COMPOSITIO MATHEMATICA

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Compositio Mathematica, tome 89, n^{o} 3 (1993), p. 243-250

<http://www.numdam.org/item?id=CM_1993__89_3_243_0>

Compositio Mathematica **89**: 243–250, 1993.

On the topology of holomorphic foliations on Hopf manifolds

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Received 6 January 1992; accepted in final form 19 October 1992

1. Introduction

The topological behavior under perturbation of complex linear flows on $\mathbb{C}^n$ near generic singular points was investigated by J. Guggenheimer (cf. [Gu]). Such flows induce holomorphic foliations on certain Hopf manifolds of dimension n which we shall call diagonal (see below). In this note we investigate the topological behavior of these foliations under perturbation.

The standard complex coordinates in $\mathbb{C}^n$ are denoted by $z = (z_1, \ldots, z_n)$. We identify $\mathbb{C}^n$ canonically with the tangent space at each of its points. If $A \in GL(n, \mathbb{C})$, X_A will denote the linear vector field defined on $\mathbb{C}^n$ by $X_A(z) = Az$, and Φ_A will denote the complex flow $\Phi_A : \mathbb{C}^n \times \mathbb{C} \to \mathbb{C}^n$ obtained by integrating X_A,

$$\Phi_A(z, t) = e^{tA}z.$$

DEFINITION. A matrix $A \in GL(n, \mathbb{C})$ lies in the *strong Poincaré domain* if and only if all eigenvalues of A are different, do not contain the origin in their convex hull and no two eigenvalues lie on the same line through the origin.

A matrix Λ in the strong Poincaré domain may without loss of generality be assumed to be diagonal with diagonal entries $(\lambda_1, \ldots, \lambda_n)$. By abuse of notation we shall identify Λ with the vector $(\lambda_1, \ldots, \lambda_n)$. If $\Lambda \in \mathbb{C}^n$ lies in the strong Poincaré domain, then there is a neighbourhood U of Λ in $\mathbb{C}^n$ such that all $\Lambda' \in U$ lie in the strong Poincaré domain.

There is a complex analog of Hartman's theorem proved by Guggenheimer (cf. [Gu]): *If Φ_Λ is the flow of the vector field $X_\Lambda(z)$ on $\mathbb{C}^n$ with Λ a diagonal matrix in the strong Poincaré domain, there there is a neighbourhood U of Λ in $\mathbb{C}^n$ such that for all $\Lambda' \in U$ a homeomorphism of $\mathbb{C}^n$ exists mapping Φ_Λ orbits to the $\Phi_{\Lambda'}$ orbits.*

The subgroup $\langle f \rangle$ of automorphisms of $\mathbb{C}^n$ generated by a contraction f: $(z_1, \ldots, z_n) \to (\mu_1 z_1, \ldots, \mu_n z_n)$ with $0 < |\mu_1| \leqslant \cdots \leqslant |\mu_n| < 1$ operates freely and properly discontinuously on $\mathbb{C}^n - \{0\}$. The quotient $X = \mathbb{C}^n - \{0\}/_{\langle f \rangle}$ is a compact, complex manifold of dimension n called a (diagonal) Hopf manifold (cf. [Hae], [Ko] §10, [Ma] and [We]).

The structure of the foliations $\bar{\mathscr{F}}$ induced by the orbits of a flow Φ_Λ on $\mathbb{C}^n - \{0\}$, Λ in the strong Poincaré domain, has been described by Arnold (cf. [Ar]): the coordinate axes are leaves and the rest of the leaves are $\mathbb{C}$-planes which wind around the axes. Because these foliations are invariant under the action of the contraction f we obtain a foliation $\mathscr{F}$ on the Hopf manifold X.

The purpose of this note is to prove the following theorem.

