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THE DIMENSIONS OF COMPLEMENTED HILBERTIAN SUBSPACES OF UNIFORMLY CONVEX BANACH LATTICES

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For X a given Banach space Dvoretzky's Theorem [1] implies that every finite dimensional $E \subset X$ contains a Hilbertian subspace F. In this paper we are interested in spaces X for which the F's can always be chosen to be uniformly complemented in X, and especially in obtaining estimates for dim F in terms of dim E. It is clearly necessary to suppose that X doesn't contain ℓ_1^n's uniformly. For Banach lattices X Johnson and Tzafriri [8] have shown that the last condition is also sufficient. The novelty of the results presented here is the estimates for dim F in terms of dim E, which are quite sharp. The main technique used in the proofs is the version of Dvoretzky's Theorem proven by Figiel, Lindenstrauss and Milman in [2]; for properly chosen ellipsoids the Levy means involved there can be estimated using the properties of p-summing operators defined on the space X.

This paper was submitted in another place in 1978, and so has been delayed in appearing. Since that time the results presented here have been considerably strengthened: Figiel and Tomczak-Jaegermann [21] extend these results to uniformly convex and k-convex spaces; Benyamini and Gordon [20] consider random factorizations of maps more general than the identity on ℓ_2^n: Pisier's theorem [22] a space not containing ℓ_1^n's must be k-convex shows all the results mentioned carry over to B-convex spaces.

The notation and terminology used here is for the most part standard. We only recall the definitions used in the statements of theorems.

A Banach lattice L is *q-concave* if there is a constant $A > 0$ with

$$\left[\sum_{i \leqslant n} \|x_i\|^q\right]^{1/q} \leqslant A \left\| \left[\sum_{i \leqslant n} |x_i|^q\right]^{1/q} \right\|$$

for all $x_1, x_2, \ldots, x_n \in L$. Similarly L is *p-convex* if there is a constant

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$B > 0$ with

$$\left\| \left[\sum_{i \leqslant n} |x_i|^p \right]^{1/p} \right\| \leqslant B \left[\sum_{i \leqslant n} \|x_i\|^p \right]^{1/p}$$

for all $x_1, x_2, \dots, x_n \in L$. In each case we write $K_q(L)$ and $K^p(L)$ for the best constants A and B appearing in the inequalities. The basic facts about *q-concave* and *p-convex* lattices may be found in [9], [13] and [12].

For $2 \leqslant s < \infty$ and E a given space we take the *s-cotype constant of E*, $\alpha_S(E)$, to be the smallest $\alpha > 0$ such that

$$\left[\sum_{i \leqslant n} \|x_i\|^s \right]^{1/s} \leqslant \alpha \left[\int_0^1 \left\| \sum_{i \leqslant n} r_i(t) x_i \right\|^s \mathrm{d}t \right]^{1/s}$$

for any $x_1, x_2, \dots, x_n \in E$. Here $r_1, r_2, \dots, r_n, \dots$ are the Rademacher functions on [0, 1]. It is obvious that $\alpha_s(\ell_s^N) = 1$, $2 \leqslant s < \infty$.

The *Banach-Mazur distance* between isomorphic spaces E and F is

$$d(E, F) = \inf\{\|u\| \, \|u^{-1}\| \colon u \colon E \to F \text{ an isomorphism}\}.$$

Let X be a fixed space and $\lambda \geqslant 1$. For $E \subset X$ a finite dimensional subspace we define $c_\lambda(E)$ to be the maximum of the dimensions of those $F \subset E$ for which

(i) $d(F, \ell_2^{\dim F}) \leqslant 2$, and

(ii) there is a projection of X onto F of norm at most λ.

In the terminology of Pelczynski and Rosenthal [16] X is called *locally π-Euclidean* if there is a constant $\lambda \geqslant 1$ and a function f on the natural numbers such that $c_\lambda(E) \geqslant n$ whenever $\dim E \geqslant f(n)$.

Finally for $1 < p < \infty$, p' is the conjugate of p $(1/p + 1/p' = 1)$.

THEOREM 1: *Let X be a space which is a subspace of quotient of a p-convex and q-concave Banach lattice L, $1 < p \leqslant 2 \leqslant q < \infty$. There is a $\lambda \geqslant 1$ so that, for $E \subset X$ any n dimensional subspace and $s \in [2, q]$,*

$$c_\lambda(E) \geqslant \lambda^{-1} \min\left\{ n^{2/p'}, \alpha_s(E)^{-2} n^{2/s} \right\}.$$

Before giving the proof we point out some instances of the theorem.

