COMPOSITIO MATHEMATICA

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Compositio Mathematica, tome 57, nº 1 (1986), p. 3-13
<http://www.numdam.org/item?id=CM_1986__57_1_3_0>

Compositio Mathematica 57 (1986) 3–13.

NON-ARCHIMEDEAN INDUCED REPRESENTATIONS OF COMPACT ZERODIMENSIONAL GROUPS

Luc Duponcheel

I. Introduction

Let $\mathscr{K}$ be a complete non-archimedean valued field which is algebraically closed. Let G be a compact zerodimensional group which has a $\mathscr{K}$-valued invariant measure m (see [1]). This paper deals with irreducible continuous unitary representations of G into non-archimedian Banach spaces over $\mathscr{K}$. It is important to be able to describe all those representations. For many groups induced representations play an essential role. In the first part of the paper we will give some theoretical information which will be useful in the second part of the paper where we will give some examples to show how induced representations can be used to describe all the irreducible representations of certain groups. We will not go into the details in the first part. Many results are very similar with results about finite groups. Most of the results are easy consequences of the more fundamental results in [2], [3] and [4]. The examples will be explained more carefully.

II. Induced irreducible representations

Let $\mathscr{K}$ be a *complete non-archimedean valued field* which is *algebraically closed*. Let G be a *compact zerodimensional group* which has a $\mathscr{K}$-valued *invariant measure* m.

II.1. DEFINITION: Let $\mathscr{U}: G \to GL(E)$ be an *irreducible continuous unitary representation* of G into a *non-archimedean Banach space* E over $\mathscr{K}$. A sequence $(E_k)_{1 \leqslant k \leqslant n}$ of *closed subspaces* of E is a *transitive system of imprimitivity* for $\mathscr{U}$ if we have:

$$(a)\ \ E = \oplus_{1 \leqslant k \leqslant n} E_k$$

$$(b)\ \ \forall k \in \{1,\dots,n\}\ \forall s \in G\ \exists l \in \{1,\dots,n\}: \mathscr{U}_s(E_k) = E_l$$

Let F be one of the $E_k - s$ and

$$H = \{ s \in G \mid \mathscr{U}_s(F) = F \}.$$

If $\mathscr{V}: H \to GL(F)$ is the *restriction* of $\mathscr{U}$ to H (into F), then we say that our representation $\mathscr{U}$ is *induced* by the representation $\mathscr{V}$. It is clear that $\mathscr{V}$ is also an irreducible representation of H. The problem of describing $\mathscr{U}$ has now changed into the (probably easier) problem of describing $\mathscr{V}$. This will be the strategy we will use when we will describe all the irreducible representations of G.

It may happen that F is a one dimensional space. In that case the representation $\mathscr{V}$ corresponds in a natural way with a *character* e of H. Our representation $\mathscr{U}$ is then isomorphic with the *left regular representation* of G into the *minimal left ideal* $L(G) * f$ of $L(G)$, the *group algebra* of G, where:

$$\begin{cases} f(s) = m(G)/m(H)e(s)^{-1} & \text{if } s \in H \\ f(s) = 0 & \text{otherwise.} \end{cases}$$

It may happen that H is a *normal* subgroup of G. In that case the *minimal two sided ideal* of $L(G)$ containing $L(G) * f$ is the ideal $L(G) * g$ where, if $G = U_{k=1}^{n} s_k H$:

$$g(s) = \sum_{k=1}^{n} f\left(s_k^{-1} s s_k\right) \quad (s \in G).$$

This g is a *minimal central idempotent* of $L(G)$. Those minimal central idempotents play an essential role in the study of the structure of the Banach algebra $L(G)$ (see [1], [2] and [4]).

In the following proposition we will give some conditions for a representation $\mathscr{U}$ to be induced by a representation $\mathscr{V}$.

