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On isospectral deformations of riemannian metrics. II


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ON ISOSPECTRAL DEFORMATIONS OF RIEMANNIAN METRICS. II

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1. Introduction

Let $M$ be an $n(\geq 2)$ dimensional compact oriented $C^\infty$ manifold without boundary. Let $g$ be a $C^\infty$ Riemannian metric on $M$, and $\text{Spec}(M, g)$ denote the set of eigenvalues of the Laplace-Beltrami operator $\Delta_g = -g^{jk}\nabla_j \nabla_k$ acting on real $C^\infty$ functions on $M$. A 1-parameter $C^\infty$ deformation $g(t)$ ($-\varepsilon < t < \varepsilon$) of a Riemannian metric on $M$ is called an isospectral deformation of $g(0)$ if $\text{Spec}(M, g(t)) = \text{Spec}(M, g(0))$ holds for every $t$. We call $g(t)$ to be trivial if there is a 1-parameter family $\eta(t)$ of diffeomorphisms of $M$ such that $g(t) = \eta(t)^*g(0)$. We have considered in [1], [2] the following problem (given in [3, p. 233]).

**PROBLEM A:** Is there a non-trivial isospectral deformation of a Riemannian metric?

So far, we have few results concerning this problem except for special cases [1]~[6]. Among others the following is known.

**THEOREM:** There are no non-trivial isospectral deformations of $g$, if

1. $(M, g)$ is $(1/n)$-pinched, that is, for each $x \in M$, there exists a positive number $A$(depending on $x$) such that $-1 - (1/n) < K/A < -1 + (1/n)$, $K$ being the sectional curvature associated with any two dimensional subspace of $T_x M$, or
2. $(M, g)$ is of non-negative constant curvature.

The case (1) was proved by Guillemin and Kazhdan [4], [5], and (2) is due to Kuwabara [2] for flat case and to Tanno [6] for the case of positive constant curvature. Moreover, for the case (2), a stronger result...
was shown as follows. Let $\mathcal{M}$ be the manifold of $C^\infty$ Riemannian metrics on $M$ with $C^\infty$ topology. If $(M, g)$ is flat or a standard sphere, there is a neighborhood $U$ of $g$ in $\mathcal{M}$ such that if $\text{Spec}(M, g) = \text{Spec}(M, g')$ and $g' \in U$ then $(M, g')$ is isometric with $(M, g)$.

In the previous paper [1], [2] we studied the problem by considering the variations of Minakshisundaram's coefficients under the deformation of the metric. We try in this paper a different approach to the problem based on Lax's idea which plays a fundamental role in theory of nonlinear waves [7]. We consider the isospectral deformations confined to Lax's sense which are called $L$-isospectral deformations, and set up the following problem.

**Problem B:** Is there a non-trivial $L$-isospectral deformation of a metric?

We see that there are no non-trivial $L$-isospectral deformations under suitable conditions.

In §2 we introduce the notion of $L$-isospectral deformations. In §3 we consider the non-existence of $L$-isospectral deformations and give a sufficient condition for it. It is shown in §4 that this condition is related to the non-existence of first integrals of the geodesic flow, and we give some results concerning the non-existence of $L$-isospectral deformations.

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## 2. $L$-isospectral deformations

Let $g(t)$ be a $C^\infty$ isospectral deformation of $g = g(0)$, that is,

$$\Delta_{g(t)} \phi_j(t) = \Delta_c \phi_j(t) = \lambda_j \phi_j(t),$$

(2.1)

and $\{\phi_j(t)\}_{j=0}^\infty$ is the system of real eigenfunctions orthonormal with respect to the inner product $(,)_t$, defined from the metric $g(t)$, namely, $(\phi, \psi)_t = \int \phi \psi \, dV(g(t))$, $dV(g(t)) = \sqrt{\det g(t)} \, dx^1 \ldots dx^n$. Moreover by Browder's theorem [8], we can choose $\phi_j(t)$ to be of $C^\infty$ class with respect to $t$.

First, we give the following lemma.

**Lemma 2.1:** Let $g(t)$ be a $C^\infty$ isospectral deformation of $g$, and $\mu = dV(g)$. Then, there is a $C^\infty$ isospectral deformation $\tilde{g}(t)$ of $g$ such that $\tilde{g}(t) = \eta(t)^* g(t)$ for a 1-parameter family $\eta(t)$ of diffeomorphisms of $M$, and $dV(\tilde{g}(t)) = \mu$. 

