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ON THE COHOMOLOGY OF TWISTOR FLAG SPACES

G.M. Henkin and Yu.I. Manin

Aldo Andreotti in memoriam

Introduction

In [1] the authors have shown that a number of fundamental physical equations for the classical fields in a domain of the complete complex Minkowski space admit a consistent reinterpretation in cohomological terms. The cohomology in question involves coherent sheaves on the null-line space. The Yang–Mills (and Maxwell) equations for the gauge (connection) fields and the Dirac equations for the sections of spinor bundles turn out to be equivalent to certain conditions on the obstructions to extending infinitesimally vector bundles and cohomology classes. A review of earlier results concerning chiefly self-dual situations can be found in [2]–[4]. The version of the Penrose transform used in [1] and here was conceived in [5] and [8].

The objective of this note is to present in the explicit form the main cohomological calculations needed to justify the assertions made in [1]. In §1 we explain notations and state the principal theorem on the cohomology of the null-line space. In §2 we display the calculations. Finally, in §3 we briefly review the applications to the field equations.

The calculations of cohomology is a traditional theme in algebraic and analytic geometry. Many cohomology groups of compact, especially algebraic projective varieties, are explicitly known. In the noncompact case the fundamental notion of strictly q-convex domain was introduced by Andreotti and Grauert in [9]. All cohomology of a strictly q-convex domain is finite-dimensional except, possibly, H^q. In

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the context of Penrose transform this case is realized in auto-dual situations where the H^1-cohomology in subdomains of P^3 corresponds to massless fields.

The spaces considered in this article are certainly not strictly q-convex. Our method of calculations which can be traced back to Andreotti–Norguet [10] consists essentially in restricting cohomology on maximal compact subspaces.

§1. Notation and results

1. Let T be a four-dimensional vector space over the field of complex numbers (Penrose's twistor space). The Grassmanian $\mathrm{Gr}(2, T)$ of two-dimensional complex subspaces of T is the Penrose model CM of the Minkowski space. A point $x \in CM$ determines the plane $S_+(x) \subset T$ and the plane $S_-(x) = S_+(x)^\perp \subset T^*$ in the dual twistor space. The corresponding vector bundles $S_\pm \to CM$ are called spinor bundles. We will not distinguish between holomorphic vector bundles and their sheaves of sections. The canonical isomorphism of the cotangent bundle $\Omega^1\mathrm{Gr}(2, T)$ with $S_+ \otimes S_-$ defines in $S^2(\Omega^1)$ the line subbundle $\Lambda^2 S_+ \otimes \Lambda^2 S_-$, i.e. "the conformal holomorphic metric".

The Plücker embedding $CM \to P(\Lambda^2 T)$: $x \mapsto \Lambda^2 S_+(x)$ identifies CM with the four-dimensional projective quadric of decomposable bitwistors. The lines of $P(\Lambda^2 T)$ lying in this quadric are precisely complex null-geodesics of the conformal metric defined above. The space of these lines L can be identified with the $(1, 3)$-flag space of T. Namely, the point x lies on the line corresponding to the flag $T_1 \subset T_3$ iff $T_1 \subset S_+(x) \subset T_3$. Thus the incidence relation graph $F \subset L \times CM$ is the $(1, 2, 3)$-flag space of T. We denote the corresponding projections by π_1: $F \to L$ and π_2: $F \to CM$.

In the following we will be concerned mostly with a non-compact piece of this picture. We choose a Stein open connected subset $U \subset CM$ with the following property: the morphism π_1: $F(U) = \pi_2^{-1}(U) \to L(U) = \pi_1\pi_2^{-1}(U)$ is Stein and has connected and simply connected fibers. Note that these fibers are intersections of null-lines with U. For a point $x \in U$ we set $L(x) = \pi_1\pi_2^{-1}(x)$. This is a two-dimensional quadric $CP^1 \times CP^1$, the base of the light cone of the point x.