II.2. Proposition: *Let $\mathscr{U}: G \to GL(E)$ be an irreducible continuous unitary representation. Let H be a closed subgroup of G. Suppose that one of the following conditions is true*:

(*a*) *There exists a group homomorphism $s \to \sigma$ from G to the automorphismgroup of H such that for every s in G and every t in H we have $\mathscr{U}_s - 1 \circ \mathscr{U}_t \circ \mathscr{U}_s = \mathscr{U}_{\sigma(t)}$*

(*b*) *There exists a group homomorphism $s \to \sigma$ from G to the charactergroup of H such that for every s in G and every t in H we have $\mathscr{U}_s - 1 \circ \mathscr{U}_t \circ \mathscr{U}_s = \sigma(t)\mathscr{U}_t$,*

then, if $(g_k)_{1 \leqslant k \leqslant n}$ are the minimal central idempotents of $L(H)$ with $g_k * E \neq 0$, the closed subspaces $(g_k * E)_{1 \leqslant k \leqslant n}$ form a transitive system of imprimitivity for $\mathscr{U}$. If g is one of those g_k and if $E' = g * E$ and if $H' = \{s \in G \mid g(\sigma(t)) = g(t)\ \forall t \in H\}$ when condition (a) holds or $H' = \{s \in G \mid \sigma(t) = 1\ \forall t \in H\}$ when condition (b) holds, then our representation $\mathscr{U}$ is induced by the representation $\mathscr{V}': H' \to GL(E')$ defined by restriction.

It may happen that g is a character e of H and that H is a semidirect factor in H'. This means that H is a normal subgroup of H' and that there exists a closed subgroup G' of H' with $HG' = H'$ and $H \cap G' = \{1\}$. In that case every s in H' has a unique decomposition $s = rt$ ($r \in H$, $t \in G'$) and $\mathscr{V}_s'' = e(r)^{-1}\mathscr{U}_t'$ where $\mathscr{U}'$ is the restriction of $\mathscr{V}'$ to G'. It is clear that $\mathscr{U}'$ is also an irreducible representation of G'. The problem of describing $\mathscr{U}$ has changed into the (probably easier) problem of describing $\mathscr{U}'$. Now it is time for some examples.

3. Examples

From now on p is a *prime number* different from two. We also suppose that p is not equal to the characteristic of the *residue class field* of $\mathscr{K}$. In that case we can easily describe the characters of $p^n\mathbb{Z}_p$ and $1 + p^n\mathbb{Z}_p$ ($n \geqslant 1$):

If e is a character of $p^n\mathbb{Z}_p$ and if $\ker(e) = p^m\mathbb{Z}_p (m \geqslant n)$ then there exists a primitive p^m-th root of unity μ in $\mathscr{K}$ such that $e(\alpha) = \mu^\alpha$ ($\alpha \in p^n\mathbb{Z}_p$). If e is a character of $1 + p^n\mathbb{Z}_p$ and if $\ker(e) = 1 + p^m\mathbb{Z}_p$ ($m \geqslant n$) then there exists primitive p^m-th root of unity μ in $\mathscr{K}$ such that $e(\alpha) = \mu^{\log(\alpha)}$ ($\alpha \in 1 + p^n\mathbb{Z}_p$)

Let G be a compact zerodimensional group. It may happen that there exists a topological group isomorphism $F: p^n\mathbb{Z}_p \to G$ between $p^n\mathbb{Z}_p$ and a closed subgroup H of G. Let $\mathscr{U}: G \to GL(E)$ be an irreducible continuous unitary representation. Let $\mathscr{A}: p^n\mathbb{Z}_p \to GL(E)$ be defined by setting $\mathscr{A}_\alpha = \mathscr{U}_{F(\alpha)}$ ($\alpha \in p^n\mathbb{Z}_p$). Proposition II.2. can be translated as follows:

III.1. Proposition: *Suppose that (with the notations of above) one of the following conditions is true:*

(*a*) *There exists a group homomorphism $s \to \sigma$ from G to $\mathbb{Z}_p \backslash p\mathbb{Z}_p$ such that for every s in G we have $\mathscr{U}_s - 1 \circ \mathscr{A}_p n \circ \mathscr{U}_s = \mathscr{A}_p n_\sigma$*

(*b*) *There exists a group homomorphiscm $s \to \sigma$ from G to $\mathscr{K}$ such that for every s in G we have $\mathscr{U}_s - 1 \circ \mathscr{A}_p n \circ \mathscr{U}_s = \sigma \mathscr{A}_p n$*

then, if $(\lambda_k)_{1 \leqslant k \leqslant n}$ are the eigenvalues of $\mathscr{A}_p n$, the closed eigenspaces $(E_{\lambda_k})_{1 \leqslant k \leqslant n}$ form a transitive system of imprimitivity for $\mathscr{U}$. If λ is one of those λ_k and if $E' = E_\lambda$ and if $H' = \{s \in G \mid \lambda^\sigma = \lambda\}$ if condition (a) holds or $H' = \{s \in G \mid \sigma = 1\}$ if condition (b) holds, then our representation $\mathscr{U}$ is induced by the representation $\mathscr{V}': H' \to GL(E')$ defined by restriction.