PROOF: It is well known that \( \text{vol}(M, g(t)) \) is left invariant under the isospectral deformation \( g(t) \) (cf. [3, p. 216]). Hence, the lemma is immediately obtained by the following lemma due to Moser [9].

**Lemma (Moser):** Let \( \mu(t) \) be a \( C^\infty \) deformation of \( n \)-form on \( M \) which is non-degenerate and \( \int_M \mu(t) = \int_M \mu(0) \) for each \( t \). Then, there is a \( C^\infty \) family \( \eta(t) \) of diffeomorphisms of \( M \) such that \( \eta(t)*\mu(t) = \mu(0) \).

By Lemma 2.1, we consider hereafter only volume-element preserving deformations, for which the infinitesimal deformation (i-deformation, for short) \( h(t) = dg(t)/dt \) satisfies (cf. [10])

\[
\text{Tr}_{g(t)} h(t) = h_{jk}(t)g^{jk}(t) = 0.
\]

We denote the set of all square integrable real functions on \( M \) by \( L^2(M) \), the inner product being \( (,)_i = (,)_0 \), and the space of distributions on \( M \) by \( \mathcal{E}'(M) \). For an isospectral deformation \( g(t) \), we introduce a linear operator \( B_i : L^2(M) \to \mathcal{E}'(M) \) for each \( t \) as follows. Suppose an element \( \phi \) of \( L^2(M) \) is expressed as \( \sum_{j=0}^{\infty} a_j(t)\phi_j(t), a_j(t) \in \mathbb{R} \). Then for \( \psi \in C^\infty(M) \), we define

\[
\langle B_i \phi, \psi \rangle = \sum_{j=0}^{\infty} a_j(t)(\phi_j(t), \psi),
\]

where \( \phi_j(t) \equiv d\phi_j(t)/dt \) and the domain \( D(B_i) \) of the operator \( B_i \) is the set of all \( \phi \in L^2(M) \) for which the right hand side of the above has a real finite value. Note that \( B_i \phi_j(t) = \phi_j(t) \in C^\infty(M) \) holds good.

Now, differentiate (2.1) with respect to \( t \), and we have

\[
\Delta_i \phi_j(t) + \Delta_j B_i \phi_j(t) - \lambda_j B_i \phi_j(t) = 0,
\]

hence,

\[
(\Delta_i' + \Delta_i B_i - B_i \Delta_i) \phi_j(t) = 0.
\]

Therefore, we get the following equation of operators on \( D(B_i) \cap C^\infty(M) \);

\[
\Delta_i' + [\Delta_i, B_i] = 0. \tag{2.2}
\]

Thus we have

**Proposition 2.2:** If \( g(t) \) is an isospectral deformation, there is a linear operator \( B_i \) satisfying (2.2), where

\[
\Delta_i' = h^{jk}\nabla_j \nabla_k + (\nabla_k h^{jk}) \nabla_j = \nabla_j (h^{jk} \nabla_k),
\]

\( \nabla \) being the covariant differentiation defined by \( g(t) \).
PROOF: (2.3) is immediately derived from variational formulas of Riemannian structure [10]. Q.E.D.

REMARK: The operator $B$, depends on the choice of the orthonormal basis of eigenfunctions \{\phi_j(t)\}.

The equation (2.2) may be called Lax's equation, which is originally studied concerning Korteweg-de Vries (KdV) equation (see Lax [7]):

$$\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} + u \frac{\partial u}{\partial x} = 0.$$  

For the Schrödinger operator $L_t = (d^2/dx^2) + (1/6)u(x, t)$, consider a third order differential operator

$$B_t = -4 \frac{d^3}{dx^3} - u \frac{d}{dx} - \frac{1}{2} \frac{du}{dx}.$$  

Then the equation $L_t' + [L_t, B_t] = 0$ is equivalent to the KdV equation and Spec($L_t$) is left invariant when $u$ changes with $t$ subject to the KdV equation. Moreover, for higher odd order differential operators $B_t$ we get a series of higher order KdV equations, and Spec($L_t$) is invariant if $u$ changes according to them.

On the basis of the above discussion, we introduce the following definition.