Fix a holomorphic vector bundle $E \to U$ endowed with a holomorphic connection ∇: $E \to E \otimes \Omega^1$, represented by the covariant differential. It was shown in [5] and [8] that all information about

(E, ∇) can be encoded in a vector bundle $E_L \to L(U)$ which is U-trivial that is $E_L \mid_{L(x)}$ is trivial for all $x \in U$. Namely, the fiber of E_L over a point of $L(U)$ is the space of the ∇-horizontal sections of E over (the part of) the corresponding null-line in CM. This construction actually defines the equivalence of the categories, compatible with internal Hom's. tensor products and restrictions of the structure group (cf. [6]).

The embedding $L \subset P(T) \times P(T^*)$ induces on L and $L(U)$ the invertible sheaves $\mathcal{O}(a, b)$. We set $E_L(a, b) = E_L \otimes \mathcal{O}(a, b)$. In the following theorem we collect the information about all cohomology groups alluded to in [1].

2. Theorem. *In the conditions stated above and for all values of* $(i; a, b)$ *stated below the following isomorphisms hold*:

$$H^i(L(U), E_L(a, b)) = \Gamma(U, E \otimes S(i; a, b))$$

where $S(i; a, b)$ *are the sheaves on* U, *shown in the following table*:

$i \backslash (a,b)$	$(-1, 0)$	$(-1, -1)$	$(-2, 0)$	$(-2, -1)$	$(-2, -2)$	$(-3, -1)$	$(-3, -2)$
1	$S_- \otimes \Lambda^2 S_+$	$\Lambda^2 S_+ \otimes \Lambda^2 S_-$	$\Lambda^2 S_+$	0	0	0	0
2	0	0	0	0	$\Lambda^2 S_+ \otimes \Lambda^2 S_-$	$(\Lambda^2 S_+)^2 \otimes \Lambda^2 S_-$	$S_+ \otimes \Lambda^2 S_+ \otimes \Lambda^2 S_+$

Moreover,

a) *Denote by* ∇_3: $E \otimes \Omega^3 \to E \otimes \Omega^4$ *the differential in the de Rham sequence of* E *induced by the connection* ∇. *Then* $H^2(L(U), E_L(-3, -3)) = \Gamma(U, \operatorname{Ker} \nabla_3)$; $H^3(L(U), E_L(-3, -3)) = \Gamma(U, E \otimes \Omega^4)/\operatorname{Im} \nabla_3$; $H^i(L(U), E_L(-3, -3)) = 0$ *for* $i \neq 2, 3$.

b) $H^i(L(U), E_L(a, b)) = 0$ *for* $i \neq 1, 2$ *and all* (a, b)*'s in the table.*

c) *Interchanging* a, b *results in interchanging* S_+, S_- *in* $S(i; a, b)$.

3. Remarks. a) Actually in the following section we will show how to calculate $H^i(L(U), E_L(a, b))$ for all (a, b)'s. But in general the results are less explicit.

b) We have $\Lambda^2 S_+ \simeq \Lambda^2 S_- \simeq \mathcal{O}(-1)$ on CM, this last sheaf being induced by the Plücker embedding. Nevertheless we prefer not to identify these sheaves since most of our computations are equally valid in the more general context of the conformally curved holomorphic space-times where there are no intrinsic isomorphism between $\Lambda^2 S_+$ and $\Lambda^2 S_-$.

§2. Proofs

1. Let G be a locally free sheaf on $L(U)$. We denote by $\pi_1^{-1}(G)$ its inverse image on $F(U)$ in the sheaf-theoretic sense and by $\pi_1^*(G) = \mathcal{O}_F \underset{\mathcal{O}_L}{\otimes} \pi_1^{-1}(G)$ the corresponding coherent sheaf. This last sheaf is locally free and is endowed with a canonical connection $\nabla_{F/L}$ along the fibers of the morphism π_1: $F(U)\to L(U)$. Let $\Omega^1 F/L$ be the sheaf of relative 1-forms. Then the relative connection $\nabla_{F/L}$ on $\pi_1^*(G)$ is well defined by the condition that sections of $\pi_1^{-1}(G)$ are horizontal. There is an exact sequence

$$\mathcal{O} \to \pi_1^{-1}(G) \to \pi_1^*(G) \xrightarrow{\nabla_{F/L}} \pi_1^*(G) \otimes \Omega^1 F/L \to 0 \tag{1}$$