If now H is a semidirect factor in H' and if G' is the closed subgroup of H' with $HG' = H'$ and $H \cap G' = \{1\}$, then our representation $\mathscr{V}'$ can be described as follows: if $\ker(\mathscr{A}) = p^m\mathbb{Z}_p$ ($m \geqslant$ in) and if $s = F(\alpha)t$ ($\alpha \in p^n\mathbb{Z}_p$, $t \in G'$) is a decomposition of $s \in H'$, then there exists a primitive p^m-th root of unity μ in $\mathscr{K}$ with $\mathscr{V}_s' = \mu^\alpha \mathscr{U}_t'$. Notice that λ has to be a primitive p^1-th root of unity of $\mathscr{K}$ ($l = m - n$) because we have $\mu^{p^m} = \lambda$.

If condition (a) holds the eigenvalues of $\mathscr{A}_p n$ are $\{\lambda^\sigma \mid s \in G\}$ which eigenvalues we obtain exactly depends on the subgroup $\{\sigma \mid s \in G\}$ of $\mathbb{Z}_p \backslash p\mathbb{Z}_p$. If condition (b) holds the eigenvalues of $\mathscr{A}_p n$ are $\{\lambda\sigma \mid s \in G\}$. All those eigenvalues are primitive p^l-th roots of unity and therefore σ has to be a primitive p^k-th root of unity with $k < l$. The eigenvalues of $\mathscr{A}_p n$ are exactly those primitive p^l-th roots of unity of $\mathscr{K}$ which can be written as a product $\lambda\sigma$ where σ runs through the p^k-th roots of unity of $\mathscr{K}$.

III.2. A first example

Let G be the group

$$\left\{ s = \begin{pmatrix} \sigma & \gamma \\ 0 & \beta \end{pmatrix} \middle| \; \alpha \in 1 + p\mathbb{Z}_p,\ \beta \in 1 + p\mathbb{Z}_p,\ \gamma \in p\mathbb{Z}_p \right\}$$

Let $\mathscr{U}$ be an irreducible representation of G. Define $F\colon p\mathbb{Z}_p \to G$ by setting $F(\gamma) = \begin{pmatrix} 1 & \gamma \\ 0 & 1 \end{pmatrix}$. Let $\mathscr{C}$ be defined by setting $\mathscr{C}_\gamma = \mathscr{U}_{F(\gamma)}$. Finally let $l \geqslant 1$ be such that $\ker(\mathscr{C}) = p^l\mathbb{Z}_p$.

For every $s = \begin{pmatrix} \alpha & \gamma \\ 0 & \beta \end{pmatrix}$ in G we have $\mathscr{U}_s - 1 \circ \mathscr{C}_p \circ \mathscr{U}_s = \mathscr{C}_{p\beta/\alpha}$. There exists a primitive p^l-th root of unity λ in $\mathscr{K}$ such that our representation $\mathscr{U}$ is induced by the representation $\mathscr{V}'\colon H' \to GL(E')$ where:

$$H' = \left\{ s = \begin{pmatrix} \alpha & \gamma \\ 0 & \beta \end{pmatrix} \in G \mid \beta/\alpha \in 1 + p^{l-1}\mathbb{Z}_p \right\}$$

and if

$$G' = \left\{ s = \begin{pmatrix} \alpha & \gamma \\ 0 & \beta \end{pmatrix} \in H' \mid \gamma = 0 \right\}$$

then

$$\mathscr{V}_s' = \lambda^{\gamma/\beta}\mathscr{U}_t', \quad \left(s = \begin{pmatrix} \alpha & \gamma \\ 0 & \beta \end{pmatrix} \in H',\ t = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \in G' \right).$$