**DEFINITION:** Let $g(t)$ be an isospectral deformation. If $B_t$ is a differential operator for each $t$, we call $g(t)$ an **isospectral deformation in Lax's sense**, or **$L$-isospectral deformation**. If $B_t$ is a $k$-th order differential operator for each $t$, we call $g(t)$ an **$L_k$-isospectral deformation**. Note that $D(B_t) = L^2(M)$ for the $L$-isospectral deformation.

**LEMMA 2.3:** Let $g(t)$ be an $L_k$-isospectral deformation. Then, the $k$-th differential operator $B_t$ is skew-symmetric, that is, 

$$B_t + B_t^* = 0,$$  

where $B_t^*$ is the formal adjoint of $B_t$ with respect to $(,)$.

**PROOF:** By differentiating $(\phi_j(t), \phi_k(t)) = \delta_{jk}$ with respect to $t$, we have

$$(B_t \phi_j(t), \phi_k(t)) + (\phi_j(t), B_t \phi_k(t)) = 0,$$

and (2.4) because the above holds for all $\phi_j$'s. Q.E.D.

As a converse of Proposition 2.2, we have the following.
PROPOSITION 2.4: Suppose there are a volume-element preserving $C^\infty$ deformation $g(t)$ of a metric and a skew-symmetric $k$-th order differential operator $B_t$ smoothly depending on $t$, which satisfy eq. (2.2). Assume that there exists a 1-parameter family of linear operators $T_t: C^\infty(M) \rightarrow C^\infty(M)$, $-\varepsilon < t < \varepsilon$, whose infinitesimal generator is $B_t$, that is, $T_t = \exp(\int_0^t B_s ds)$ and $T_0 = \text{Identity}$. Then the deformation $g(t)$ $(-\varepsilon < t < \varepsilon)$ is an isospectral deformation of $g(0)$.

PROOF: Let $\{\psi_j\}$ be a set of orthonormal eigenfunctions associated with $\text{Spec}(M, g(0)) = \{\lambda_j\}$, and set $\phi_j(t) = T_t \psi_j$. Then $\{\phi_j(t)\}_{j=0}^\infty$ forms an orthonormal basis of $L^2(M)$ for each $t$. In fact,

$$\frac{d}{dt} (\phi_j(t), \phi_k(t)) = (B_t \phi_j(t), \phi_k(t)) + (\phi_j(t), B_t \phi_k(t)) = 0,$$

hence $(\phi_j(t), \phi_k(t)) = (\psi_j, \psi_k) = \delta_{jk}$ holds. Set

$$\Delta_t \phi_j(t) = \sum_{k=0}^\infty a_{jk}^t(\phi_k(t), \phi_j(t)),$$

$$a_{jk}^t(\phi_k(t), \phi_j(t)) = \lambda_j \delta_{jk}.$$

The coefficients $a_{jk}^t(\phi_k(t), \phi_j(t))$ are $C^\infty$ functions and

$$\frac{d}{dt} a_{jk}^t(\phi_k(t), \phi_j(t)) = (\Delta_t \phi_j(t) + \Delta_t B_t \phi_j(t), \phi_k(t)) + (\Delta_t \phi_j(t), B_t \phi_k(t)) =$$

$$= ((\Delta_t + [\Delta, B_t]) \phi_j(t), \phi_k(t)) = 0.$$

Therefore $a_{jk}^t = \lambda_j \delta_{jk}$ and accordingly $\text{Spec}(M, g(t)) = \{\lambda_j\}$. Q.E.D.

A fundamental example of $L$-isospectral deformation is a trivial deformation, that is,

LEMMA 2.5: A trivial deformation is an $L_1$-isospectral deformation.