In the proof of the theorem we will use this sequence for $G = E_L(a, b)$. In the notations of §1 there is a canonical isomorphism $\pi_1^* E_L = \pi_2^* E \overset{\text{def}}{=} E_F$. Besides, $\pi_1^*(E_L(a, b)) = E_F(a, b) = E_F \otimes \mathcal{O}_F(a, b)$, where $\mathcal{O}_F(a, b) = \pi_1^* \mathcal{O}_L(a, b)$. Finally, $\nabla_{F/L}$: $\pi_2^* E \to \pi_2^* E \otimes \Omega^1 F/L$ can be obtained by lifting ∇: $E \to E \otimes \Omega^1 U$ on $F(U)$ and then restricting it on the fibers of π_1 (see [6] for details).

The cohomology $H^i(U, G)$ is calculated in three stages. First we compare $H^i(U, G)$ with $H^i(F(U), \pi_1^{-1}(G))$ using the Leray spectral sequence of the morphism π_1. Then we calculate $R^q \pi_{2*} \pi_1^{-1}(G)$ using $(1)_G$. Finally we compute $H^i(F(U), \pi_1^{-1}(G))$ using the Leray sequence of π_2.

2. Lemma. *The canonical morphism $H^i(L(U), G) \to H^i(F(U), \pi_1^{-1}(G)$, is an isomorphism for all i.*

Proof. Using the properties of the morphism π_1: $F(U)\to L(U)$ postulated in §1, one easily checks that $\pi_{1*}\pi_1^{-1}(G) = G$ and $R^q \pi_{1*} \pi_1^{-1}(G) = 0$ for $q > 0$. Thus the Leray spectral sequence degenerates. (Cf. also [4]).

3. Lemma. *There is an isomorphism*

$$\Omega^1(F/L) = \pi_2^*[\Lambda^2 S_+ \otimes \Lambda^2 S_-](1, 1) \tag{2}$$

Proof. In [6] it was shown that $\pi_{2*}\Omega^1(F/L) = \Omega^1 U$. Now

$$\begin{aligned} \Omega^1 U &= S_+ \otimes S_- = \Lambda^2 S_+ \otimes \Lambda^2 S_- \otimes S_+^* \otimes S_-^* \\ &= \pi_{2*}(\pi_2^*[\Lambda^2 S_+ \otimes \Lambda^2 S_-](1, 1)) \end{aligned}$$

Hence there is an isomorphism

$$\pi_{2*}\Omega^1(F/L) \tilde{\rightarrow} \pi_{2*}(\pi_2^*[\Lambda^2 S_+ \otimes \Lambda^2 S_-](1, 1))$$

It is not difficult to check that it induces a morphism of sheaves on $F(U)$ which is an isomorphism.

4. Lemma. *The following list exhausts all the nonvanishing segments of the higher direct images sequence $R\pi_{2*}(1)_G$ for $G = E_L(a, b)$:*

$(a = -1,\ b \geq -1)$: $\pi_{2*}(\pi_1^*(G) \otimes \Omega^1 F/L) =$
$= E \otimes S^{b+1}(S_-^*) \otimes \Lambda^2 S_+ \otimes \Lambda^2 S_- \tilde{\rightarrow} = \tilde{\rightarrow} R^1\pi_{2*}\pi_1^{-1}(G)$

$(a = -1,\ b \leq -3)$: $R^1\pi_{2*}(\pi_1^*(G) \otimes \Omega^1 F/L) =$
$= E \otimes S^{-b-3}(S_-) \otimes \Lambda^2 S_+ \otimes (\Lambda^2 S_-)^2 \tilde{\rightarrow} R^2\pi_{2*}\pi_1^{-1}(G)$

$(a = -2,\ b \geq 0)$: $R^1\pi_{2*}\pi_1^{-1}(G) \tilde{\rightarrow} R^1\pi_{2*}\pi_1^*(G) = E \otimes S^b(S_-^*) \otimes \Lambda^2 S_+$

$(a = -2, b \leq -2)$: $R^2\pi_{2*}\pi_1^{-1}(G) \tilde{\rightarrow} R^2\pi_{2*}\pi_1^*(G) =$
$E \otimes S^{-b-2}(S_-) \otimes \Lambda^2 S_+ \otimes \Lambda^2 S_-$