Define $D\colon p\mathbb{Z}_p \to G'$ by setting $D(a) = \begin{pmatrix} \exp(a) & 0 \\ 0 & \exp(a) \end{pmatrix}$. Let $\mathscr{A}$ be defined by setting $\mathscr{A}_a = \mathscr{U}'_{D(a)}$. Finally let $n \geqslant 1$ be such that $\ker(\mathscr{A}) = \mathrm{p}^{\mathrm{n}}\mathbb{Z}_{\mathrm{p}}$. For every $t = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$ in G' we have $\mathscr{U}_t' - 1 \circ \mathscr{A}_p \circ \mathscr{U}_t' = \mathscr{A}_p$. There exists a primitive p^n-th root of unity ν in $\mathscr{K}$ such that if

$$G'' = \left\{ t = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \in G' \mid \alpha = 1 \right\}$$

then

$$\mathscr{U}'_t = \nu^{\log(\alpha)}\mathscr{U}''_r, \quad \left(t = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \in G', \; r = \begin{pmatrix} 1 & 0 \\ 0 & \beta/\alpha \end{pmatrix} \in G''\right).$$

Define $E: p^{l-1}\mathbb{Z}_p \to G''$ by setting $E(b) = \begin{pmatrix} 1 & 0 \\ 0 & \exp(b) \end{pmatrix}$. Let $\mathscr{B}$ be defined by setting $\mathscr{B}_b = \mathscr{U}''_{E(b)}$. Finally let $m \geqslant l-1$ be such that $\ker(\mathscr{B}) = p^m\mathbb{Z}_p$. There exists a primitive p^m-th root of unity μ in $\mathscr{K}$ such that our representation $\mathscr{U}$ is induced by the character e of H' where:

$$e(s) = \lambda^{\gamma/\beta}\mu^{\log(\beta/\alpha)}\nu^{\log(\alpha)}, \quad \left(s = \begin{pmatrix} \alpha & \gamma \\ 0 & \beta \end{pmatrix} \in H'\right).$$

It is clear that if $l \leqslant 2$ then $\mathscr{U}$ itself is the character e of G given by the formula above. The minimal central idempotent g of $L(G)$ corresponding with our irreducible representation $\mathscr{U}$ is given by:

$$\begin{cases} g(s) = \left[\lambda^{\gamma/\beta}\mu^{\log(\beta/\alpha)}\nu^{\log(\alpha)}\right]^{-1} & \text{if } \beta/\alpha \in 1 + p^{l-1}\mathbb{Z}_p \\ & \text{and } \gamma \in p^{l-1}\mathbb{Z}_p \\ g(s) = 0 & \text{otherwise.} \end{cases}$$

III.3. A second example

Let G be the group

$$\left\{ s = \begin{pmatrix} 1 & \epsilon & \varphi \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix} \middle| \; \delta \in p\mathbb{Z}_p, \; \epsilon \in p\mathbb{Z}_p, \; \varphi \in p\mathbb{Z}_p \right\}.$$

Let $\mathscr{U}: G \to GL(E)$ be an irreducible representation of G. Define $F: p\mathbb{Z}_p \to G$ by setting

$$F(\varphi) = \begin{pmatrix} 1 & 0 & \varphi \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Let $\mathscr{C}$ be defined by setting $\mathscr{C}_\varphi = \mathscr{U}_{F(\varphi)}$. Finally let $l \geqslant 1$ be such that $\ker(\mathscr{C}) = p^l\mathbb{Z}_p$.

For every

$$s = \begin{pmatrix} 1 & \epsilon & \varphi \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix}$$

in G we have $\mathscr{U}_s - 1 \circ \mathscr{C}_p \circ \mathscr{U}_s = \mathscr{C}_p$. There exists a primitive p^l-th root of unity λ in $\mathscr{K}$ such that:

$$\mathscr{U}_s = \lambda^{\varphi}\mathscr{U}_t, \quad \left(s = \begin{pmatrix} 1 & \epsilon & \varphi \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix}, t = \begin{pmatrix} 1 & \epsilon & 0 \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix} \right).$$