(a) The hypothesis of the theorem implies that X is cotype q [13], and so for some constant $a > 0$ and $\alpha = \min(2/p', 2/q)$,

$$c_\lambda(E) \geqslant a(\dim E)^\alpha \qquad (*)$$

whenever $X \subset E$ is finite dimensional. In particular X is locally π-Euclidean. This result is also stated in [8], though no estimate for $c_\lambda(E)$ is given.

(b) The lattice $L = L_p(\mu)$ is both *p-convex* and *p-concave*, $1 < p < \infty$, and consequently $(*)$ holds with $\alpha = \min(2/p', 2/p)$. For $2 \leqslant p < \infty$ this is well-known, and follows from the results of [2] and [14]. Taking $E = \ell_p^n$ and using the results of [2] shows that this lower bound for $c_\lambda(E)$ is best possible for L an L_p-space.

(c) In case $2 \leqslant s \leqslant q$ and $d(E, \ell_s^n) \leqslant n^{1/s - 1/p'}$

$$\alpha_s(E)^{-2} n^{2/s} \geqslant d(E, \ell_s^n)^{-2} n^{2/s} \geqslant n^{2/p'},$$

and hence $(*)$ is true with $\alpha = 2/p'$. For E a 2-isomorph of ℓ_s^n, $2 \leqslant s \leqslant \min(p', q)$, this gives a lower bound for $c_\lambda(E)$ depending only on the convexity of L. For Hilbertain subspaces of $L_p(\mu)$-spaces $1 < p < 2$ this lower estimate cannot be improved; by [2] ℓ_p^n, $1 < p < 2$, contains a Hilbert subspace of dimension $c_1 n$, but no complemented Hilbert subspaces of dimension greater than $c_2 n^{2/p'}$. For L a p-convex lattice $(1 < p \leqslant 2)$ with some non-trivial concavity, every n dimensional Hilbert subspace is $c_3 n^{1/p - 1/2}$-complemented [11].

Below X, L, p and q have the same meaning as in the statement of Theorem 1. The proof is preceeded by three short lemmas, the first mentioned by Pisier in [18].

The lattice structure enters into the proof only through Lemma 1.

LEMMA 1: *If u: $X \to G$ is q'-integral then u' is p'-summing and*

$$\pi_{p'}(u') \leqslant K^p(L) K_q(L) i_{q'}(u).$$

PROOF: It is enough to show that for u: $L \to G$ q'-summing,

$$i_p'(u') \leqslant K_p(L) K^q(L) \pi_q'(u).$$

By Proposition 3.1 of [11] u' maps the closed unit ball of G' into an order bounded set and

$$\left\| \sup_{\|x'\| \leqslant 1} |u'(x')| \right\| \leqslant K_q(L) \pi_q'(u).$$

Since L' is *p'-concave* with $K_p'(L') = K^p(L)$ [9], the same proposition gives

$$i_{p'}(u') \leqslant K^p(L) \left\| \sup_{\|x'\| \leqslant 1} |u'(x')| \right\|.$$

LEMMA 2: *If $2 \leqslant s < \infty$, H is an n dimensional Hilbert space and u: $H \to G$ any map, there is a subspace $A \subset H$ with* $\dim A \geqslant n/2$ *and*

$$\|u|A\| \leqslant (2/n)^{1/s} \pi_{(2,s)}(u).$$

PROOF: If the conclusion fails inductively choose m vectors $x_1, x_2, \dots, x_m \in H$ to satisfy

$$\|x_i\| = 1, \qquad \|u(x_i)\| > (2/n)^{1/s}\pi_{(2,s)}(u)$$

and

$$x_k \in [x_1, x_2, \dots, x_{k-1}]^{\perp}, \qquad 1 \leqslant k \leqslant m.$$

It is clearly possible to choose $m = [n/2 + 1] \geqslant n/2$ such vectors. But since the x_i's are orthonormal

$$\pi_{(2,s)}(u) \geqslant \left[\sum_{i \leqslant m} \|u(x_i)\|^s\right]^{1/s} > m^{1/s}(2/n)^{1/s}\pi_{(2,s)}(u),$$

a contradiction.

For G a finite dimensional space and $\| \ \|_2$ a Hilbertian norm on G, G_2 denotes G under $\| \ \|_2$.