THEOREM. *Let X be a diagonal Hopf manifold, Λ a diagonal matrix Λ in the strong Poincaré domain and $\mathscr{F}$ the foliation on X induced by the linear vector field $X_\Lambda(z) = \Lambda z$ on the universal covering $\mathbb{C}^n - \{0\}$ of X. Then there is a neighbourhood $U \subset \mathbb{C}^n$ of Λ, such that any foliation $\mathscr{F}'$ on X induced by a vector field $X_{\Lambda'}(z) = \Lambda' z$, $\Lambda' \in U$, is topologically inequivalent to $\mathscr{F}$, i.e., there is no homeomorphism $h: X \to X$ mapping the leaves of $\mathscr{F}$ to the leaves of $\mathscr{F}'$.*

Notation:
$\mathbb{R}_0^+ := \{x \in \mathbb{R} \mid x \geqslant 0\}$,
$W^n := \mathbb{C}^n - \{0\}$,
$pr: W^n \to X$, the canonical projection,
if $\mathscr{F}$ is a foliation on X, $\bar{\mathscr{F}}$ denotes the lifted foliation $pr^*(\mathscr{F})$ on W^n,
$E_i := pr(\{z \in \mathbb{C}^n \mid z_j = 0, j \neq i\})$.
Notation for the two dimensional case:
$S_r^3 := \{(z_1, z_2) \in \mathbb{C}^2 \| z_1|^2 + |z_2|^2 = r^2\}$,
$P := \{(z_1, z_2) \in \mathbb{C}^2 \| z_1| \leqslant 1$ and $|z_2| \leqslant 1\}$,
∂P: the boundary of P,
$B := \{(z_1, z_2) \in \mathbb{C}^2 \| z_1| = 1$ and $|z_2| = 1\} \subset \partial P$, B is homeomorphic to $S^1 \times S^1$,
$A_j := \{(z_1, z_2) \in \mathbb{C}^2 \mid z_j = 0\}$, $j = 1, 2$, the coordinate axes,
$C_j := S_{\sqrt{2}}^3 \cap A_j$, $j = 1, 2$, the intersection circles,
$Z_1 := \{(s, e^{it}) \in \mathbb{C}^2 \mid s, t \in \mathbb{R}\}$,
$Z_2 := \{(e^{it}, s) \in \mathbb{C}^2 \mid s, t \in \mathbb{R}\}$,
$B \cap Z_j$ has two components we fix one and denote it by G_j, $j = 1, 2$.

2. Proof of the theorem

We shall prove the theorem in the two dimensional case first (Proposition 5) and then reduce the general case to this situation.

LEMMA 1. *Let X_Λ be a linear complex vector field on $\mathbb{C}^2$ with $\Lambda = (\lambda_1, \lambda_2)$ in the strong Poincaré domain and let $\bar{\mathscr{F}}$ be the foliation induced by X_Λ on W^2.*

Then each leaf of $\bar{\mathscr{F}}$ which is not a coordinate axis has a unique point on B.

Proof. Take any leaf L which is not a coordinate axis and a point $p := (z_1(p), z_2(p)) \in L \cap \partial P$. Without loss of generality we may assume $|z_1(p)| = 1$, $|z_2(p)| \leqslant 1$. The flow through the point p can be described by

$$z_1(T) = e^{\lambda_1 T} z_1(p), \quad z_2(T) = e^{\lambda_2 T} z_2(p), \quad \text{with } T \in \mathbb{C}.$$

Of course $|z_1(T)| = 1$ if and only if $T \in (i/\lambda_1)\mathbb{R}$. Since $\lambda_1 \notin \lambda_2 \mathbb{R}$, there is a unique value $t_0 \in \mathbb{R}$ such that $|e^{(\lambda_2/\lambda_1)it_0}| = |z_2(p)|^{-1}$.

Let X_Λ be a linear complex vector field on $\mathbb{C}^2$ with $\Lambda = (\lambda_1, \lambda_2)$ in the strong Poincaré domain and let $\bar{\mathscr{F}}$ be the foliation induced by X_Λ on W^2. The contraction $f: (z_1, z_2) \mapsto (\mu_1 z_1, \mu_2 z_2)$ maps B on $B_\mu := f(B) \subset \partial(f(P))$ and again each leaf of $\bar{\mathscr{F}}$ with the exception of the axes intersects B_μ in a unique point. There is a bijection *flow*: $B \to B_\mu$ defined in the following way: If $p \in B$ then here is a unique leaf L of $\bar{\mathscr{F}}$ such that $p \in L \cap B$. The application *flow* maps p to the unique point $\tilde{p} \in L \cap B_\mu$. The gluing of the leaves of $\bar{\mathscr{F}}$ when we map W^2 onto the Hopf surface X by the canonical projection *pr* is described by the map: $M: B \xrightarrow{flow} B_\mu \xrightarrow{f^{-1}} B$. A short computation shows that M corresponds to a rotation of the torus B: Note that the vector fields (λ_1, λ_2) and $c(\lambda_1, \lambda_2)$ with $c \in \mathbb{C}^*$ induce the same foliation on W^2. Hence we can restrict ourselves to the case $(1, \lambda)$.

