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On determinantal ideals which are set-theoretic complete intersections

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Let $A$ be an $r \times s$ ($r \leq s$) matrix with entries in a commutative noetherian ring $R$ with identity. We shall denote by $(A)$ the ideal generated by its subdeterminants of order $r$. If $(A)$ is a proper ideal of $R$, then the height of $(A)$, abbreviated as $h(A)$, is at most $s - r + 1$ (see [1], Theorem 3). In this paper we prove that there exist elements $f_1, \ldots, f_{s-r+1} \in (A)$ such that $\text{rad}(A) = \text{rad}(f_1, \ldots, f_{s-r+1})$ (where $\text{rad}(I)$ means the radical of the ideal $I$) in each of the following situations:

1. $A = \begin{pmatrix} a_{ii} \end{pmatrix}$ is an $r \times s$ matrix such that $a_{ij} = a_{kl}$ if $i + j = k + l$.
2. $A$ is an $r \times (r+1)$ partly symmetric matrix, where partly symmetric means that the $r \times r$ matrix obtained by omitting the last column is symmetric.
3. $A = \begin{pmatrix} a_{pq} & b_{pq} & c_{pq} \\ b_{pq} & c_{pq} & a_{pq} \\ c_{pq} & a_{pq} & b_{pq} \end{pmatrix}$ where $(a, b, c)$ is an ideal of height 3 and $p_i, q_i, r_i$ are positive integers not necessarily distinct.

It follows that if $h(A)$ is as large as possible, $s - r + 1$, then the above determinantal ideals are set-theoretic complete intersections.

It is interesting to compare these results with the following theorem due to M. Hochster (never published).

**Theorem:** Let $t < r < s$ be integer, and let $k$ be a field of characteristic 0. Let $A = k[X_{ij}]$ be the ring of polynomials in $rs$ variables, and let $I_t(X)$ be the ideal generated by the $t \times t$ minors of the $r \times s$ matrix $(X_{ij})$. Then $I_t(X)$ is not set theoretically a complete intersection.

Let $A = \|a_{ij}\|$ be an $r \times s$ given matrix, where $a_{ij} \in R$ and $r \leq s$. In

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this section we assume that $a_{ij} = a_{kl}$ if $i + j = k + l$, hence we may write

$$A = \begin{vmatrix} a_{11} & a_{21} & \cdots & a_{1r} \\ a_{21} & a_{22} & \cdots & a_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ a_{r1} & a_{r2} & \cdots & a_{rr} \end{vmatrix}$$

We shall denote by $(A)$ the ideal generated by the $r$-rowed minors of $A$ and if $0^r = (0_{\sigma_1}, \ldots, 0_{\sigma_r})$ is a set of $r$ integers such that $1 \leq \sigma_1 < \sigma_2 < \cdots < \sigma_r \leq s$, we put

$$A_\sigma = \begin{vmatrix} a_{\sigma_1} & a_{\sigma_2} & \cdots & a_{\sigma_r} \\ a_{\sigma_1+1} & a_{\sigma_2+1} & \cdots & a_{\sigma_{r+1}} \\ \vdots & \vdots & \ddots & \vdots \\ a_{\sigma_1+r-1} & a_{\sigma_2+r-1} & \cdots & a_{\sigma_{r+r-1}} \end{vmatrix}$$

and $d_\sigma = \det A_\sigma$.

If $i = r, \ldots, s$ let $\mathcal{A}_i$ be the ideal generated by the $d_\sigma$ with $\sigma \leq i$; then $\mathcal{A}_s = (A)$ and, with a self explanatory notation, $\mathcal{A}_i = (\mathcal{A}_{i-1}, d_\sigma)_{\sigma=i}$ (where $\mathcal{A}_{r-1} = (0)$).

Next for all $i = r, \ldots, s$, let $f_i$ be the determinant of the $i \times i$ matrix

$$M_i = \begin{vmatrix} a_{11} & \cdots & a_{ir} & \cdots & a_{i} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{r1} & \cdots & a_{r} & \cdots & a_{i+r-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & \cdots & a_{i} & 0 & 0 \end{vmatrix}$$

It is clear that $\mathcal{A}_r = (f_i)$ and $f_i \in \mathcal{A}_i$ for all $i = r, \ldots, s$.

