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Addendum and errata “Hyperbolic tessellations, modular symbols, and elliptic curves over complex quadratic fields”


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Addendum and errata

Hyperbolic tessellations, modular symbols, and elliptic curves over complex quadratic fields

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Addendum

On page 315 of the original paper [1], a table of twelve “missing conductors” was given. These were ideals \( f \) for which we expected to find an elliptic curve with conductor \( f \) and certain specific traces of Frobenius, as predicted by the Main Conjecture on page 298, but had not yet found such a curve. Twelve such curves have now been found, and, in Table 1, we give their details to complete the tables in [1]. (We reiterate that the tables of curves in [1] are not closed under isogeny.) For each curve, we give its conductor \( f \), and the coefficients \( a_1, a_2, a_3, a_4 \) and \( a_6 \) of a minimal Weierstrass equation.

In the case of \( f = (17 + 11i) \), the curve above corresponds to the first newform in \( V^+(17 + 11i) \) listed in Table 3.2.2 of [1]; a curve corresponding to the second newform was already given in Table 3.2.3.

Table 1.

<table>
<thead>
<tr>
<th>Field</th>
<th>( f )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
<th>( a_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbb{Q}(i) ) ((i = \sqrt{-1}))</td>
<td>( (17 + 11i) ) ( (19 + 8i) )</td>
<td>(-1)</td>
<td>(-1 - i)</td>
<td>(-i)</td>
<td>(55 - 67i)</td>
<td>(-31 + 57i)</td>
</tr>
<tr>
<td>( \mathbb{Q}(\theta) ) ((\theta = \sqrt{-2}))</td>
<td>( (6 + 6\theta) ) ( (5 + 10\theta) ) ( (12 + 7\theta) ) ( (3 + 12\theta) )</td>
<td>(\theta)</td>
<td>(1 - \theta)</td>
<td>(\theta)</td>
<td>(4 - 3\theta)</td>
<td>(4 - 2\theta)</td>
</tr>
<tr>
<td>( \mathbb{Q}(\xi) ) ((\xi = \frac{1}{2}(1 + \sqrt{-3}))</td>
<td>( (14 + 7\xi) ) ( (21) )</td>
<td>(1 - \xi)</td>
<td>(1 - \xi)</td>
<td>(-\xi)</td>
<td>(11 - 7\xi)</td>
<td>(-5 - 9\xi)</td>
</tr>
<tr>
<td>( \mathbb{Q}(\alpha) ) ((\alpha = \frac{1}{2}(1 + \sqrt{-7}))</td>
<td>( (14) )</td>
<td>(-1)</td>
<td>(-2 + \alpha)</td>
<td>(-\alpha)</td>
<td>(-10 + \alpha)</td>
<td>(-8 - \alpha)</td>
</tr>
<tr>
<td>( \mathbb{Q}(\xi) ) ((\xi = \frac{1}{2}(1 + \sqrt{-11}))</td>
<td>( (6\xi) ) ( (2 + 7\xi) ) ( (6 + 6\xi) )</td>
<td>(1 - \alpha)</td>
<td>(-1 - \alpha)</td>
<td>(-\alpha)</td>
<td>(-9 + 5\alpha)</td>
<td>(15 - 2\alpha)</td>
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</table>

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Thanks are due to R.G.E. Pinch, who found the curves with $f = (3 + 12\theta)$ and $f = (2 + 7\alpha)$. The rest were found by the author using programs written in Algol68, run on the ICL 2980 computer at the South West Universities Regional Computing Centre.

Errata

- Table 3.2.3: The line with $f = (16)$ should have a $\sqrt{}$ in the column headed “CM(1)?”.
- Table 3.5.2: The line with $a = (3 - 6\alpha)$ should read

$$
(3 - 6\alpha) - 1 - 1 + -4 4 0 0 -6 -2 -2 6 6 -4 -4
$$

- Table 3.5.3: The four lines with $f = (8\alpha)$ should be linked in the last column (by 2-isogenies).

References