Let

$$\lambda = x + iy, \quad T = t_1 + it_2; \quad \mu_1 = \rho_1 e^{i\psi_1}, \quad \mu_2 = \rho_2 e^{i\psi_2}.$$

We take a point $p := (z_1(p), z_2(p)) \in B$, i.e., $|z_1(p)| = |z_2(p)| = 1$.

We compute $M: B \xrightarrow{flow} B_\mu \xrightarrow{f^{-1}} B$: choose a value T such that $|z_1(T)| = |\mu_1|$, $|z_2(T)| = |\mu_2|$.

Then $|z_1(T)| = |e^T| = e^{t_1} = |\mu_1| = \rho_1$ implies that $t_1 = \log \rho_1$, and $|z_2(T)| = |e^{\lambda T}| = e^{xt_1 - yt_2} = |\mu_2| = \rho_2$ implies that $xt_1 - yt_2 = \log \rho_2$. Hence

$$t_2 = \frac{xt_1 - \log \rho_2}{y} = \frac{x \log \rho_1 - \log \rho_2}{y}.$$

It follows that

$$(\mu_1^{-1} \mathrm{e}^T z_1(p), \mu_2^{-1} e^{\lambda T} z_1(p)) \in B, \quad \text{i.e., } |\mu_1^{-1} e^T| = 1 = |\mu_2^{-1} e^{\lambda T}|.$$

Therefore the map M corresponds to a rotation about

$$(\alpha, \beta) := (\arg(\mu_1^{-1} e^T), \arg(\mu_2^{-1} e^{\lambda T})).$$

We represent the values of the arg function in $(-\pi, \pi]$. If $a \in \mathbb{R}$, then $a \bmod 2\pi$ is represented in the same interval. We obtain

$$\begin{aligned}(\alpha, \beta) &= (t_2 - \psi_1, xt_2 + yt_1 - \psi_2) \bmod 2\pi \\ &= \left(\frac{x \log \rho_1 - \log \rho_2}{y} - \psi_1, \frac{(x^2 + y^2)\log \rho_1 - x \log \rho_2}{y} - \psi_2\right) \bmod 2\pi.\end{aligned} \tag{1}$$

REMARK 2. It is well known that if ϕ_1, ϕ_2 are topological maps of $S^1 \times S^1$ and the induced maps ϕ_1^*, ϕ_2^* on the "mapping class group" $H_1(S^1 \times S^1, \mathbb{Z})$ are equal, then ϕ_1, ϕ_2 are homotopic (cf. [Ro] p. 26).

LEMMA 3. *Let $\psi: S^3_{\sqrt{2}} \to S^3_{\sqrt{2}}$ be a topological map which maps the set $\{C_1, C_2\}$ of intersection circles and the torus B on themselves, respectively. Then ψ^* operates as one of the following matrices on $H_1(G_1 \times G_2, \mathbb{Z})$:*

$$\begin{pmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{pmatrix}, \begin{pmatrix} \pm 1 & 0 \\ 0 & \mp 1 \end{pmatrix}, \begin{pmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \pm 1 \\ \mp 1 & 0 \end{pmatrix}.$$

Proof. The sphere $S^3_{\sqrt{2}}$ is the union of two solid tori

$$D_1 := \{(z_1, z_2) \in S^3_{\sqrt{2}} \mid |z_1| \leqslant 1, |z_2| \geqslant 1\}, \quad D_2 := \{(z_1, z_2) \in {}^3_{\sqrt{2}} \mid |z_1| \geqslant 1, |z_2| \leqslant 1\}$$

with $D_1 \cap D_2 = B$. The intersection circles C_j are the souls of D_j, $j = 1, 2$. The map $f: (z_1, z_2) \mapsto (z_2, z_1)$ on $\mathbb{C}^2$ fixes $S^3_{\sqrt{2}}$ and B, but exchanges the intersection circles. We deduce that $f^* = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$. We will show that if $\psi(C_i) = C_i$ then

$$\text{either } \psi^* = \begin{pmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{pmatrix} \quad \text{or } \psi^* = \begin{pmatrix} \pm 1 & 0 \\ 0 & \mp 1 \end{pmatrix}.$$