$(a \geq 0,\ b \geq 0)$: $\pi_{2*}\pi_1^{-1}(G) = \operatorname{Ker} \pi_{2*}\nabla_{F/L}(a, b): \pi_{2*}\pi_1^*(G) =$
$= E \otimes S^a(S_+^*) \otimes S^b(S_-^*) \rightarrow \pi_{2*}(\pi_1^*(G) \otimes \Omega^1 F/L) =$
$= E \otimes S^{a+1}(S_+^*) \otimes S^{b+1}(S_-^*) \otimes \Lambda^2 S_+ \otimes \Lambda^2 S_- \rightarrow$
$\rightarrow R^1\pi_{2*}\pi_1^{-1}(G) = \operatorname{Coker} \pi_{2*}\nabla_{F/L}(a, b)$

$(a \leq -3,\ b \geq 0)$: $R^1\pi_{2*}\pi_1^{-1}(G) = \operatorname{Ker} R^1\pi_{2*}\nabla_{F/L}(a, b)$: $R^1\pi_{2*}\pi_1^*(G) =$
$= E \otimes S^{-a-2}(S_+) \otimes S^b(S_-^*) \otimes \Lambda^2 S_+ \rightarrow R^1\pi_{2*}(\pi_1^*(G) \otimes \Omega^1 F/L) =$
$= E \otimes S^{-a-3}(S_+) \otimes S^{b+1}(S_-^*) \otimes (\Lambda^2 S_+)^2 \otimes \Lambda^2 S_- \rightarrow$
$\rightarrow R^2\pi_{2*}\pi_1^{-1}(G) = \operatorname{Coker} R^1\pi_{2*}\nabla_{F/L}(a, b)$

$(a \leq -3,\ b \leq -3)$: $R^2\pi_{2*}\pi_1^{-1}(G) = \operatorname{Ker} R^2\pi_{2*}\nabla_{F/L}(a, b)$: $R^2\pi_{2*}\pi_1^*(G) =$
$= E \otimes S^{-a-2}(S_+) \otimes S^{-b-2}(S_1) \otimes \Lambda^2 S_+ \otimes \Lambda^2 S_- \rightarrow$
$\rightarrow R^2\pi_{2*}(\pi_1^*(G) \otimes \Omega^1 F/L)$
$= E \otimes S^{-a-3}(S_+) \otimes S^{-b-3}(S_-) \otimes (\Lambda^2 S_+)^2 \otimes (\Lambda^2 S_-)^2 \rightarrow$
$\rightarrow R^3\pi_{2*}\pi_1^{-1}(G) = \operatorname{Coker} R^2\pi_{2*}\nabla_{F/L}(a, b)$

(*Note the missing entries*: *for* $(-1, -2)$ *and* $(-2, -1)$ *everything vanishes*; *for* $(a \geq 0,\ b \leq -3)$ *interchange* S_+ *and* S_-).

Proof. The second and the third sheaves in the sequence $(1)_G$ are coherent and for $G = E_L(a, b)$ they are isomorphic respectively to $\pi_1^*(E)(a, b)$ and $\pi_1^*(E \otimes \Lambda^2 S_+ \otimes \Lambda^2 S_-)(a+1,\ b+1)$ (cf. Lemma 3, (2)). Hence to calculate $R^q\pi_{2*}$ of these sheaves it suffices to know $R^q\pi_{2*}\mathcal{O}_F(a, b)$. But $F(U) = P(S_+)_U \times P(S_-)$ is a relative quadric, whose cohomology is well known. We divide the (a, b)-plane into four quadrants by the lines $a = -1$ and $b = -1$. On these lines all $R^q\pi_{2*}\mathcal{O}_F(a, b)$ vanish. In each quadrant $R^q \neq 0$ only for one value of q. These sheaves are listed below:

$$\pi_{2*}\mathcal{O}_F(a,\ b) = S^a(S_+^*) \otimes S^b(S_-^*), \text{ if } a \geq 0,\ b \geq 0;$$