PROOF: Let $g(t) = \eta(t)^* g(0)$ for a 1-parameter family $\eta(t)$ of volume preserving diffeomorphisms of $M$. Then, we have for each eigenfunction,

$$\phi_j(x, s) = \phi_j(\eta(s - t)x, t) = \eta(s - t)^* \phi_j(x, t).$$

Therefore, we get $\phi_j(t) = X_t \phi_j(t)$, where $X_t = d\eta(t)/dt$ is a vector field satisfying $\nabla_t X_t^j = 0$ (cf. [11]). Thus $B_t = X_t$ is a first order differential operator and satisfies (2.2) and (2.4). Q.E.D.
3. Non-existence of L-isospectral deformations

Let \( g(t) \) be a \( C^\infty \) deformation with \( g(0) = g \). We consider the equation (2.2) at \( t = 0 \) (the suffix 0 being omitted). A \( k \)-th order differential operator \( B \) on \( (M, g) \) is expressed as

\[
B = a^{i_1 \ldots i_k}_{(k)} \nabla_{i_1} \ldots \nabla_{i_k} + a^{i_1 \ldots j_{k-1}}_{(k-1)} \nabla_{j_1} \ldots \nabla_{j_{k-1}} + \ldots + a^{0}_{(0)},
\]

(3.1)

where \( a^{i_1 \ldots i_m}_{(m)} \) are components of a contravariant symmetric \( m \)-tensor. For this operator \( B \), we have

\[
B^* = (-1)^k a^{i_1 \ldots i_k}_{(k)} \nabla_{i_1} \ldots \nabla_{i_k} + \text{(lower order terms)}.
\]

Therefore, \( k \) is odd because \( B \) is skew-symmetric (Lemma 2.3). Thus we have only to consider odd order differential operators \( B \).

First, we deal with \( L_1 \)-isospectral deformations, and have the following which is the converse of Lemma 2.5.

**Proposition 3.1:** There are no non-trivial \( L_1 \)-isospectral deformations.

**Proof:** Let \( B \) is a first order skew-symmetric differential operator, namely, \( B = a^i \nabla_i + (1/2)(\nabla_i a^i) \). Then, we have from (2.2),

\[
(h^{jk} - 2 \nabla^j a^k) \nabla_j \nabla_k + \{ \nabla^k h^j_k - \nabla_k \nabla^k a^j - \nabla^j \nabla_i a^i - a^k R^j_k \} \nabla_j + \frac{1}{2} \Delta(\nabla_i a^i) = 0,
\]

where \( R_{jk} \) is the Ricci curvature tensor of \( (M, g) \). Therefore, we get \( h^{jk} = \nabla^j a^k + \nabla^k a^j \), that is, \( h(=\frac{dg(t)}{dt}(0)) \) is a trivial \( i \)-deformation (see [1]).

Thus, if \( g(t) \) is an \( L_1 \)-isospectral deformation, then \( h(t) \) is trivial with respect to \( g(t) \) for each \( t \). Hence the proposition is obtained by the following lemma.

**Lemma (Koiso [12, Lemma 2.9]):** If \( h(t) = \frac{dg(t)}{dt} \) is trivial for each \( t \), then \( g(t) \) is a trivial deformation.

Next, we consider \( L_k \)-isospectral deformations for \( k(\text{odd}) \geq 3 \). Substituting the differential operator \( B \) given by (3.1) into eq. (2.2), we get a necessary and sufficient condition that the coefficients \( a_{(m)} \) and \( h \) should be satisfied. The computation, however, is so complicated that we cannot write it explicitly.

As a necessary condition, we have the following.

**Proposition 3.2:** If \( g(t) \) is an \( L_k \)-isospectral deformation for \( k(\text{odd}) \geq 3 \), then the highest order coefficients of \( B \) satisfy

\[
\nabla^p a^{i_1 \ldots j_k}_{(k)} + \nabla^j a^{i_1 \ldots j_k}_{(k)} + \ldots + \nabla^{jk} a^{i_1 \ldots j_{k-1}}_{(k-1)} = 0.
\]

(3.2)
PROOF: By straightforward calculations, eq. (2.2) leads to
\[(\nabla^\nu a_{(i_1\ldots i_k)})\nabla_j a_{j_1\ldots j_k} + \text{(lower order terms)} = 0.\]
Thus we get (3.2). \(\text{Q.E.D.}\)

Let \(S_k\) be the space of all \(C^\infty\) contravariant symmetric \(k\)-tensor fields on \(M\) endowed with \(C^\infty\) topology. For a \(C^\infty\) Riemannian metric \(g\), we define \(V_g : S_k \to S_{k+1}\) by
\[(V_g^k a)_{i_1\ldots i_k+1} = \nabla^l a_{i_1\ldots i_k}^l + \nabla^l a_{i_1\ldots i_k}^l + \ldots + \nabla^l a_{i_1\ldots i_k}^l,
\]
where \(V\) is the covariant differentiation defined by \(g\). Let \(R\) be the manifold of all \(C^\infty\) Riemannian metrics with \(C^\infty\) topology, and
\[N_k = \{g \in R; (V_g^k)^{-1}(0) = \{0\}\}.
\]