**Theorem 1.1:** With the above notations, we have:

$$\text{rad}(\mathcal{A}_i) = \text{rad}(\mathcal{A}_{i-1}, f_i)$$

for all $i = r, \ldots, s$.

**Proof:** Since $(\mathcal{A}_{i-1}, f_i) \subseteq \mathcal{A}_i$ we need only to prove that $\mathcal{A}_i \subseteq \text{rad}(\mathcal{A}_{i-1}, f_i)$. This is true if $i = r$, hence we may assume $i > r$. Now $\mathcal{A}_i = (\mathcal{A}_{i-1}, d_\sigma)_{\sigma=i}$, so it is enough to show that $d_\sigma \in \text{rad}(\mathcal{A}_{i-1}, f_i)$ for all $\sigma$ such that $\sigma_r = i$. Let $\sigma = (\sigma_1, \ldots, \sigma_r = i)$; then
Hence, by expanding the determinant along the last column, we get
\[ d_\sigma = \sum_{k=0}^{r-1} a_{i+k} c_k \] where \( c_k \) is the cofactor of \( a_{i+k} \) in \( A_\sigma \). Denote by \( \lambda_m \) \((m = 1, \ldots, i)\) the \( m \)-th row of \( M_i \) and let \( 1 \leq \tau_1 < \tau_2 < \cdots < \tau_{i-r} \leq i-1 \), where \( \{\tau_1, \ldots, \tau_{i-r}\} \) is the complement of \( \{\sigma_1, \ldots, \sigma_r = i\} \) in \( \{1, 2, \ldots, i\} \).

Then if \( j = 1, \ldots, i-r \) we have \( j \leq \tau_j \leq \tau_{i-r} - (i-r-j) \leq i-1 - i + r + j = r + j - 1 \).

Denote by \( N_i \) the matrix obtained from \( M_i \) by replacing, for all \( j = 1, \ldots, i-r \), the row \( \lambda_j \) by \( \sum_{k=0}^{r-1} \lambda_{j+k} c_k \); since, as we have seen, \( j \leq \tau_j \leq r + j - 1 \), in this linear combination \( \lambda_j \) has coefficient \( c_{\tau_j} \). It follows that

\[ \det N_i = \left( \prod_{j=1}^{i-r} c_{\tau_j-j} \right) f_i. \]

Denote by \( m_{pq} \) the entries of the matrix \( M_i \) and by \( n_{pq} \) those of \( N_i \); then \( m_{j+k,l} = a_{j+k+l-1} \) (where \( a_t = 0 \) if \( t > i + r - 1 \)), hence \( n_{j,l} = \sum_{k=0}^{r-1} a_{j+k+l-1} c_k \) for all \( j = 1, \ldots, i-r \) and \( l = 1, \ldots, i-j+1 \). It follows that for all \( j = 1, \ldots, i-r \) if \( 1 \leq l \leq i-j+1 \), \( n_{\tau_j, l} \) is the determinant of the matrix obtained by replacing the last column of \( A_\sigma \) by the \((j+l-1)\)-th column of \( A \). Therefore we get:

1. \( n_{\tau_j} = 0 \) if \( j + l - 1 \in \{\sigma_1, \ldots, \sigma_{r-1}\} \).
2. \( n_{\tau_j} = d_\sigma \) if \( j + l - 1 = i \), or, which is the same, \( l = i-j+1 \).
3. \( n_{\tau_j} \in \mathbb{A}_{i-1} \) if \( j + l - 1 \in \{\tau_1, \ldots, \tau_{i-r}\} \) and this because \( \tau_{i-r} \leq i-1 \) and \( \sigma_{r-1} \leq i-1 \).