From this fact the claim will follow immediately because if $\psi(C_1) = C_2$ we replace ψ by $f \circ \psi$ and apply the above result. Hence assume that $\psi(C_j) = C_j$, $j = 1, 2$, and that $\psi^*([G_1]) = n_1[G_1] + n_2[G_2]$, $n_1, n_2 \in \mathbb{Z}$. We look at the two natural embeddings $e_j: B \to S^3_{\sqrt{2}} - C_j$, $j = 1, 2$, and the diagrams

$$\begin{array}{ccc} B & \xrightarrow{e_j} & S^3_{\sqrt{2}} - C_j \\ \psi \downarrow & & \downarrow \psi \\ B & \xrightarrow{e_j} & S^3_{\sqrt{2}} - C_j \end{array}$$

and

$$\begin{array}{ccc} H_1(B, \mathbb{Z}) & \xrightarrow{e_j^*} & H_1(S^3_{\sqrt{2}} - C_j, \mathbb{Z}) \\ \psi^* \downarrow & & \downarrow \psi^* \\ H_1(B, \mathbb{Z}) & \xrightarrow{e_j^*} & H_1(S^3_{\sqrt{2}} - C_j, \mathbb{Z}). \end{array}$$

The circles C_1 and G_1 induce the same generator $\alpha_1 \in H_1(S^3_{\sqrt{2}} - C_2, \mathbb{Z})$ and we must have $\psi^*(\alpha_1) = \pm\alpha_1$. Hence $\psi^* \circ e_2^*([G_1]) = \pm\alpha_1 = e_2^* \circ \psi^*([G_1]) = e_2^*(n_1[G_1] + n_2[G_2]) = n_1\alpha_1$.

If we take the embedding e_1 and the intersection circle C_2 instead of e_2, C_1 we conclude that $n_2 = 0$. The result $\psi^*(\alpha_2) = \pm\alpha_2$ is obtained by a repetition of the previous argument. □

PROPOSITION 4. *Let X be the diagonal Hopf surface induced by the contraction $f: (z_1, z_2) \mapsto (\mu_1 z_1, \mu_2 z_2)$ with $\mu_1 = \rho_1 e^{i\psi_1}$, $\mu_2 = \rho_2 e^{i\psi_2}$, and let $\mathscr{F}$ be the foliation on X induced by the vector field $X_\Lambda(z) = \Lambda z$ with $\Lambda = (1, \lambda)$, $\lambda = x + iy \in \mathbb{C} - \mathbb{R}$ on W^2. Then the set*

$$R_{\mathscr{F}} := \{|\alpha|, |\beta|\} = \left\{ \left| \frac{x \log\rho_1 - \log\rho_2}{y} - \psi_1 \bmod 2\pi \right|, \right.$$
$$\left. \left| \frac{(x^2 + y^2)\log\rho_1 - x\log\rho_2}{y} - \psi_2 \bmod 2\pi \right| \right\}$$

is a topological invariant of $\mathscr{F}$ on X.

Proof. Let $\mathscr{F}$, $\mathscr{F}'$ be two foliations on X, induced by X_Λ, $X_{\Lambda'}$ with Λ, Λ' in the strong Poincaré domain, and let $h: X \to X$ be a homeomorphism such that $h^*(\mathscr{F}) = \mathscr{F}'$. Remember that $\bar{\mathscr{F}} = \mathrm{pr}^*(\mathscr{F})$ and $\bar{\mathscr{F}}' = pr^*(\mathscr{F}')$. The map h induces a topological map $\tilde{h}: W^2 \to W^2$ on the universal covering of X such that $\tilde{h}^*(\bar{\mathscr{F}}) = \bar{\mathscr{F}}'$. These foliations induce rotation maps M, M' on B. Put $B' := \tilde{h}(B)$. Now we deform $\tilde{h}$ continuously along the flow until B' lies on B and again call by an abuse of notation the resulting map $\tilde{h}$. The following diagram is commutative:

$$\begin{array}{ccc} B & \xrightarrow{\tilde{h}} & B \\ M \downarrow & & \downarrow M' \\ B & \xrightarrow{\tilde{h}} & B \end{array}$$

which means that M and M' are topologically conjugate.

