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A NOTE ON ENRIQUES' SURFACES IN CHARACTERISTIC 2

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0. Introduction

Let X be a non-singular complete algebraic surface defined over an algebraically closed field k of characteristic p. X is called unirational if there exists a generically surjective rational mapping φ from the projective space $\mathbf{P}^2$ to X. Recently, Blass has shown that an Enriques' surface in characteristic 2 is unirational if and only if it is either classical or supersingular in the sense of Bombieri and Mumford (see Blass [1], and Bombieri and Mumford [3]). And he asked whether the surfaces in a certain class are Zariski surfaces (for the definition, see Section 1). In this note, we show that they are not Zariski surfaces, and give in characteristic 2 examples of unirational surfaces of purely inseparable type which are not Zariski surfaces. The author thinks that these are the first examples of unirational surfaces of purely inseparable type which are not Zariski surfaces (see Blass [2]). Finally, we prove that the other Enriques' surfaces which are either classical or supersingular are unirational elliptic or quasi-elliptic surfaces of base change type (for the definition, see Section 1, and also see Katsura [5]).

Notations

Let X be a non-singular projective surface defined over an algebraically closed field k of characteristic p. We denote by O_X the structure sheaf of X. We denote by F the Frobenius map which acts on O_X by $f \to f^p$ for $f \in O_X$. Let $\mathscr{F}$ be a sheaf on X. We denote by $H^i(X, \mathscr{F})$ the i-th cohomology group of $\mathscr{F}$. We denote by Ω_X^i the sheaf of regular i-forms. We set $\chi(O_X) = \sum_{i=0}^{2}(-1)^i \dim_k H^i(X, O_X)$. We denote by $B_i(X)$ (resp. $c_1(X)$, $c_2(X)$) the i-th Betti number (resp. the first Chern class, the second Chern number) of X. $q(X)$ means the dimension of the Albanese variety of X.

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1. Preliminaries and results

Let X be a non-singular complete algebraic surface defined over an algebraically closed field k of characteristic p.

DEFINITION 1.1: X is called a *unirational surface of purely inseparable type* if there exists a generically surjective purely inseparable rational mapping φ from the projective plane $\mathbf{P}^2$ to X. In particular, X is called a *Zariski surface* if there exists a rational mapping φ as above with degree $\varphi = p$.

DEFINITION 1.2: Let $\pi: X \to C$ be an elliptic (resp. quasi-elliptic) surface with a non-singular complete curve C defined over k. X is said to be a *unirational elliptic* (resp. *quasi-elliptic*) *surface of base change type* if there exist a curve C' and a morphism f from C' to C such that the fiber product $S \times_C C'$ is rational.

Using a result in Bombieri and Mumford [3], we can easily prove that any unirational quasi-elliptic surface is of base change type.

Now, let the characteristic of the field k be equal to 2. We consider an Enriques' surface X defined over k. By Bombieri and Mumford [3], Enriques' surfaces are divided into three classes as follows:

1) *classical* if $\dim H^1(X, O_X) = 0$,
2) *supersingular* in the sense of Bombieri and Mumford if $\dim H^1(X, O_X) = 1$ and F is zero on $H^1(X, O_X)$.
3) *singular* if $\dim H^1(X, O_X) = 1$ and F is bijective on $H^1(X, O_X)$.

In any case, X has a canonical principal covering space $\tilde{X} \to X$ of degree 2 such that $\tilde{X}$ is a "K3-like" surface. In [4], Crew proved that the surfaces in Class 3) are not unirational. Blass divided the surfaces in Classes 1) and 2) into four classes, that is,

Class 1) i) X is classical and $\tilde{X}$ is birationally isomorphic to a K3 surface.
ii) X is classical and $\tilde{X}$ is birationally isomorphic to a rational surface.

Class 2) i) X is supersingular and $\tilde{X}$ is birationally isomorphic to a K3 surface.
ii) X is supersingular and $\tilde{X}$ is birationally isomorphic to a rational surface.

And he proved that all these surfaces are unirational. We first prove the following.