So we get for all \( j = 1, \ldots, i-r \); \( n_{\tau_j} \in \mathbb{A}_{i-1} \) if \( l = 1, \ldots, i-j \) and \( n_{\tau_{i-r}, i-j+1} = d_\sigma \). Then we can write

\[ \det N_i = \det \begin{vmatrix} \cdot & \cdot & \cdots & \cdot & \cdots & \cdot & \cdots & \cdot & \cdots & \cdot & \cdots & \cdot \\ n_{\tau_{i-r}} & n_{\tau_{i-r-2}} & \cdots & n_{\tau_{i-r}} & \cdots & n_{\tau_{i-r-2}} & \cdots & n_{\tau_{i-r}} & \cdots & \cdot \\ n_{\tau_{i-r-2}} & n_{\tau_{i-r-2}} & \cdots & n_{\tau_{i-r-2}} & \cdots & n_{\tau_{i-r-2}} & \cdots & n_{\tau_{i-r-2}} & \cdots & \cdot \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \cdot \\ a_i & a_{i+1} & \cdots & a_{i+r-1} & \cdots & a_i & \cdots & a_{i+r-1} & \cdots & \cdot \end{vmatrix}. \]
By expanding the determinant along the first $r$ columns we get
\[
\text{det } N_i = \pm \text{det } \begin{vmatrix}
0 & 0 & \cdots & 0 & \cdots & 0 & d_\sigma \\
. & . & \cdots & . & \cdots & . & . \\
0 & 0 & \cdots & 0 & \cdots & d_\sigma & 0 \\
. & . & \cdots & . & \cdots & . & . \\
0 & 0 & \cdots & 0 & d_\sigma & . & . \\
. & . & \cdots & . & \cdots & . & . \\
1_i & 1_{i+1} & \cdots & 1_{i+r-1} & \cdots & . & . \\
\end{vmatrix}
\mod \mathfrak{A}_{i-1}.
\]

By expanding the determinant along the first $r$ columns we get
\[
\text{det } N_i = \pm \text{det } \begin{vmatrix}
a_{\sigma_1} & a_{\sigma_1+1} & \cdots & a_{\sigma_1+r-1} \\
a_{\sigma_2} & a_{\sigma_2+1} & \cdots & a_{\sigma_2+r-1} \\
. & . & \cdots & . \\
1_i & 1_{i+1} & \cdots & 1_{i+r-1} \\
\end{vmatrix}
\text{det } \begin{vmatrix}
0 & 0 & \cdots & d_\sigma \\
0 & 0 & \cdots & d_\sigma \\
. & . & \cdots & . \\
d_\sigma & . & \cdots & . \\
\end{vmatrix}
\mod \mathfrak{A}_{i-1}.
\]

but clearly $A_\sigma$ is a symmetric matrix, hence $\text{det } N_i = \pm d_\sigma^{r+r+1} \mod \mathfrak{A}_{i-1}$. It follows that $d_\sigma \in \text{rad}(\mathfrak{A}_{i-1}, f_i)$, since, as we have seen, $\text{det } N_i \in (f_i)$; this completes the proof.

**Corollary 1.2:** With $A$ and $f_1, \ldots, f_s$ as before, we have:

\[
\text{rad}(A) = \text{rad}(f_1, \ldots, f_s).
\]

**Proof:** By Theorem 1.1,

\[
\text{rad}(A) = \text{rad}(\mathfrak{A}_s) = \text{rad}(\mathfrak{A}_{s-1}, f_s) = \text{rad}(\text{rad}(\mathfrak{A}_{s-1}) + \text{rad}(f_s)) \\
= \text{rad}(\text{rad}(\mathfrak{A}_{s-2}, f_{s-1}) + \text{rad}(f_s)) = \text{rad}(\mathfrak{A}_{s-2}, f_{s-1}, f_s) \\
= \cdots = \text{rad}(\mathfrak{A}_r, f_{r+1}, \ldots, f_s) = \text{rad}(f_r, \ldots, f_s).
\]

**Remark 1.3:** If the elements of the matrix $A$ are indeterminates over an algebraically closed field $k$, the ideal $(A)$ is the defining ideal of the locus $V$ of chordal $[r - 2]$'s of the normal rational curve of $P^{s+r-2}$, where if $p \geq 2$ a chordal $[p - 1]$ of a manifold is one which meets it in $p$ independent points (see [4] pag. 91 and 229). $V$ is a projective variety in $P^{s+r-2}$ of dimension $2r - 3$ and order $\binom{s}{r+1}$; hence the codimension of $V$ is $s + r - 2 - 2r + 3 = s - r + 1$ and the above result proves that $V$ is set-theoretic complete intersection. The case $r = 2$ is the main result in [5].
In this section $A$ is a partly symmetric $r \times (r + 1)$ matrix whose elements belong to $R$. Therefore we may write

$$A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1r} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2r} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{r1} & a_{r2} & \cdots & a_{rr} & b_r \end{vmatrix}$$

where the matrix $S = \|a_{ij}\|$ is $r \times r$ symmetric.