**Lemma 3.3:**
(1) \(N_k\) is an open subset of \(R\).
(2) \(R \supseteq N_1 \supseteq N_3 \supseteq \ldots \supseteq N_{2m-1} \supseteq N_{2m+1} \supseteq \ldots\)

**Proof:** (1) Define \(\hat{V}_g^k : S_k \setminus \{0\} \to S_{k+1}\) by \(\hat{V}_g^k(g, a) = \hat{V}_g^k a\). Then we have \(N_k = R \setminus (\ker(\hat{V}_g^k)),\) where \(\pi : R \times (S_k \setminus \{0\}) \to R\) is the projection. It is easy to see that \(\hat{V}_g^k\) is continuous and \(\pi\) is an open mapping. Hence \(N_k\) is open in \(R\).
(2) We show \((R \setminus N_{2m-1}) \subset (R \setminus N_{2m+1})\). Let \(g \in (R \setminus N_{2m-1})\) and \(\hat{V}_g^{2m-1} a = 0\). Then, obviously, \(\hat{V}_g^{2m+1}(a \otimes g^{-1}) = 0\) holds, where \(a \otimes g^{-1}\) denotes the symmetrization of \(a \otimes g^{-1}\). \(\text{Q.E.D.}\)

We have the following proposition by virtue of Proposition 3.2.

**Proposition 3.4:** If the metric \(g\) belongs to \(N_k, k(\text{odd}) \geq 3\), then there are no non-trivial \(L_k\)-isospectral deformations of \(g\).

**Proof:** Assume \(B\) is the \(k\)-th order differential operator satisfying (2.2). If \(g \in N_k\), then it follows from Proposition 3.2 and Lemma 3.3, (2) that the operator \(B\) reduces to be of first order. Since the set \(N_k\) is open, the isospectral deformation must be trivial by virtue of Proposition 3.1. \(\text{Q.E.D.}\)

**Remark:** We conjecture that for each positive odd integer \(k\), the set \(N_k\) is dense in \(R\). It is known that the statement is valid for the case of \(k = 1\) (cf. Ebin [13, Proposition 8.3]).
Set $\mathcal{N}_\infty = \bigcap_{k:\text{odd}} \mathcal{N}_k$. Noting that $\mathcal{N}_\infty$ is not necessarily open, we get the following.

**Proposition 3.5:** If the metric $g$ belongs to $\mathcal{N}_\infty$, there are no non-trivial $L$-isospectral $i$-deformations of $g$.

4. Relation with first integrals of geodesic flows

Consider the cotangent bundle $T^*M$ with the natural symplectic structure. Let $(x^i, p_i)$ be the local coordinate system of $T^*M$ naturally induced from the coordinates $(x^i)$ of $M$. For a Riemannian metric $g$ on $M$, define a function $H_g$ on $T^*M$ by

$$H_g = \frac{1}{2} g^{jk} p_j p_k.$$  

The Hamiltonian flow on $T^*M$ defined by $H_g$ is called the geodesic flow, and the image of its integral curves projected on $M$ are geodesics of $(M, g)$.

Let $P_k$ ($k$: positive integer) be the set of all polynomial functions on $T^*M$ which are homogeneous of degree $k$ in $(p_i)$. We define a one-one correspondence $\Phi : S_k \rightarrow P_k$ by

$$\Phi(a) = \frac{1}{k} a_{i_1 \ldots i_k} p_{i_1} \ldots p_{i_k}.$$  

Then, we have the following (cf. [5, Proposition 3.1]).

**Lemma 4.1:** For each positive integer $k$, the equation $\nabla^k \Phi a = 0$ is equivalent to

$$\{ \Phi(a), H_g \} = 0.$$

Here $\{,\}$ is the Poisson bracket defined from the symplectic structure of $T^*M$.