THEOREM 1: *The surfaces in Classes* 1) i) *and* 2) i) *are not Zariski surfaces.*

REMARK 1.3: Examples of such Enriques' surfaces are given in Bombieri and Mumford [3]. As is stated in Blass [1], we have a purely inseparable rational mapping of degree 2^2 from the projective plane $\mathbf{P}^2$ to the surfaces in Classes 1) i) and 2) i), using the result of Rudakov and Shafarevich [8]. And so, Theorem 1 gives in characteristic 2 examples of unirational surfaces of purely inseparable type which are not Zariski surfaces.

By Bombieri and Mumford [3], any Enriques' surface has a structure of an elliptic or quasi-elliptic surface. For any such structure, we have the following.

THEOREM 2: *The surfaces in Classes* 1) ii) *and* 2) ii) *are Zariski elliptic or quasi-elliptic surfaces of base change type, that is, they are transformed into rational surfaces by the base change of the purely inseparable morphism of degree* 2.

THEOREM 3: *Any Enriques' surface in Class* 1) i), 2) i) *or* 3) *cannot have a structure of a quasi-elliptic surface, that is, it has a structure of an elliptic surface.*

2. Proof of Theorem 1

First, we consider Class 1) i). Let $\mathscr{U} = \{U_i\}$ $(i = 1, 2, \ldots, n)$ be an affine open covering of X. Let $\alpha = \{f_{ij}\}$ be a non-zero 1-cocycle with respect to $\mathscr{U}$ which corresponds to a non-zero element of $Pic^\tau(X)$. Since $Pic^\tau(X) = \mathbf{Z}/2\mathbf{Z}$, we have

$$f_{ij}^2 = f_i/f_j \text{ on } U_i \cap U_j, \tag{2.1}$$

where f_i's are suitable elements of $\Gamma(U_i, O_X^*)$.

We have a purely inseparable double covering

$$\pi: \tilde{X} \to X, \tag{2.2}$$

defined by

$$z_i^2 = f_i \text{ on } U_i, \tag{2.3}$$

and

$$z_i = f_{ij} z_j \text{ on } U_i \cap U_j. \tag{2.4}$$

Suppose that X is a Zariski surface. Then, there exists a purely inseparable rational mapping of degree 2 from the projective plane $\mathbf{P}^2$ to X.

By some blowing-ups, we have a purely inseparable morphism φ of degree 2:

$$\varphi : \tilde{\mathbf{P}}^2 \to X. \tag{2.5}$$

Then, we have an (not always affine) open covering

$$\varphi^{-1}(\mathscr{U}) = \{\varphi^{-1}(U_i) \mid i = 1, 2, \ldots, n\}$$

of $\tilde{\mathbf{P}}^2$, and we have an injection

$$H^1(\varphi^{-1}(\mathscr{U}), O^*_{\tilde{\mathbf{P}}^2}) \hookrightarrow H^1(\tilde{\mathbf{P}}^2, O^*_{\tilde{\mathbf{P}}^2}) \tag{2.6}$$

(cf. Kodaira and Morrow [6]).

Since $H^1(\tilde{\mathbf{P}}^2, O^*_{\tilde{\mathbf{P}}^2})$ has no torsion and $\varphi^*\alpha$ is a torsion, we have $\varphi^*\alpha = 0$. This means that there exists a suitable element F_i in $\Gamma(\varphi^{-1}(U_i), O^*_{\tilde{\mathbf{P}}^2})$ $(i = 1, 2, \ldots, n)$ such that

$$\varphi^* f_{ij} = F_i / F_j \text{ on } \varphi^{-1}(U_i) \cap \varphi^{-1}(U_j). \tag{2.7}$$

Using (2.1), we have

$$\varphi^* f_i / F_i^2 = \varphi^* f_j / F_j^2 \text{ on } \varphi^{-1}(U_i) \cap \varphi^{-1}(U_j). \tag{2.8}$$