Let $B = \begin{vmatrix} A \\ b_1 \ldots b_{r0} \end{vmatrix}$, $f_1 = \det S$ and $f_2 = \det B$; next, for all $i = 1, \ldots, r + 1$, denote by $A_i$ the matrix which results when the $i$-th column of $A$ is deleted, and put $d_i = \det A_i$. Then $f_1 = d_{r+1}$, $(A) = (d_1, \ldots, d_{r+1})$ and $f_2 \in (A)$.

**Theorem 2.1:** With the above notations we have:

$$\text{rad}(A) = \text{rad}(f_1, f_2).$$

**Proof:** Since $(f_1, f_2) \subseteq (A)$ and $d_{r+1} = f_1$, it is enough to prove that $(d_1, \ldots, d_r) \subseteq \text{rad}(f_1, f_2)$. Let $i$ be any integer, $1 \leq i \leq r$; by expanding the determinant of $A_i$ along the last column, we get $d_i = \sum_{k=1}^{r} b_k c_{ki}$ where $c_{ki}$ is the cofactor of $b_k$ in $A_i$. Denote by $B'$ the matrix obtained by replacing the $i$-th row of $B$ by the linear combination of the first $r$ rows of $B$ with coefficients $c_{1i}$, $c_{2i}, \ldots, c_{ri}$. Then it is clear that $\det B' = c_{ii} \det B$ and the $i$-th row of $B'$ is:

$$\left( \sum_{k=1}^{r} a_{k1} c_{ki}, \ldots, \sum_{k=1}^{r} a_{kr} c_{ki}, \sum_{k=1}^{r} b_k c_{ki} \right).$$

But $\sum_{k=1}^{r} a_{kj} c_{ki}$ is the determinant of the matrix obtained by replacing the last column of $A_i$, by the $j$-th column of $A$. Hence $\sum_{k=1}^{r} a_{kj} c_{ki} = 0$ if $j \neq i$, while $\sum_{k=1}^{r} a_{ki} c_{ki} = \pm f_1$. Therefore we get:

$$c_{ij} f_2 = \det B' = \det \begin{vmatrix} a_{11} & \cdots & a_{1r} & b_1 \\ \cdot & \cdots & \cdot & \cdot \\ a_{i-1,1} & \cdots & a_{i-1,r} & b_{i-1} \\ 0 & \cdots & 0 & d_i \\ a_{i+1,1} & \cdots & a_{i+1,r} & b_{i+1} \\ \cdot & \cdots & \cdot & \cdot \\ a_{r1} & \cdots & a_{rr} & b_r \\ b_1 & \cdots & b_r & 0 \end{vmatrix} \mod f_1.$$
By expanding this determinant along the first $r$ columns we get:

$$c_{if_2} = \pm d_i \det A_i \mod f_1;$$

But $S$ is symmetric, hence $c_{if_2} = \pm d_i \det A_i = \pm d_i^2 \mod f_1$, and the theorem is proved.

**EXAMPLE 2.2:** Let $V$ be the rational cubic scroll in $\mathbb{P}^4$; then it is well known that $V$ is the locus where $rk \begin{bmatrix} X_0 & X_1 & X_3 \\ X_1 & X_2 & X_4 \end{bmatrix} = 1$. Hence the above theorem shows that $V$ is set-theoretic complete intersection.

### 3

In this last section we will be interested in a particular $2 \times 3$ matrix. Suppose $a$, $b$ and $c$ are elements of the ring $R$, such that the ideal they generate is of height 3; next let $p_i$, $q_i$, $r_i$ ($i = 1, 2$) positive integers not necessarily distinct. Let us consider the $2 \times 3$ matrix

$$A = \begin{bmatrix} a^{p_1} & b^{q_1} & c^{r_1} \\ b^{q_2} & c^{r_2} & a^{p_2} \end{bmatrix}$$

and put $p = p_1 + p_2$, $q = q_1 + q_2$, $r = r_1 + r_2$ and $f_1 = b^{q_1} a^{p_2} - c^r$, $f_2 = a^p - b^{q_2} c^{r_1}$, $f_3 = a^{p_1} c^{r_2} - b^q$.