$$R^1\pi_{2*}\mathcal{O}_F(a,\ b) = S^{-a-2}(S_+) \otimes \Lambda^2 S_+ \otimes S^b(S_-^*), \text{ if } a \leq -2,\ b \geq 0;$$

$$R^2\pi_{2*}\mathcal{O}_F(a,\ b) = S^{-a-2}(S_+) \otimes S^{-b-2}(S_-) \otimes \Lambda^2 S_+ \otimes \Lambda^2 S_-, \quad \text{if } a \leq -2,\ b \leq -2;$$

$$R^1\pi_{2*}\mathcal{O}_F(a,\ b) = S^a(S_+^*) \otimes S^{-b-2}(S_-) \otimes \Lambda^2 S_-, \text{ if } a \geq 0,\ b \leq -2.$$

The lemma is then checked by the direct observation.

The morphisms $R^i\pi_{2*}\nabla_{F/L}(a, b)$ deserve some further comments. They are differential operators of the first order invariantly defined in terms of (E, ∇). For example, the map

$$\pi_{2*}\nabla_{F/L}(0, 0)\colon E \to E \otimes S_+^* \otimes S_-^* \otimes \Lambda^2 S_+ \otimes \Lambda^2 S_- = E \otimes \Omega^1 U$$

is just ∇. Similarly, using $\Omega^1 U = S_+ \otimes S_-$, we can define isomorphisms

$$R^2\pi_{2*}\mathcal{O}_F(-3, -3) = S_+ \otimes \Lambda^2 S_+ \otimes S_- \otimes \Lambda^2 S_- \simeq \Omega^3 U,$$

$$R^2\pi_{2*}\mathcal{O}_F(-2, -2) \otimes \Lambda^2 S_+ \otimes \Lambda^2 S_- = (\Lambda^2 S_+)^2 \otimes (\Lambda^2 S_-)^2 \simeq \Omega^4 U$$

and identify the map $R^2\pi_{2*}(\nabla_{F/L}(-3, -3))\colon E \otimes \Omega^3 \to E \otimes \Omega^4$ with (a constant multiple of) ∇_3. Elementary group-theoretic considerations show that in all cases the space of the invariant operators of the given type is at most one-dimensional. Calculations in local coordinates help to identify them explicitly whenever necessary.

5. THE END OF THE PROOF. Comparing the data in lemma 4 with the table in §1 we see that the cohomology $H^{\cdot}(L(U),\ E_L(a, b)) = H^{\cdot}(F(U),\ \pi_1^{-1}E_L(a, b))$ is the abuttment of the Leray spectral sequence of π_2 whose E_2-term is

$$E_2^{p,q} = H^p(U,\ E \otimes S(q;\ a,\ b))$$

since $R^q\pi_{2*}\pi_1^{-1}E_L(a,b) = E \otimes S_-(q;a,b)$ for (a,b)'s in the table. But $S(q;a,b)$ is coherent and U is Stein. Thus only $E_2^{0,q}$ matters and we are done.

The point is that for those values all $R^i\pi_{2*}\nabla_{F/L}(a,b)$ vanish. We are interested also in the case $(a,b) = (-3,-3)$, which involves the non-trivial differential operator ∇_3 as was remarked earlier. Looking at d_2 we see that still $E_2^{0,2} = E_\infty^{0,2} = H^2$. Furthermore,

$$H^3 = E_\infty^{0,3} = E_3^{0,3} = \operatorname{Ker}(d_2\colon H^0(U, \operatorname{Coker} \nabla_3) \to H^2(U, \operatorname{Ker} \nabla_3)).$$

The differential d_2 is readily identified as the composite map $H^0(U, \operatorname{Coker} \nabla_3) \to H^1(U, \operatorname{Im} \nabla_3) \xrightarrow{\sim} H^2(U, \operatorname{Ker} \nabla_3)$ which finishes the proof of the theorem.

We remark that outside the "safe" domain of (a,b) where $R^i\pi_{2*}\nabla_{F/L}(a,b) = 0$ we would need some information about $H^i(U, \operatorname{Ker} D)$ and $H^i(U, \operatorname{Coker} D)$, where $D = R^i\pi_{2*}\nabla_{F/L}(a,b)$. One easily sees that $H^i(U, \operatorname{Coker} D) = H^{i+2}(U, \operatorname{Ker} D)$ for $i \geq 1$ when U is Stein. Besides in some cases $\operatorname{Ker} D$ can be calculated.