Therefore, we have a function g on $\tilde{\mathbf{P}}^2$ such that

$$g = \varphi^* f_i / F_i^2 \text{ on } \varphi^{-1}(U_i). \tag{2.9}$$

Since g has no poles on $\tilde{\mathbf{P}}^2$, g must be a non-zero constant c. Replacing F_i by $c^{1/2} F_i$ $(i = 1, 2, \ldots, n)$, we can assume $g = 1$. So we have

$$\varphi^* f_i = F_i^2 \text{ on } \varphi^{-1}(U_i). \tag{2.10}$$

It is easy to see that z_i's and F_i's are not contained in $k(X)$. Therefore, we have

$$k(\tilde{X}) = k(X)(z_i) \text{ and } k(\tilde{\mathbf{P}}^2) = k(X)(F_i) \tag{2.11}$$

for some i. By the defining equations (2.3) and (2.10), we can conclude that $k(\tilde{X})$ is isomorphic to $k(\tilde{\mathbf{P}}^2)$, which contradicts the fact that $\tilde{X}$ is birationally isomorphic to a K3 surface.

Now, we consider Class 2) i). Let $\mathscr{U} = \{U_i\}$ $(i = 1, 2, \ldots, n)$ be an affine open covering of X. Let $\beta = \{f_{ij}\}$ be a non-zero 1-cocycle with respect to $\mathscr{U}$ which corresponds to a non-zero element of $H^1(X, O_X)$. Since the Frobenius morphism acts trivially on $H^1(X, O_X)$, we have

$$f_{ij}^2 = f_i - f_j \text{ on } U_i \cap U_j, \tag{2.12}$$

where f_i's are elements of $\Gamma(U_i, O_X)$. We have a purely inseparable double covering

$$\pi : \tilde{X} \to X, \tag{2.13}$$

defined by

$$z_i^2 = f_i \text{ on } U_i, \tag{2.14}$$

and

$$z_i = z_j + f_{ij} \text{ on } U_i \cap U_j. \tag{2.15}$$

Suppose that X is a Zariski surface. Then we have a purely inseparable rational mapping of degree 2 from the projective plane $\mathbf{P}^2$ to X. By some blowing-ups, we have a purely inseparable morphism φ of degree 2:

$$\varphi : \tilde{\mathbf{P}}^2 \to X. \tag{2.16}$$

We have an (not always affine) open covering

$$\varphi^{-1}(\mathscr{U}) = \{\varphi^{-1}(U_i) \mid i = 1, 2, \ldots, n\},$$

and it is easy to see that for the first cohomology group we have an injection

$$H^1(\varphi^{-1}(\mathscr{U}), \mathcal{O}_{\tilde{\mathbf{P}}^2}) \hookrightarrow H^1(\tilde{\mathbf{P}}^2, \mathcal{O}_{\tilde{\mathbf{P}}^2}) \tag{2.17}$$

(cf. Kodaira and Morrow [6]).

Since $H^1(\tilde{\mathbf{P}}^2, \mathcal{O}_{\tilde{\mathbf{P}}^2}) = 0$, we have $H^1(\varphi^{-1}(\mathscr{U}), \mathcal{O}_{\tilde{\mathbf{P}}^2}) = 0$. Therefore, there exists a suitable element F_i in $\Gamma(\varphi^{-1}(U_i), \mathcal{O}_{\tilde{\mathbf{P}}^2})$ $(i = 1, 2, \ldots, n)$ such that

$$\varphi^* f_{ij} = F_i - F_j \text{ on } \varphi^{-1}(U_i) \cap \varphi^{-1}(U_j). \tag{2.18}$$

Using (2.12) and (2.18), we have

$$\varphi^* f_i - F_i^2 = \varphi^* f_j - F_j^2 \text{ on } \varphi^{-1}(U_i) \cap \varphi^{-1}(U_j). \tag{2.19}$$