We want to show that if $(A) = (f_1, f_2, f_3)$ then rad$(A)$ is equal to the radical of an ideal generated by 2 elements; but first we shall give some remarks which are useful in the following.

Let $k$ be any integer, $0 \leq k \leq q$; then we can write

(1) \hspace{1cm} kq_1 = tq + s \hspace{1cm} \text{where } 0 \leq s \leq q - 1.

Hence we have $kq = kq_1 + kq_2 = tq + s + kq_2$; it follows that

(2) \hspace{1cm} q_2(q - k) = (q_2 - k + t)q + s \hspace{1cm} \text{for all } k = 0, \ldots, q.$
Now, since \( q^2(q - k) \geq 0 \), we have \( (q^2 - k + t)q + s \geq 0 \); but \( s < q \) by (1), hence

\[
q^2 - k + t \geq 0 \quad \text{for all } k = 0, \ldots, q.
\]

Then we have also

\[
0 \leq (q - k)r_1 + r_2(q^2 - k + t) = (q - k)r + tr_2 - q_1r_2 \quad \text{for all } k = 0, \ldots, q.
\]

This allows us to consider the element

\[
g = \sum_{k=0}^{q} (-1)^{q-k} \binom{q}{k} a^{kp_1}b^{k^2(q - k)}c^{r(q-k)+tr_2-q_1r_2}.
\]

**Theorem 3.1:** With the above notations we have:

\[
\text{rad}(A) = \text{rad}(g, f_3).
\]

**Proof:** We have

\[
f_1^q = (b^{q_1}a^{p_2} - c^r)^q = \sum_{k=0}^{q} (-1)^{q-k} \binom{q}{k} a^{kp_1}b^{k^2(q - k)}c^{r(q-k)+tr_2-q_1r_2} \quad \text{mod } f_3,
\]

since by (1) \( kq_1 = tq + s \) for all \( k = 0, \ldots, q \) we get

\[
f_1^q = \sum_{k=0}^{q} (-1)^{q-k} \binom{q}{k} a^{kp_1}b^{k^2(q - k)}c^{r(q-k)+tr_2-q_1r_2} \quad \text{mod } f_3,
\]

or \( f_1^q = c^{q_1r_2}g \mod f_3 \). On the other hand

\[
f_2^q = (a^p - b^{q_2}c^r)^q = \sum_{k=0}^{q} (-1)^{q-k} \binom{q}{k} a^{kp_1}b^{q_2(q - k)}c^{r(q-k)},
\]

hence, using (2) and (3) we get

\[
f_2^q = \sum_{k=0}^{q} (-1)^{q-k} \binom{q}{k} a^{kp_1(q_2-k+t)}b^{r(q-k)+tr_2-q_1r_2} \quad \text{mod } f_3.
\]

But \( kp + p_1(q_2 - k + t) = kp_2 + p_1q_2 + tp_1 \), hence, using (4), we get \( f_2^q = a^{p_1q_2}g \mod f_3 \). This proves that \( (A) \subseteq \text{rad}(g, f_3) \).

Next we have seen that \( f_1^q = c^{q_1r_2}g \mod f_3 \); hence \( c^{q_1r_2}g \in (A) \). Let \( \exists g \)
be a minimal prime ideal of \((A)\), then \(h(\mathfrak{P}) \leq 2\) by [1, Theorem 3], so \(c \not \in \mathfrak{P}\), because if \(c \in \mathfrak{P}\) then \((a, b, c) \subseteq \mathfrak{P}\) which is a contradiction since we have assumed \(h(a, b, c) = 3\). It follows that \(g \in \text{rad}(A)\); this completes the proof.