**Proof:** For $\Phi(a) = (1/k) a_{i_1 \ldots i_k} p_{i_1} \ldots p_{i_k}$, we have

$$\{ \Phi(a), H_g \} = \frac{1}{k} \frac{\partial a_{i_1 \ldots i_k}}{\partial x^j} p_{i_1} \ldots p_{i_k} g^{lm} p_m -$$

$$- \frac{1}{k} a_{j_1 \ldots i_k} p_{i_1} \ldots p_{i_k} \frac{\partial g^{km}}{\partial x^j} p_k p_m =$$

$$= \frac{1}{k} (\nabla^m a_{i_1 \ldots i_k}) p_m p_{i_1} \ldots p_{i_k}.$$  

Thus the lemma is proved. Q.E.D.
DEFINITION: A $C^\infty$ function $f$ on $T^*M$ is called the first integral of the geodesic flow if $\{f, H_g\} = 0$, and $f$ is not constant on any open set of any level surface of $H_g$. Moreover, if $f$ belongs to $P_k$, we call $f$ the first integral of degree $k$.

From the above lemma, we have for odd $k$,

$$\mathcal{N}_k = \{g \in \mathcal{R}; \text{the geodesic flow has no first integral of degree } k\}.$$

We have the following theorem from Propositions 3.4 and 3.5.

Theorem 4.2: There are no non-trivial $L$-isospectral i-deformations (resp. $L_k$-isospectral deformations for odd integer $k \geq 3$) of $g$, if the geodesic flow defined by $g$ has no first integrals (resp. first integrals of degree $k$).

By Anosov [14] the geodesic flow defined by the metric of negative curvature is ergodic and has no first integrals. Thus we have

Corollary 4.3: If $(M, g)$ is of negative sectional curvature, there are no non-trivial $L$-isospectral deformations of $g$.

Remark: In [4] Guillemin and Kazhdan showed that if $(M, g)$ is of negative sectional curvature and $g(t)$ is an isospectral deformation of $g$, then there is a $C^1$ function $f$ on $T^*M$ such that

$$H'_g + \{H_g, f\} = 0,$$

where $H'_g = (1/2)h^{jk}p_jp_k$. Moreover if $(M, g)$ is $(1/n)$-pinched, it is shown that the function $f$ satisfying (4.1) belongs to $P_1$ and accordingly $h = (dg/dt)(0)$ is trivial. We note that the equation (2.2) may be regarded as a quantum version of eq. (4.1).

Finally, we consider the case where the metric does not belong to $\mathcal{N}_k$, and have the following theorem.

Theorem 4.4: Let $k$ be a positive odd integer, and assume that every first integral of odd degree $\leq k$ of the geodesic flow defined by the metric $g$ is expressed as a linear combination of the products of the first integrals of degree one and $H_g$. Then there are no non-trivial $L_k$-isospectral i-deformations of $g$.

Proof: We prove the theorem by induction on $k$. For the case $k = 1$, the statement reduces to Proposition 3.1. For general odd $k$, suppose $h$
is an $L_k$-isospectral $i$-deformation of $g$, and
\[ \Lambda' + [\Lambda, B] = 0, \]
where
\[ \Lambda' = \nabla_j (h^{jk} \nabla_k), \]
\[ B = a^{i_1 \cdots i_s} \nabla_{i_1} \cdots \nabla_{i_s} + \text{(lower order terms)}. \]

By Proposition 3.2, Lemma 4.1, and the assumption of the theorem, we have
\[ a = \sum_{k=2r+s}^r \frac{1}{s!} \left( \xi_1 \cdots \xi_s \right), \]
where $\xi_1, \ldots, \xi_s$ are the Killing vectors on $(M, g)$. Set $\Omega_k = \xi_k \nabla_j$, $k = 1, \ldots, s$, and
\[ B_1 = \sum_{k=2r+s}^r (\Lambda' \Omega_1 \cdots \Omega_k) \]
corresponding to $a$, where ( ) denotes the symmetrization. We see easily that $B_1$ is a skew-symmetric $k$-th differential operator, and $[\Lambda, B_1] = 0$. Moreover, we have $B = B_1 + B_2$, where $B_2$ is a skew-symmetric $(k - 2)$-th differential operator, and
\[ \Lambda' + [\Lambda, B_2] = 0 \]
holds good. Thus $h$ is an $L_{k-2}$-isospectral $i$-deformation of $g$. Therefore $h$ is trivial by the assumption of induction. Q.E.D.

We conjecture that the assumption of the theorem is satisfied for every Riemannian symmetric spaces.

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