Therefore, we have a function g on $\tilde{\mathbf{P}}^2$ such that

$$g = \varphi^* f_i - F_i^2 \text{ on } \varphi^{-1}(U_i). \tag{2.20}$$

Since g has no poles on $\tilde{\mathbf{P}}^2$, it must be a constant c. Replacing F_i by $F_i + c^{1/2}$ $(i = 1, 2, \ldots, n)$, we can assume $g = 0$. So we have

$$\varphi^* f_i = F_i^2 \text{ on } \varphi^{-1}(U_i). \tag{2.21}$$

It is easy to see that z_i's and F_i's are not contained in $k(X)$. And so, we have

$$k(\tilde{X}) = k(X)(z_i) \text{ and } k(\tilde{\mathbf{P}}^2) = k(X)(F_i) \tag{2.22}$$

for some i. By (2.14) and (2.21), we can conclude that $k(\tilde{X})$ is isomorphic to $k(\tilde{\mathbf{P}}^2)$, which contradicts the fact that $k(\tilde{X})$ is birationally isomorphic to a K3 surface. q.e.d.

3. Elliptic or quasi-elliptic fibrations

In this section, we prove Theorems 2 and 3. To begin with, we prove the following proposition.

PROPOSITION 3.1: *Let $\pi : X \to \mathbf{P}^1$ be an elliptic or quasi-elliptic surface over the projective line $\mathbf{P}^1$ defined over an algebraically closed field k of characteristic $p > 0$. Suppose $q(X) = 0$ and $H^1(X, O_X) \neq 0$. Moreover, suppose that the Frobenius map is nilpotent on $H^1(X, O_X)$. Then there exists a rational 1-form ω on $\mathbf{P}^1$ such that $\pi^*\omega$ is a non-zero regular 1-form on X.*

PROOF: By Leray's spectral sequence, we have $H^1(X, O_X) \cong H^0(\mathbf{P}^1, R^1\pi_* O_X)$. By Bombieri and Mumford [3], we have $R^1\pi_*(O_X) = L \oplus T$, where L is an invertible sheaf and T is supported precisely at the points $P \in \mathbf{P}^1$ at which $\pi^{-1}(P)$ is a wild fiber. And we have degree $L = -\chi(O_X) - \text{length } T$. Since we have $B_1 = 2q = 0$ and $\chi(O_X) = (c_1^2 + c_2)/12 = (2 + B_2)/12$ is positive, degree L is negative. Therefore, we have $H^1(X, O_X) = H^0(\mathbf{P}^1, T)$.

Since $H^1(X, O_X)$ is not zero, this elliptic fiber space has some wild fibers. Let t be the coordinate of $\mathbf{P}^1$. Then we can assume that $E = \pi^{-1}(\infty)$ is a regular fiber and $E' = \pi^{-1}(0)$ is a wild fiber. We consider the exact sequence

$$0 \to O_X(-E) \to O_X \to O_E \to 0. \tag{3.1}$$

We have the long exact sequence

$$\begin{aligned} 0 \to k \xrightarrow{\sim} k \to H^1(X, O_X(-E)) \to H^1(X, O_X) \to H^1(E, O_E) \to \\ \to H^2(X, O_X(-E)) \to H^2(X, O_X) \to 0. \end{aligned} \tag{3.2}$$

Since $\chi(O_X)$ is positive and $H^1(X, O_X)$ is not zero, we see that $\dim H^0(X, \Omega_X^2)$ is positive. The base curve is a projective line, we have

$$\dim H^2(X, O_X(-E)) > \dim H^2(X, O_X). \tag{3.3}$$

Since $\dim H^1(E, O_E) = 1$, we have an isomorphism:

$$H^1(X, O_X(-E)) \xrightarrow{\sim} H^1(X, O_X). \tag{3.4}$$

On the other hand, since E is linearly equivalent to E', we have an isomorphism:

$$\times\, t : O_X(-E) \xrightarrow{\sim} O_X(-E'). \tag{3.5}$$

Therefore we have an isomorphism:

$$\times\, 1/t : H^1(X, O_X(-E')) \xrightarrow{\sim} H^1(X, O_X(-E)). \tag{3.6}$$

