=1}^{k} e(a_{i_n n})\Big) \\ &= \sum_{i_1=1}^{k} \sum_{i_2=1}^{k} \cdots \sum_{i_n=1}^{k} \zeta_{i_1} P(e(a_{i_1 1}), e(a_{i_2 2}), \ldots, e(a_{i_n n})) = \\ &= \sum_{i=1}^{k} \zeta_i (P(e(a_{i1}), e(a_{i2}), \ldots, e(a_{in})). \end{aligned}\right\} \tag{7}$$

Put $D = Q$ and define the map ϕ from the basis $\{e(d); d \in D\}$ of $c_0(D)$ to $Y - \{0\}$ by

$$\phi(e(d)) = P(e(d_1), e(d_2), \ldots, e(d_n))\ (d = (d_1, d_2, \ldots, d_n) \in D).$$

Let $k \in N$ and let a_i $(1 \leq i \leq k)$ be distinct elements of D where $a_i = (a_{i1}, a_{i2}, \ldots, a_{in})$ $(1 \leq i \leq k)$. Let $|\zeta_i| \leq 1$ $(1 \leq i \leq k)$. By (6) we have

$$a_{ij} \neq a_{rj}\ (1 \leq j \leq n; 1 \leq i, r \leq k; i \neq r)$$

and it follows that

$$\Big\|\sum_{i=1}^{k} \zeta_i e(a_{ij})\Big\| \leq 1\ (1 \leq j \leq n; |\zeta_i| \leq 1\ (1 \leq i \leq k)).$$

By (7) it follows that

$$\begin{aligned} \Big\|\sum_{i=1}^{k} \zeta_i \phi(e(a_i))\Big\| &= \Big\|\sum_{i=1}^{k} \zeta_i P(e(a_{i1}), e(a_{i2}), \ldots, e(a_{in}))\Big\| \\ &= \Big\|P\Big(\sum_{i_1=1}^{k} \zeta_{i_1} e(a_{i_1 1}), \sum_{i_2=1}^{k} e(a_{i_2 2}), \ldots, \sum_{i_n=1}^{k} e(a_{i_n n})\Big)\Big\| \\ &\leq \|P\| \cdot \Big\|\sum_{i_1=1}^{k} \zeta_{i_1} e(a_{i_1 1})\Big\| \Big\|\sum_{i_2=1}^{k} e(a_{i_2 2})\Big\| \cdots \Big\|\sum_{i_n=1}^{k} e(a_{i_n n})\Big\| \leq \|P\|. \end{aligned}$$

Consequently ϕ admits a bounded linear extension L to all $c_0(D)$. Since card $D > d$ this completes the proof for $m = n$. Q.E.D.

COROLLARY 2: *Let $X = c_0(\Gamma)$ where Γ is an infinite set and let Y be a Banach space. Suppose that the range of every bounded linear map from X to Y is separable. Then the range of every analytic map from $B_1(X)$ to Y is separable.*

PROOF: If Γ is countable there is nothing to prove so assume that Γ is uncountable. Suppose that there is an analytic map from $B_1(X)$ to Y with nonseparable range. By Theorem there are an uncountable set Δ and a bounded linear map $A: c_0(\Delta) \to Y$ which maps $c_0(\Delta)$ isomorphically onto $R(A)$. Since $c_0(\Delta)$ is up to isometry determined by card Δ assume with no loss of generality that either $\Delta \subset \Gamma$ or $\Gamma \subset \Delta$. If $\Delta \subset \Gamma$ define $B: X \to Y$ by $B = A \circ P$ where P is the projection from X onto $c_0(\Delta)$ defined by $(Px)(\gamma) = x(\gamma)\,(\gamma \in \Delta; x \in X)$. $B: X \to Y$ is a bounded linear map whose range $R(B) = R(A)$ is nonseparable, a contradiction. Let $\Gamma \subset \Delta$. Since Γ is uncountable X is a nonseparable subspace of $c_0(\Delta)$ and by the properties of A, $A(X)$ is nonseparable. Consequently $A|X: X \to Y$ is a bounded linear map with nonseparable range, a contradiction. Q.E.D.

REMARK 2: Let Γ be an uncountable set and let $1^p(\Gamma)$ $(1 \le p < \infty)$ be the Banach space of all scalar-valued functions x on Γ such that $\|x\| = (\Sigma_{\gamma \in \Gamma} |x(\gamma)|^p)^{1/p} < \infty$. Since every bounded linear map from $1^2(\Gamma)$ to $1^1(\Gamma)$ is compact [10] it follows that the range of every bounded linear map from $1^2(\Gamma)$ to $1^1(\Gamma)$ is separable. On the other hand, the range of the bounded 2-homogeneous polynomial $P: 1^2(\Gamma) \to 1^1(\Gamma)$ defined by $P(x) = y$ where $y(\gamma) = x(\gamma)^2$ $(\gamma \in \Gamma, x \in 1^2(\Gamma))$ is nonseparable since P is surjective. This shows that Corollary 2 does not hold in general. We ask under which conditions on a Banach space X does the assertion of Corollary 2 hold.

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