LEMMA 3: *Let $E \subset X$ be any n dimensional subspace. There is a Hilbertian norm $\| \ \|_2$ on E and an operator v: $X \to E_2$ such that, if u: $E_2 \to X$ is the formal inclusion, then $vu = 1_E$ and $\pi_q(u) = i_q'(v) = n^{1/2}$.*

PROOF: By Theorem 1.1 of [10] there is an isomorphism w: $\ell_2^n \to E$ so that $\pi_q(w) = 1$ and $i_q'(w^{-1}) = n$. Define $\| \ \|_2$ on E by $\|x\|_2 = n^{-1/2}\|w^{-1}(x)\|$. Clearly $\pi_q(u) = i_q'(u^{-1}) = n^{1/2}$. For v take any map v: $X \to E_2$ with $v|E = u^{-1}$ and $i_q'(v) = i_q'(u^{-1})$ (such an extension exists by the defining factorization of q'-integral maps).

PROOF OF THEOREM 1: Given $E \subset X$ of dimension n, choose $\| \ \|_2$, u and v as in Lemma 3. We claim there is a constant $a > 0$ (depending only on q and L) and a subspace $B \subset E$ with $\dim B \geqslant n/4$ such that, if $u_1 = u \mid B_2$ and v_1 is v followed by the orthogonal projection of E_2 onto B_2, then

$$v_1 u_1 = 1_B, \tag{1}$$

$$\pi_q(u_1) \leqslant n^{1/2}, \tag{2}$$

$$\pi_{p'}(v_1') \leqslant a n^{1/2}, \tag{3}$$

$$\|u_1\| \leqslant a\alpha_s(E) n^{1/2 - 1/s}, \tag{4}$$

and

$$\|v_1\| \leqslant a n^{1/p - 1/2}. \tag{5}$$

Trivially, $\pi_{(2,q)}(u) \leqslant \pi_q(u)$. For $2 \leqslant s < q$ the proof of a result of Maurey ([15], Proposition 74, p. 90) implies

$$\pi_{(2,s)}(u) \leqslant c(q)\alpha_s(E)\pi_q(u) \leqslant c(q)\alpha_s(E)n^{1/2},$$

where $c(q)$ is Khintchin's constant. By Lemma 2 there is an $A \subset E$ with $\dim A \geqslant n/2$ and

$$\|u|A_2\| \leqslant (2/n)^{1/s}\pi_{(2,s)}(u) \leqslant c_2\alpha_s(E)n^{1/2-1/2}.$$

By Lemma 1

$$\pi_{(2,p')}(v') \leqslant \pi_{p'}(v') \leqslant c_L i_{q'}(v) = C_L n^{1/2}$$

and thus, again by Lemma 2, there is a $B \subset E$ with $\dim B \geqslant (\dim A)/2 \geqslant n/4$ and

$$\|v'|B_2\| \leqslant (4/n)^{1/p'}\pi_{(2,p')}(v') \leqslant c_4 n^{1/p-1/2}.$$

Properties (1), (2) and (3) follow immediately from the corresponding properties of u and v (and Lemma 1).

The remainder of the proof now follows using the results and techniques of [2]. Let $S \subset B_2$ be the unit sphere $\| \; \|_2 = 1$ and dm be the normalized, rotational invariant measure on S. Recall that the Levy mean of a continuous real valued function f on S is the number M_f such that

$$m\big(x \in S : f(x) \geqslant M_f\big) = m\big(x \in S : f(x) \leqslant M_f\big).$$

Let M be the Levy mean of $x \to \|u_1(x)\| = \|x\|$ on S and $M^{\#}$ be the Levy mean of $x \to \|v_1'(x)\|$ on S (of course $B_2' = B_2$ naturally). Equality (1) implies that for $x \in S$,

$$1 = \langle u_1(x), v_1'(x)\rangle \leqslant \|u_1(x)\|\,\|v_1'(x)\|,$$

and consequently

$$1 \leqslant MM^{\#}. \tag{6}$$

We now claim that there is a constant $b > 0$, depending only on p, q and L, such that

$$M \leqslant b \quad \text{and} \quad M^{\#} \leqslant b. \tag{7}$$

To prove the first let $a(q)$ be the constant satisfying

$$\|z\|_2 = a(q)\left[\int |(x, z)|^q m(\mathrm{d}x)\right]^{1/q}, \qquad z \in B_2;$$

$a(q)$ is the q-summing norm of the identity on B_2 and $n/4 \leqslant \dim B$, so $a(q) \geqslant c_5 n^{1/2}$ for some constant c_5 depending only on q (cf. [4]). By Pietsch's integral representation theorem [17] there is a probability measure μ on S with

$$\|u_1(x)\| \leqslant \pi_q(u_1)\left[\int |(x, z)|^q \mu(\mathrm{d}z)\right]^{1/q}, \qquad x \in B_2.$$

Thus

$$M^q \leqslant 2\int \|u_1(x)\|^q m(\mathrm{d}x)$$

$$\leqslant 2\pi_q(u_1)^q \int\int |(x, z)|^q m(\mathrm{d}x)\mu(\mathrm{d}z)$$

$$= 2\pi_q(u_1)^q a(q)^{-q}$$

$$\leqslant 2n^{q/2} c_5^{-q} n^{-q/2},$$

the last by (2). The inequality $M^\# \leqslant b$ follows similarly from (3).