Finally in the case of convex U we can prove that $H^i(L(U), \mathscr{F}) = 0$ for all coherent analytic sheaves and $i \geq 3$ using the methods of [9]. Actually in this case $L(U)$ is 2-convex (not strictly).

§3. Applications and remarks

1. To translate into the cohomological language the field equations on U we proceeded in [1] as follows. Denote by $(L^{(k)}, \mathcal{O}_L^{(k)})$ the k-th infinitesimal neighbourhood of L in $P(T) \times P(T^*)$, $L^{(0)} = L$. It is not difficult to check that $\mathcal{O}_L^{(k)}/\mathcal{O}_L^{(k-1)} \simeq \mathcal{O}_L(-k,-k)$.

Suppose we have constructed a series of infinitesimal extensions $E_L^{(i)}$ of E_L to $L^{(i)}$ such that $E_L^{(i)}$ induces $E_L^{(j)}$ on $L^{(j)}$ for $j < i$. Then the exact sequence

$$0 \to E_L(a-1-j, b-1-j) \to E_L^{(j+1)}(a,b) \to E_L^{(j)}(a,b) \to 0$$

induces the coboundary maps

$$\delta(E_L(a,b); j,k)\colon H^k(L(U), \; E_L^{(j)}(a,b)) \to H^{k+1}(L(U), \\ E_L(a-j-1, b-j-1))$$

which are interpreted as some physical operators on U with the help of our theorem. Specifically, here is a summary of some results of [1].

a) There is a unique extension $E_L^{(2)}$.

b) There is a canonical isomorphism $H^1(E_L(-1,0)) = H^1(E_L^{(1)}(-1,0))$ and the operator

$$\delta(E_L(-1,0);\,1,\,1)\colon H^1(E_L^{(1)}(-1,\,0)) = \Gamma(E \otimes S_- \otimes \Lambda^2 S_+) \to$$
$$\to H^2(E_L(-3,\,-2)) = \Gamma(E \otimes S_+ \otimes \Lambda^2 S_+ \otimes \Lambda^2 S_-)$$

is the two-component Dirac operator on the Yang–Mills background. (Henceforth we omit $L(U)$ and U in the notations of cohomology groups).

c) The operator

$$\delta(E_L(-2,\,0);\,0,\,1)\colon H^1(E_L(-2,\,0)) = \Gamma(\Lambda^2 S_+) \to$$
$$\to H^2(E_L(-3,\,-1)) = \Gamma((\Lambda^2 S_+)^2 \otimes \Lambda^2 S_-)$$

is (the conformal version of) the Laplace operator on the Yang–Mills background.

d) The obstruction to the third extension of $E_L^{(2)}$

$$\omega(E_L^{(2)}) \epsilon H^2(\mathrm{End}\ E_L(-3,\,-3)) = \Gamma(\mathrm{Ker}\ \nabla_3) \subset \Gamma(\mathrm{End}\ E \otimes \Omega^3)$$

is (up to a constant) the current of the Yang–Mills field $(E,\,\nabla)$ i.e. the 3-form $\nabla * F_\nabla$, where F_∇ is the curvature and $*$ is the Hodge operator.

2. To prove these assertions and their refinements given in [1] we actually need some additional information about the cohomology of $L(U)$. For example, to prove a) we check that the cup-squaring map $H^1(\mathrm{End}\ E_L(-1,\,-1)) \to H^2(\mathrm{End}\ E_L(-2,\,-2))$ vanishes. This is used to construct the unique $E_L^{(1)}$ extendable to $L^{(2)}$. We need the multiplicative structure as well to write equations, not just operators, since we must express on $L(U)$ such things as "the product of the mass-matrix by a spinor". And of course some maps should be identified explicitly.

To complete the given picture, one can compute everything using the Dolbeault cocycles (cf. [1], [7]). Otherwise one can calculate "point-by-point" as in [6]. We shall describe the details elsewhere.

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