Thus by (3.4) and (3.6) we have an isomorphism:

$$\varphi : H^1(X, O_X(-E')) \to H^1(X, O_X(-E)) \to H^1(X, O_X). \tag{3.7}$$

Now we consider the exact sequence

$$0 \to O_X(-E') \to O_X \to O_{E'} \to 0. \tag{3.8}$$

We have the exact sequence

$$0 \to H^0(X, O_X) \to H^0(E', O_{E'}) \xrightarrow{\delta} H^1(X, O_X(-E')). \tag{3.9}$$

Since E' is a wild fiber, we have $\dim H^0(E', O_{E'}) \geq 2$. This means that there exists an element α of $H^0(E', O_{E'})$ such that $\delta(\alpha)$ is not zero. We represent α by a Čech cocycle $\{g_i\}$ with respect to an affine open covering $\{U_i\}$ $(i \in I)$. We have

$$g_i - g_j = f_{ij}t \text{ on } U_i \cap U_j, \tag{3.10}$$

where f_{ij} is a regular function on $U_i \cap U_j$ $(i, j \in I)$. By our choice, $\delta(\{g_i\}) = \{f_{ij}t\}$ is not zero in $H^1(X, O_X(-E'))$. Therefore $\varphi(\{f_{ij}t\}) = \{f_{ij}\}$ is not zero in $H^1(X, O_X)$. By assumption, F is nilpotent on $H^1(X, O_X)$. Therefore there exists an integer n such that

$$f_{ij}^{p^n} = f_i - f_j, \tag{3.11}$$

and $\{f_{ij}^{p^{n-1}}\}$ is not zero in $H^1(X, O_X)$, where f_i is a regular function on U_i $(i \in I)$. By (3.10) and (3.11), we have

$$f_j - (g_j/t)^{p^n} = f_i - (g_i/t)^{p^n} \text{ on } U_i \cap U_j. \tag{3.12}$$

Thus $f_i - (g_i/t)^{p^n}$ $(i \in I)$ define a rational function h on X. It has a pole only on E'. Putting $U = \mathbf{P}^1 - \{t = 0\}$, we see that h is a regular function on $\pi^{-1}(U)$. Since $\pi_* O_X = O_{\mathbf{P}^1}$, we can conclude that h is a function of t. We write it by $h(t)$. Derivating $h(t)$, we have

$$dh(t) = df_i \text{ on } U_i. \tag{3.13}$$

By (3.11), df_i defines a non-zero regular 1-form on X. Hence, we have a rational 1-form $dh(t)$ on $\mathbf{P}^1$ such that $dh(t)$ is a non-zero regular 1-form on X. q.e.d.

PROPOSITION 3.2: *Suppose that the characteristic of k is equal to 2. Let X be an Enriques' surface defined over k. Then we have the following.*

a) If X is classical, then there exists a rational 1-form ω on $\mathbf{P}^1$ such that $\pi^*\omega$ is a non-zero regular 1-form on X. By a suitable coordinate t of $\mathbf{P}^1$, we can express it by the form dt/t.

b) If X is supersingular in the sense of Bombieri and Mumford, there exists a rational 1-form ω on $\mathbf{P}^1$ such that $\pi^*\omega$ is a non-zero regular 1-

form on X. By a suitable coordinate t of $\mathbf{P}^1$, we can express it by the form dt.

PROOF: In the case of a), X has an elliptic or quasi-elliptic fibration $\pi: X \to \mathbf{P}^1$ with two multiple fibers of multiplicity two. Let t be a local coordinate of $\mathbb{P}^1$. We can assume that multiple fibers exist on $t = 0$ and $t = \infty$. Then, $\pi^*(dt/t)$ is regular on $X\backslash\pi^{-1}(0) \cup \pi^{-1}(\infty)$. Let P be a point on $\pi^{-1}(0)$ on which the reduced divisor $\pi^{-1}(0)_{\mathrm{red}}$ is non-singular. Using the local parameter s which defines $\pi^{-1}(0)_{\mathrm{red}}$ at P, we have

$$t = us^{2n}, \tag{3.14}$$

where u is a unit in the complete local ring $\hat{O}_P$ and $2n$ is the multiplicity of $\pi^{-1}(0)$ at P. Then we have $\pi^*(dt/t) = du/u$. This is regular at P. By the same way, we can prove that $\pi^*(dt/t)$ is regular at the non-singular points on $\pi^{-1}(\infty)_{\mathrm{red}}$. Therefore $\pi^*(dt/t)$ is regular except at a finite number of points. Since the irregular points of rational 1-form is a divisor, we can conclude that $\pi^*(dt/t)$ is regular.

b) is the direct consequence of Proposition 3.1. We have the normal form dt, using the construction in the proof. We omit the detail. q.e.d.