By Theorem 2.6 of [2] (and the remarks following) there is an absolute constant $c > 0$ and an $F \subset E$ with

$\|\ \|$2-equivalent to $M\|\ \|_2$ on F, (8)

the norm $x \to \|v_1'(x)\|$ 2-equivalent to $M^\#\|\ \|_2$ on F, and (9)

$$\dim F \geqslant cn \min\{\|u_1\|^{-1}M, \|v_1\|^{-1}M^\#\}^2. \qquad (10)$$

By (6) and (7) M and $M^\#$ are at least b^{-1} so, using (4) and (5),

$$\dim F \geqslant c_6 \min\{\alpha_s(E)^{-2} n^{2/s}, n^{2/p'}\}.$$

Finally, let w: $B_2 \to F_2$ be the orthogonal projection. Since $\|v_1'(x)\| \leqslant 2b\|x\|_2$ for $x \in F$, the projection wv_1 has norm at most $2b$ as an operator from X into F_2. But $\|y\| \leqslant 2b\|y\|_2$ for $y \in F$, so $\|wv_1\| \leqslant 4b^2$ as an operator from X into F. This concludes the proof. □

A review of the proof of Theorem 1 shows that, once the Hilbert norm $\|\ \|_2$ and the operators u, v have been chosen, the key inequalities are the upper estimates for M and $M^\#$ given in terms of $\pi_q(u)$ and $\pi_{p'}(v')$. Such estimates are available in several other instances.

Given $1 \leqslant p \leqslant \infty$ a space X *contains* ℓ_p^n*'s uniformly* if there is a sequence $(E_n)_{n \geqslant 1}$ of finite dimensional subspaces of X with $\sup_n d(E_n, \ell_p^n) < \infty$. If in addition there are projections u_n: $X \to E_n$ with $\sup_n \|u_n\| < \infty$, then X *contains uniformly complemented* ℓ_p^n's.

THEOREM 2: *Let L be a Banach lattice not containing ℓ_∞^n's uniformly, and let $X \subset L$. Then either*

(*a*) *X contains uniformly complemented ℓ_1^n's,*

or

(*b*) *X is locally π-Euclidean.*

In the second case there are positive constants λ and α such that

$$c_\lambda(E) \geqslant \lambda^{-1}(\dim E)^\alpha$$

for all finite dimensional $E \subset X$.

THEOREM 3: *There is an absolute constant $c > 0$ with the following property. If E is an n dimensional space with a monotone symmetric basis, there is an m dimensional $F \subset E$ and a projection w: $E \to F$ with*

(*a*) $d(F, \ell_2^m) \leqslant 2$,

(*b*) $m \geqslant c^{-1} d(E, \ell_2^n)^{-2} n$,

and

(*c*) $\|w\| \leqslant c \log n$.

Again the first part of Theorem 2 is stated without proof in [8]. The conclusion of Theorem 3 is of interest only in case $d(E, \ell_2^n)^{-2}n$ is substantially larger than $(\log n)^2$; by John's Theorem [7] $d(E, \ell_2^n) \leqslant n^{1/2}$ for every n dimensional space, and every m dimensional $F \subset E$ is at least $M^{1/2}$-complemented (of [3]).

PROOF OF THEOREM 2: Assume that X' doesn't contain ℓ_∞^n's uniformly. The arguments of Pisier in [18] show that there is a constant $c > 0$ and indices p and q, $1 < p \leqslant 2 \leqslant q < \infty$, so that $\pi_{p'}(v') \leqslant c i'_q(v)$ for every q'-integral map on X. Once this is established as a substitute for Lemma 1, the proof can proceed exactly as before.