**Example 3.2:** Let \(k\) be an arbitrary field, \(t\) transcendental over \(k\). Let \(n_1, n_2, n_3\) natural numbers with greatest common divisor 1, and let \(C\) be the affine space curve with the parametric equations \(X = t^{n_1}\), \(Y = t^{n_2}\), \(Z = t^{n_3}\). Let \(c_i\) be the smallest positive integer such that there exist integers \(r_{ij} \geq 0\) with \(c_1n_1 = r_{12}n_2 + r_{13}n_3\), \(c_2n_2 = r_{21}n_1 + r_{23}n_3\), \(c_3n_3 = r_{31}n_1 + r_{32}n_2\). In [2] it is proved that if \(C\) is not a complete intersection then \(r_{ij} > 0\) for all \(i, j\) and \(c_1 = r_{21} + r_{31}\), \(c_2 = r_{12} + r_{32}\), \(c_3 = r_{13} + r_{23}\).

Further, if \(f_1 = X^{c_1}Y^{n_2} - Z^{c_2}\), \(f_2 = X^{c_1}Y^{n_3}Z^{n_3}\) and \(f_3 = X^{c_1}Z^{c_3} - Y^{c_2}\), then the vanishing ideal \(I(C) \subseteq k[X, Y, Z]\) of \(C\) is \(I(C) = (f_1, f_2, f_3)\). Then it is easy to see that \(I(C)\) is the ideal generated by the \(2 \times 2\) minors of the matrix

\[
\begin{pmatrix}
X^{c_1} & Y^{n_2} & Z^{n_3} \\
Y^{n_2} & Z^{c_2} & X^{c_3}
\end{pmatrix}
\]

It follows, by Theorem 3.1, that \(C\) is set-theoretic complete intersection. This result has been proved in [3] by completely different methods; see also [6].

Finally we remark that if \(C = \{(t^5, t^7, t^8) \in \mathbb{A}^3(k)\}\) then the matrix is

\[
\begin{pmatrix}
X & Y^2 & Z \\
Y & Z^2 & X^2
\end{pmatrix}
\]

which is not partly symmetric; so the conclusion that \(C\) is set-theoretic complete intersection cannot be drawn from Theorem 2.1.

**Example 3.3:** Let \(n, p\) be non-negative integers; we have seen (see Example 3.2) that if

\[
C = \{(t^{2n+1}, t^{2n+1+p}, t^{2n+1+2p}) \in \mathbb{A}^3(k)\},
\]

the vanishing ideal \(I(C)\) in \(k[X_1, X_2, X_3]\) is generated by \(X_1^{n+p}X_2 - X_3^{n+p+1}\), \(X_1^{n+p+1} - X_2X_3^n\) and \(X_1X_3 - X_2^n\). Let \(\bar{C}\) be the projective closure of \(C\) in \(\mathbb{P}^3\). Since \(C\) has only one point at the infinity, it is well known that the homogeneous ideal of \(\bar{C}\) in \(k[X_0, X_1, X_2, X_3]\) is generated by the polynomials \(X_1^{n+p}X_2 - X_0^pX_3^{n+1}\), \(X_1^{n+p+1} - X_0^pX_2X_3^n\) and \(X_1X_3 - X_2^n\)· It is immediately seen that this ideal is generated by the \(2 \times 2\) minors of the matrix

\[
\begin{pmatrix}
X_1^{n+p} & X_2 - X_0^pX_3^{n+1} \\
X_2 & X_3^n - X_0^pX_2X_3^n
\end{pmatrix}
\]
Thus, by Theorem 2.1, \( \tilde{C} \) is set-theoretic complete intersection of the
two hypersurfaces \( X_1 X_3 - X_2^2 \) and \( X_2^{p_0} X_2^{n+1} + X_1^{2n+2p+1} - 2X_0^{p} X_1^{n+p} X_2 X_3^n \).

**Example 3.4:** If \( C = \{(t^3, t^7, t^8) \in \mathbb{A}^3(k)\} \), the vanishing ideal
\( I(\tilde{C}) \subseteq k[X_0, X_1, X_2, X_3] \) of the projective closure \( \tilde{C} \) of \( C \) in \( \mathbb{P}^3 \), needs
five generators and our methods do not apply in order to see if \( \tilde{C} \) is
set-theoretic complete intersection.

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