Define $E: p\mathbb{Z}_p \to G$ by setting

$$E(\epsilon) = \begin{pmatrix} 1 & \epsilon & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Let $\mathscr{B}$ be defined by setting $\mathscr{B}_\epsilon = \mathscr{U}_{E(\epsilon)}$. Finally let $m \geqslant 1$ be such that $\ker(\mathscr{B}) = p^m\mathbb{Z}_p$. For every

$$s = \begin{pmatrix} 1 & \epsilon & \varphi \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix}$$

in G we have $\mathscr{U}_s - 1 \circ \mathscr{B}_p \circ \mathscr{U}_s = \lambda^{p\delta}\mathscr{B}_p$. There exists a primitive p^m-th root of unitary μ in $\mathscr{K}$ such that our representation $\mathscr{U}$ is induced by the representation $\mathscr{V}': H' \to GL(E')$ where

$$H' = \left\{ s = \begin{pmatrix} 1 & \epsilon & \varphi \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix} \in G \mid \delta \in p^{l-1}\mathbb{Z}_p \right\}$$

and if

$$G' = \left\{ s = \begin{pmatrix} 1 & \epsilon & \varphi \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix} \in H \mid \epsilon = 0 \text{ and } \varphi = 0 \right\}$$

then

$$\mathscr{V}_s' = \lambda^{\varphi}\mu^{\epsilon}\mathscr{U}_t', \quad \left(s = \begin{pmatrix} 1 & \epsilon & \varphi \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix} \in H', t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix} \in G' \right).$$

Define $D: p^{l-1}\mathbb{Z}_p \to \sigma'$ by setting

$$D(\delta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix}.$$

Let $\mathscr{A}$ be defined by setting $\mathscr{A}_\delta = \mathscr{U}'_{D(\delta)}$. Finally let $n \geqslant l-1$ be such that $\ker(\mathscr{A}) = p^n\mathbb{Z}_p$. There exists a primitive p^n-th root of unity ν in $\mathscr{K}$ such that our representation $\mathscr{U}$ is induced by the character e of H' where:

$$e(s) = \lambda^{\varphi}\mu^{\epsilon}\nu^{\delta}, \qquad \left(s = \begin{pmatrix} 1 & \epsilon & \varphi \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix} \in H' \right).$$

It is clear that if $l \leqslant 2$ then $\mathscr{U}$ itself is the character e of G given by the formula above. The minimal central idempotent g of $L(G)$ corresponding with our irreducible representation $\mathscr{U}$ is given by:

$$\begin{cases} g(s) = \left[\lambda^{\varphi}\mu^{\epsilon}\nu^{\delta}\right]^{-1} & \text{if } \delta \in p^{l-1}\mathbb{Z}_p \text{ and } \epsilon \in p^{l-1}\mathbb{Z}_p \\ g(s) = 0 & \text{otherwise.} \end{cases}$$

Notice that $l \leqslant m$ and $l \leqslant n$.

III.4. A third example

Let G be the group

$$\left\{ s = \begin{pmatrix} \alpha & \epsilon & \varphi \\ 0 & \beta & \delta \\ 0 & 0 & \gamma \end{pmatrix} \middle| \; \alpha \in 1 + p\mathbb{Z}_p,\ \beta \in 1 + p\mathbb{Z}_p,\ \gamma \in 1 + p\mathbb{Z}_p, \right.$$
$$\left. \delta \in p\mathbb{Z}_p,\ \epsilon \in p\mathbb{Z}_p,\ \varphi \in p\mathbb{Z}_p \right\}.$$

Let $\mathscr{U}: G \to GL(E)$ be an irreducible representation of G. Let N be the subgroup

$$\left\{ s = \begin{pmatrix} \alpha & \epsilon & \varphi \\ 0 & \beta & \delta \\ 0 & 0 & \gamma \end{pmatrix} \in G \mid \alpha = 1,\ \beta = 1,\ \gamma = 1 \right\}.$$