PROOF OF THEOREM 3: Let $(e_i)_{i \leqslant n}$ be a monotone symmetric basis for E, set

$$\|x\|_2 = \left[\sum_{i \leqslant n} |x_i|^2\right]^{1/2} \quad \text{for} \quad x = \sum_{1 \leqslant n} x_i e_i$$

and write u: $E_2 \to E$, v: $E \to E_2$ for the formal identities. Every map g of the form $g(e_i) = \varepsilon_i e_{\pi(i)}$, with $|\varepsilon_i| = 1$ for each i and π a permutation of $\{1, 2, \ldots, n\}$, is an isometry of both E and E_2; further the only maps $E_2 \to E$ which commute with all such g are scalar multiples of u. By an averaging argument (cf. [5], Lemma 5.2) $\alpha(u)\alpha^*(v) = n$ for every Banach ideal norm α. Thus we may assume, normalizing $\| \ \|_2$ if necessary, that

$$i_\infty(u) = n^{1/2} \quad \text{and} \quad \pi_1(v) = n^{1/2}.$$

Let a and b be the best constants satisfying

$$a^{-1}\|x\|_2 \leqslant \|x\| \leqslant b\|x\|_2, \qquad x \in E.$$

Another averaging argument shows

$$ab = d(E, \ell_2^n).$$

M and $M^\#$ are defined as in the proof of Theorem 1. By that proof, for any $q \geqslant 2$,

$$M \leqslant 2^{1/q}\pi_q(\ell_2^n)^{-1}\pi_q(u) \quad \text{and} \quad M^\# \leqslant 2^{1/q}\pi_q(\ell_2^n)^{-1}\pi_q(v'),$$

where $\pi_q(\ell_2^n)$ denotes the q-summing norm of the identity on ℓ_2^n. Using the expression given in [4] for $\pi_q(\ell_2^n)$ and Stirling's formula there is an absolute constant $a > 0$ such that $\pi_q(\ell_2^n)^{-1} \leqslant a(q/n)^{1/2}$ for all $q \geqslant 2$.

Any map w into an n dimensional space satisfies $\pi_q(w) \leqslant n^{1/q} i_\infty(w)$ (cf. [11], Corollary 1.7). Consequently, combining inequalities yields

$$\begin{aligned} M &\leqslant 2^{1/q}a(q/n)^{1/2}\pi_q(u) \\ &\leqslant 2an^{1/q}q^{1/2}n^{-1/2}i_\infty(u) \\ &= 2an^{1/q}q^{1/2} \end{aligned}$$

and similarly

$$\begin{aligned} M^\# &\leqslant 2an^{1/q}q^{1/2}i_\infty(v')n^{-1/2} \\ &= 2an^{1/q}q^{1/2}\gamma_1(v)n^{-1/2} \\ &\leqslant 2an^{1/q}q^{1/2}\pi_1(v)n^{-1/2} \\ &= 2an^{1/q}q^{1/2}, \end{aligned}$$

the last inequality by [6], Lemma 3.3, since E has a monotone unconditional basis. Now taking $q = \log n$,

$$MM^{\#} \leqslant c_1 \log n$$

for some absolute constant c_1. Trivially, $M \geqslant a^{-1}$ and $M^{\#} \geqslant b^{-1}$. Using the method of Figuel-Lindenstrauss-Milman as in the proof of Theorem 1 produces an m dimensional $F \subset E$, which is 2-isomorphic to ℓ_2^m, $c_2 \log n$ complemented in E and with

$$m \geqslant c_2 n \min\{b^{-1}M, a^{-1}M^{\#}\}^2 \geqslant c_2 n d(E, \ell_2^n)^{-2}. \qquad \square$$

There are a number of natural questions about complemented Hilbert subspaces. Let X be a space with some Rademacher type. Is there a constant $\lambda \geqslant 1$, depending on X, with

$$c_\lambda(E) \geqslant \lambda^{-1} d(E, \ell_2^n)^{-2} n$$

for all n dimensional $E \subset X$? For X a *p-convex* and *q-concave* lattice it is known [11] that $d(E, \ell_2^n) \leqslant n^{1/p - q/q}$ for $E \subset X$ having dimension n. For such lattices Theorem 1 gives an apparently stronger result, although in this case it is likely that the correct distance estimate is

$$d(E, \ell_2^n) \leqslant c_L \max\{n^{1/p - 1/2}, n^{1/2 - 1/q}\}.$$

The lattice structure enters into the proofs of our results only through the inequality

$$\pi_s(v') \leqslant c i_r(v) \qquad (\#)$$

for operators on X. For X the Schatten p-trace class of operators on ℓ_2, Pisier [18] has shown that $(\#)$ fails for every non-trivial pair $1 < r \leqslant 2 \leqslant s < \infty$. We know of no non-trivial lower estimates for $c_\lambda(E)$ if $E \subset C_p$, $1 < p < 2$, although sharp upper estimates for $d(E, \ell_2^n)$ are available [19]. Finally, we know of no space X on which $(\#)$ is true which is not a subspace of a quotient of a Banach lattice having some Rademacher type.

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