REMARK 3.3: Lang calculates the dimension of $H^0(X, \Omega_X^1)$ (see Lang [7]).

PROOF OF THEOREM 2: $\pi: X \to \mathbf{P}^1$ be an elliptic or quasi-elliptic fibration on an Enriques' surface X in Class 1) ii) or 2) ii). Then we have the following diagram:

$$\begin{tikzcd} X \arrow[d, "\pi"'] & \tilde{\mathbf{P}}^2 \arrow[l, "\varphi"'] \arrow[dl, "f"] \\ \mathbf{P}^1 & \end{tikzcd} \tag{3.15}$$

where $\tilde{\mathbf{P}}^2$ is suitable blowing-ups of the projective space $\mathbf{P}^2$, φ is a generically surjective purely inseparable morphism of degree 2 and f is a morphism such that $\pi \circ \varphi = f$.

By Proposition 3.2, we have a non-zero rational 1-form ω on $\mathbf{P}^1$ such that $\pi^*\omega$ is a regular 1-form on X. Hence, we have a regular 1-form $\varphi^*\pi^*\omega$ on $\tilde{\mathbf{P}}^2$. On the other hand, since there exists no regular 1-form on $\tilde{\mathbf{P}}^2$, $\varphi^*\pi^*\omega$ must be zero. Therefore, we have $f^*\omega = 0$. Hence, the function field $k(\mathbf{P}^1)$ is not algebraically closed in the function field $k(\tilde{\mathbf{P}}^2)$.

We consider the normalization C of $\mathbf{P}^1$ in $k(\tilde{\mathbf{P}}^2)$. Then, we have the following diagram:

$$\begin{tikzcd} X \arrow[d, "\pi"'] & X \times_{\mathbf{P}^1} C \arrow[l, "\varphi_1"'] \arrow[d] & \tilde{\mathbf{P}}^2 \arrow[l, "\varphi_2"'] \arrow[dl, "f_2"] \\ \mathbf{P}^1 & C \arrow[l, "f_1"'] & \end{tikzcd} \tag{3.16}$$

where f_1 is a normalization morphism, and we have $\varphi = \varphi_1 \circ \varphi_2$, and $f = f_1 \circ f_2$. Since we have $\deg \varphi = 2$ and $\deg f_1 > 1$ we have

$$\deg f_1 = 2, \deg \varphi_1 = 2, \deg \varphi_2 = 1. \tag{3.17}$$

Hence, $X \times_{\mathbf{P}^1} C$ is birationally isomorphic to a rational surface. Thus, the elliptic or quasi-elliptic surface $\pi: X \to \mathbf{P}^1$ is a unirational elliptic or quasi-elliptic surface of base change type. q.e.d.

Finally, we prove Theorem 3.

PROOF OF THEOREM 3: Suppose that $\pi: X \to \mathbf{P}^1$ is a quasi-elliptic fibration on an Enriques' surface in Class 1) i), 2) i) or 3). Then, by the base change of the purely inseparable morphism of degree 2, X is transformed into a rational surface Y (cf. Bombieri and Mumford [3]), which contradicts Theorem 1 for the surfaces in Classes 1) i) and 2) i). For a surface X in Class 3), as is stated in Section 1, we have an étale covering $\tilde{X}$ of X of degree 2. Then, we have a rational mapping from Y to $\tilde{X}$ such that the mapping $Y \to X$ is factored as $Y \to \tilde{X} \to X$ (cf. Serre [9]), which contradicts the fact that the mapping $Y \to X$ is a purely inseparable morphism of degree 2. q.e.d.

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