There exist $l \geqslant 1$, $m \geqslant 1$ and $n \geqslant 1$ and primitive p^l-th, p^m-th and p^n-th roots of unity λ, μ and ν in $\mathscr{K}$ such that when

$$K = \left\{ s = \begin{pmatrix} 1 & \epsilon & \varphi \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix} \in N \mid \delta \in p^{l-1}\mathbb{Z}_p \right\}$$

and e is the character

$$e(s)=\lambda^{\varphi}\mu^{\epsilon}\nu^{\delta}, \left(s=\begin{pmatrix} 1 & \epsilon & \varphi \\ 0 & 1 & \delta \\ 0 & 0 & 1 \end{pmatrix} \in K \right),$$

then $e^{-1} * E \neq 0$. From now on we suppose that $m \geqslant n$, if $m \leqslant n$ then we have to work with a character defined in the same way as the character e of $\mathscr{K}$ but ϵ and δ are changed. The details are left to the reader.

Using proposition II.2. we see that our representation $\mathscr{U}$ is induced by the representation $\mathscr{V}'$: $H' \to GL(E')$ where:

$$H' = \left\{ s=\begin{pmatrix} \alpha & \epsilon & \varphi \\ 0 & \beta & \delta \\ 0 & 0 & \gamma \end{pmatrix} \in G \mid \right.$$

$$\gamma/\alpha \in 1 + p^{l-1}\mathbb{Z}_p,\ \gamma/\beta \in 1 + p^{n-l+1}\mathbb{Z}_p,$$

$$\left. \beta/\alpha \in 1 + p^{m-l+1}\mathbb{Z}_p,\ \delta + i(\beta-\alpha)/p^{m-l} \in p^{l-1}\mathbb{Z}_p \right\}$$

$$V_s' = \lambda^{\varphi/\gamma-\epsilon\delta/\gamma\beta}\mu^{\epsilon/\beta}\nu^{\delta/\gamma+i(\beta-\alpha)/\gamma p^{m-l}}\mathscr{V}_t'$$

$$\left(s=\begin{pmatrix} \alpha & \epsilon & * \\ 0 & \beta & \delta \\ 0 & 0 & \gamma \end{pmatrix},\ t=\begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & i(\alpha-\beta)/p^{m-1} \\ 0 & 0 & \gamma \end{pmatrix} \right).$$

Here i is an integer not divisible by p such that $\lambda^{l} = \mu^{p^{m-l}}$. The element t of H' can be decomposed as a product

$$\begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & i(\alpha-\beta)/p^{m-l} \\ 0 & 0 & \gamma \end{pmatrix}$$

$$= \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \beta/\alpha & i(1-\beta/\alpha)/p^{m-l} \\ 0 & 0 & 1 \end{pmatrix}.$$

if $m \geqslant n$ then the last factor belongs to H'. Therefore the first factor also belongs to H'. It follows easily from this that $n = l$ if $l \leqslant 2$ and $l \leqslant n \leqslant 2(l-1)$ otherwise.

Define $D\colon p\mathbb{Z}_p \to H'$ by setting

$$D(a) = \begin{pmatrix} \exp(a) & 0 & 0 \\ 0 & \exp(a) & 0 \\ 0 & 0 & \exp(a) \end{pmatrix}$$

Let $\mathscr{A}$ be defined by setting $\mathscr{A}_a = \mathscr{V}'_{D(a)}$. Finally let $t \geqslant 1$ be such that $\ker(\mathscr{A}) = p^t\mathbb{Z}_p$.

For every

$$s = \begin{pmatrix} \alpha & \epsilon & \varphi \\ 0 & \beta & \delta \\ 0 & 0 & \delta \end{pmatrix}$$

in H' we have $\mathscr{V}_s - 1 \circ \mathscr{A}_p \circ \mathscr{V}_s' = \mathscr{A}_p$. There exists a primitive p^t-th root of unity τ in $\mathscr{K}$ such that:

$$\mathscr{V}_s' = \lambda^{\varphi/\gamma - \epsilon\delta/\gamma\beta}\mu^{\epsilon/\beta}\nu^{\delta/\gamma + \iota(\beta-\alpha)/\gamma p^{m-l}}\tau^{\log(\alpha)}\mathscr{V}_t'$$

$$\left(s = \begin{pmatrix} \alpha & \epsilon & \varphi \\ 0 & \beta & \delta \\ 0 & 0 & \gamma \end{pmatrix}, t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \beta/\alpha & i(1-\beta/\alpha)/p^{m-l} \\ 0 & 0 & \gamma/\alpha \end{pmatrix} \right).$$