According to a result of Herman (cf. [He] XIII. Prop. 1.4, remark 1.5), the rotation number (α, β) of a rotation on $S^1 \times S^1$ is unchanged by conjugation with a topological map which is homotopic to the identity on $S^1 \times S^1$. This implies that the change of the rotation number (α, β) by conjugation with a topological map of B depends only on its homotopy class. Lemma 3 applies to our map $\tilde{h}$. Hence, if the rotation number of M equals (α, β) then the rotation number of M' must be one of the following

$$(\pm\alpha, \ \pm\beta), (\mp\alpha, \ \pm\beta), (\pm\beta, \ \pm\alpha), (\mp\beta, \ \pm\alpha). \qquad \square$$

PROPOSITION 5. *Let X be a diagonal Hopf surface and $\mathscr{F}$ a foliation on X induced by the linear complex vector field $X_\Lambda(x) = \Lambda z$, with $\Lambda = (\lambda_1, \lambda_2)$ in the strong Poincaré domain. Then there is a neighbourhood $U \subset \mathbb{C}^2$ of (λ_1, λ_2), such that for any foliation $\mathscr{F}'$ induced by a vector field $X_{\Lambda'}$ with $(\lambda'_1, \lambda'_2) \in U$, there is no homeomorphism $h: X \to X$ with $h^*(\mathscr{F}') = \mathscr{F}$.*

Proof. Without loss of generality we may assume that $\Lambda = (1, \lambda)$. Hence our claim follows from Proposition 4 and the following fact: By (1) we have a differentiable map

$$\begin{aligned} rot: \quad & \mathbb{C} - \mathbb{R} \ \to \mathbb{R}/2\pi\mathbb{Z} \times \mathbb{R}/2\pi\mathbb{Z} \\ & \lambda = x + iy \mapsto \quad (\alpha, \ \beta) \end{aligned}$$

which is locally a diffeomorphism:

$$\det\left(\frac{\partial(\alpha, \ \beta)}{\partial(x, \ y)}\right) = \frac{(y^2 + x^2)\log^2 \rho_1 - 2x \log \rho_1 \log \rho_2 + \log^2 \rho_2}{y^3} = 0$$

has for a given real $y \neq 0$ only non real x as solutions. $\square$

REMARK 6. The neighbourhood U around Λ in Proposition 5 can be chosen such that for all $\mathscr{F}'$ induced by $\Lambda' \in U$ there are only finitely many $\mathscr{F}''$ induced by a $\Lambda'' \in U$ which are topologically equivalent to $\mathscr{F}'$.

Proof of the theorem: (a) The complex planes $P_{ij} := \{z \in \mathbb{C}^n \mid z_l = 0, \ l \neq i, j\}$ are mapped by pr on Hopf surfaces X_{ij} in X. The foliation $\mathscr{F}_{ij} := \mathscr{F} \mid X_{ij}$ is induced by $X(z) = X_\Lambda \mid P_{ij} \cong (\lambda_i z_i, \lambda_j z_j)$ on $\mathbb{C}^2 - \{0\}$.

(b) If we have two foliations $\mathscr{F}$ and $\mathscr{F}'$ on X and a homeomorphism $h: X \to X$ with $h^*(\mathscr{F}') = \mathscr{F}$, then we claim that for i, j there are k, l such that $h(X_{ij}) = X_{kl}$ and $h^*(\mathscr{F}'_{kl}) = \mathscr{F}_{ij}$.

The leaves E_i are compact for all i. Therefore h maps the set of the E_i's on itself. The set of leaves of $\mathscr{F}'_{i,j}$ consists of E_i, E_j and the $\mathbb{C}$-planes which wind around E_i, E_j and have exactly these two compact leaves in its closure. If h maps E_i, E_j on E_k, E_l, respectively, then a leaf which has E_i and E_j in its closure is mapped on a leaf which has E_k and E_l in its closure. Hence the claim follows.

(c) We denote the set of embedded Hopf surfaces $X_{ij} \subset X$ by HS. It follows from (b) that each homeomorphism $h: X \to X$ with $h^*(\mathscr{F}') = \mathscr{F}$ induces a permutation $Perm(h)$ of HS.

(d) Assume there there is no neighbourhood of Λ in $\mathbb{C}^n$ with the properties claimed in the theorem. Then there exists a sequence $\{\Lambda^r\}$ which converges to Λ in $\mathbb{C}^n$, for which each $\Lambda^r \neq \Lambda$, and a sequence of homeomorphisms $h^r: X \to X$ with $(h^r)^*(\mathscr{F}^r) = \mathscr{F}$, where $\mathscr{F}^r$ denotes the foliation induced by X_{Λ^r}. The set HS is finite and $\{h^r\}$ is infinite. Choosing an appropriate subsequence of $\{h^r\}$ we may assume therefore without loss of generality that $Perm(h^r)$ is independent of r. This implies that $\{h^r\}$ induces for each pair i, j a sequence of homeomorphisms $\{h^r_{ijkl}\}$ with $h^r_{ijkl}: X_{ij} \to X_{kl}$ and $(h^r_{ijkl})^*(\mathscr{F}^r_{kl}) = \mathscr{F}_{ij}$. Proposition 4 implies that $R_{\mathscr{F}_{ij}} = R_{\mathscr{F}^r_{kl}}$ for all i, j. On the other side, the sequence $\{(\lambda^r_k, \lambda^r_l)\}$ converges to $\{(\lambda_k, \lambda_e)\}$. Therefore by Proposition 5 and Remark 6 we may assume without loss of generality that $(\lambda^r_k, \lambda^r_l)$ is independent of r for all k, l. Hence Λ^r is constant. Since $\{\Lambda^r\}$ converges to Λ we obtain $\Lambda^r = \Lambda$ for all r, a contradiction. □

3. An observation on the structure of B

Given a transversally holomorphic foliation $\mathscr{F}$ on a manifold X and a submanifold S. If the leaves of $\mathscr{F}$ intersect S transversally in all points of S, then $\mathscr{F}$ induces on S a complex structure (cf. [GHS]).

In our discussion of the two dimensional case we have introduced a differentiable torus B in W^2 which is intersected transversally in all points by the leaves of the foliation $\bar{\mathscr{F}}$ induced by the vector field $X_\Lambda(z)$, Λ in the strong Poincaré domain. The image $pr(B)$ is again a torus now in the Hopf surface X^2 and transversally intersected in all points by the leaves of the corresponding foliation $\mathscr{F}$. We inquire into the relation between the foliation $\mathscr{F}$ and the complex structure of $pr(B)$.

PROPOSITION 6. *Let $\mathscr{F}$ be the foliation on X^2 induced by the vector field $X_\Lambda(z) = \Lambda z$ where $\Lambda = (1, \lambda)$, $\lambda = x + iy \in \mathbb{C} - \mathbb{R}$ on W^2. Then the complex structure induced on pr(B) is conformally equivalent to $\mathbb{C}^*/_{\langle e^{2\pi i\lambda}\rangle}$.*

Proof. Apart from the coordinate axes every leaf has some intersection points with the punctured plane $Pl = \{(z_1, z_2) \in \mathbb{C}^2 \mid z_1 = 1, z_2 \in \mathbb{C}^*\}$. Take a leaf L and a point $p = (1, z_2(0)) \in L \cap Pl$. The flow through this point is given by $z_1(T) = e^T z_1(0) = e^T$, $z_2(T) = e^{\lambda T} z_2(0)$. We obtain the other intersection points of L with Pl, $(1, z_2(T))$, by the equation $1 = e^T 1$, and hence

$$(1, e^{\lambda 2\pi i k} z_2(0)) = (1, (e^{2\pi i \lambda})^k z_2(0)).$$

We map these isolated points in Pl onto the unique intersection point of L with B and obtain eventually a holomorphic covering map:

$$\begin{array}{ll} Pl \cong \mathbb{C}^* & \\ {\scriptstyle \exp} \downarrow & \searrow \\ \mathbb{C}^*/_{\langle e^{2\pi i \lambda}\rangle} \cong B. & \end{array} \qquad \square$$

Acknowledgement

The author thanks Prof. A. Haefliger for helpful discussions.

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