Define $F\colon p^{l-1}\mathbb{Z}_p \to H'$ by setting

$$F(c) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \exp(c) \end{pmatrix}.$$

Let $\mathscr{C}$ be defined by setting $\mathscr{C}_c = \mathscr{V}'_{F(c)}$. Finally let $r \geqslant l-1$ be such that $\ker(\mathscr{C}) = p^r\mathbb{Z}_p$. For every

$$s = \begin{pmatrix} \alpha & \epsilon & \varphi \\ 0 & \beta & \delta \\ 0 & 0 & \gamma \end{pmatrix}$$

in H' we have

$$\mathscr{V}_s' - 1 \circ \mathscr{C}_{p^{l-1}} \circ \mathscr{V}_s' = \nu^{(1-\exp(p^{l-1}))\delta/\exp(p^{l-1})\beta}\mathscr{C}_{p^{l-1}}.$$

There exists a primitive p^r-th root of unity ρ in $\mathscr{K}$ such that our representation $\mathscr{U}$ is induced by the representation $\mathscr{V}''\colon H'' \to \mathrm{GL}(E'')$ where

$$H'' = \left\{ s = \begin{pmatrix} \alpha & \epsilon & \varphi \\ 0 & \beta & \delta \\ 0 & 0 & \gamma \end{pmatrix} \in H' \,|\, \delta \in p^{n-l+1}\mathbb{Z}_p \right\}$$

if

$$G''=\left\{s=\begin{pmatrix}\alpha & \epsilon & \varphi\\ 0 & \beta & \delta\\ 0 & 0 & \gamma\end{pmatrix}\in H' \mid \alpha=1,\ \gamma=1,\ \epsilon=0,\right.$$

$$\left.\varphi=0,\ \delta=i(\alpha-\beta)/p^{m-l}\right\}$$

then

$$\mathscr{V}_s''=\lambda^{\varphi/\gamma-\epsilon\delta/\gamma\beta}\mu^{\epsilon/\beta}\nu^{\delta/\gamma+i(\beta-\alpha)/\gamma p^{m-l}}\rho^{\log(\gamma/\alpha)}\tau^{\log(\alpha)}\mathscr{U}_t'',$$

$$\left(s=\begin{pmatrix}\alpha & \epsilon & \varphi\\ 0 & \beta & \delta\\ 0 & 0 & \gamma\end{pmatrix}\in H'',\right.$$

$$\left.t=\begin{pmatrix}1 & 0 & 0\\ 0 & \beta/\alpha & i(1-\beta/\alpha)/p^{m-1}\\ 0 & 0 & 1\end{pmatrix}\in G''\right).$$

Define $E\colon P^{m-l+1}\mathbb{Z}_p\to H''$ by setting

$$E(b)=\begin{pmatrix}1 & 0 & 0\\ 0 & \exp(b) & i(1-\exp(b)/p^{m-l}\\ 0 & 0 & 1\end{pmatrix}.$$

Let $\mathscr{B}$ be defined by setting $\mathscr{B}_b=\mathscr{U}''_{E(b)}$. Finally let $s\geqslant m-l+1$ be such that $\ker(\mathscr{B})=p^s\mathbb{Z}_p$. There exists a primitive p^s-th root of unity σ in $\mathscr{K}$ such that our representation $\mathscr{U}$ is induced by the character e of H'' where:

$$e(s)=\lambda^{\varphi/\gamma-\epsilon\delta/\gamma\beta}\mu^{\epsilon/\beta}\nu^{\delta/\gamma+i(\beta-\alpha)/\gamma p^{m-l}}\rho^{\log(\gamma/\alpha)}\sigma^{\log(\beta/\alpha)}\tau^{\log(\alpha)},$$

$$\left(s=\begin{pmatrix}\alpha & \epsilon & \varphi\\ 0 & \beta & \delta\\ 0 & 0 & \gamma\end{pmatrix}\in H''\right).$$

It is clear that when $l\leqslant m\leqslant 2$ then $\mathscr{U}$ itself is the character e of G given by the formula above. Notice that $n\leqslant r$.

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(Oblatum 4-V-1984 & 25-X-1984)

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