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HOMOLOGY FUNCTORS ON SHAPE CATEGORIES

by Friedrich W. BAUER

One of the first appearances of categories and functors on the mathematical scene was in connection with attempts of an axiomatic treatment of homology and cohomology theories. It turned out that a few very simple properties can be used to characterize (co-)homology at least for nice spaces (like for example finite polyhedra). Then very soon, one found it necessary to abolish one of these axioms, the «dimension axiom», in order to get access to the wide variety of «generalized (co-)homology theories» like different kinds of K-theories, cobordism theories, etc.

Let $\underline{E} = \{E_n\}$ be a CW-spectrum (cf. [1]); then G. W. Whitehead defined homology with coefficients in $\underline{E}$ for a based CW-space X by

$$(1) \qquad \underline{E}_n(X) = \pi_n(\underline{E} \wedge X) = \varinjlim_k \pi_{n+k}(E_k \wedge X), \quad n \in Z.$$

In the case

$\underline{E} = \underline{K}(G)$ (= Eilenberg-Mac Lane spectrum for an abelian group G),

we get back ordinary homology $H_n(X;G)$.

On the other hand it is well-known that there are many different homology functors (which readily deserve this name) on larger categories of topological spaces or pairs of spaces (like

$\mathfrak{A}_{CM}$ (= category of compact metric pairs)).

The main objective of this talk is to indicate that there is, roughly speaking, only one generalized homology (with fixed coefficient spectrum) on $\mathfrak{A}_{CM}$ (or equivalently on Com = category of based compacta) having reasonable properties. To this end we do not change the definition of *homology* (1) but the *category* in which we are working and go over to the strong shape category $\bar{K} = \overline{Com}$.

So, the homology in question is defined by:

(2) $\bar{\underline{E}}_n(X) = \varinjlim \bar{\pi}_{n+k}(E_k \bar{\Lambda} X)$, where $\bar{\pi}_m(X, x_0) = \overline{Com_h}(S^n, X)$

and where $\bar{\Lambda}$ denotes the smash-product which turns out to be appropriate for the purposes of strong shape theory.

In the course of our investigation of (2) we will meet several old friends, like:

1) Steenrod-Sitnikov homology ${}^S H_n(X;G)$ (in fact achieving a generalization of J. Milnor's axiomatic characterization [7] of this important homology), as well as

2) the Brown-Douglas-Fillmore [6] K-homology $\mathcal{E}_*$ on Com^f (= finite dimensional compacta), and finally

3) shape-singular homology (for connected X):

$$\tilde{\underline{E}}_n(X) = \bar{\underline{E}}_n(|\bar{S}(X)|),$$

where

$$\bar{S}: \bar{K} \to \mathcal{S}_E \quad (= \text{category of Kan-complexes})$$

is the shape-singular complex-functor, defined in complete analogy to the ordinary singular-complex-functor $S: Top \to \mathcal{S}_E$.

This talk is based on results which are laid down in [5] where proofs and further references can be found.

1. STRONG SHAPE THEORY.

We do not attempt to give a detailed account on the construction of a strong shape theory, referring to the relevant expositions (e.g. [3,4]). Here, we will confine ourselves to the following conceptual remarks:

Let $\underline{K}$ be a suitable category of topological spaces (e.g. $\underline{K} = Top_0$, Com (= based compact metric spaces), Com^f (= finite dimensional spaces in Com), etc.) and let $\underline{P} \subset \underline{K}$ be a (traditionally supposed to be full) subcategory of «good» spaces.

In shape theory, we are dealing with those invariants of a topological space X which are determined by mappings of X into «good» spaces.

It turns out very soon that this general program allows many different - and by no means equivalent - interpretations. We will indicate three possible realizations of this general idea:

1) Let $\underline{K}$ be a homotopy category of topological spaces, i. e. $\underline{K} = Top_{0\,h}$. A shape mapping $\hat{f}: X \to Y$ is a transformation which assigns to each morphism $g \in \underline{K}(Y, P)$, $P \in \underline{P}$, a $\hat{f}(g) \in \underline{K}(X, P)$ such that for a commutative triangle $rg' = g''$ in $\underline{K}$ one has

$$\hat{f}(rg') = \hat{f}(g'') = r\hat{f}(g').$$

This yields a category $\bar{\underline{K}}$, having the same objects as $\underline{K}$, with such shape mappings as morphisms. By making the right choice of $\underline{K}$ and $\underline{P}$ (e. g., $\underline{K} = Com_h$, $\underline{P}$ = compact ANRs) this gives us precisely K. Borsuk's shape category (in the description of S. Mardesič).

2) Repeat the construction in 1 but put $\underline{K} = Top_0$ (rather than the homotopy category). Thus $rg' = g''$ means now *strict* commutativity, not as in 1, an equality of homotopy classes.

While the category $\bar{\underline{K}}$ of 1 is for most purposes of algebraic topology too weak, the $\bar{\underline{K}}$ of 2 is on the contrary much too rigid.

3) In order to do algebraic topology, one has to take into account the fact that $\underline{K} = Top_0$ is a category equipped with the concept of a homotopy (rather than just with an equivalence relation «$\simeq$»). But at the same time we must avoid to go over to the homotopy category $\underline{K}_h$ right away. So, in strong shape theory we start off with a category like $\underline{K} = Top_0$, not with a homotopy category (this resembles the procedure in 2). A $\hat{f} \in \bar{K}(X, Y)$ is again, as in 2, an assignment

$$(g: Y \to P \in \underline{P}) \longmapsto (\hat{f}(g): X \to P \in \underline{P}),$$

but now we do not simply consider commutative triangles $rg' = g''$ (neither in $\underline{K}$ nor in $\underline{K}_h$) but pairs $(r, \omega): g' \to g''$ as morphisms in a category $\underline{P}_Y$ (having continuous $g: Y \to P \in \underline{P}$ as objects) where $\omega: rg' \simeq g''$ is a given homotopy. Now $\hat{f}$ is required to be a functor $\hat{f}: \underline{P}_Y \to \underline{P}_X$ between these categories, where we assume that $\hat{f}(r, \omega) = (r, \omega')$ for a suitable (but well-defined) homotopy $\omega': r\hat{f}(g') \simeq \hat{f}(g'')$.

To illustrate the essential difference between this procedure and that of 1 or 2, we observe that it may very well happen that ω is the constant homotopy, i.e., that $rg' = g''$, although ω' *is not* constant. Moreover it turns out to become necessary to endow $\underline{P}_Y$, $\underline{P}_X$ with the structure of a

2-category and $\hat{f}$ with the structure of a 2-functor which, in addition to what we have indicated, involves higher homotopies (between homotopies, etc).

We do not intend to enter into these technical details, referring to [3, 4].

By investing some more work, one could perform the whole construction for some kind of abstract 3-categories $\underline{K}$ coming together with a prescribed 3-subcategory $\underline{P}$.

In our concrete example, the 2-morphisms are (suitably adjusted) homotopies and the 3-morphisms homotopies between homotopies. Here one has to take care of the fact that the original definition of a homotopy between two maps $f', f'': X \to Y$ does not automatically turn $\underline{K}(X, Y)$ (X, Y fixed) into a category, because, although composition of homotopies can be defined, one does not have an identity nor is this composition associative.

In all three cases, we have a functor $h: \underline{K} \to \bar{\underline{K}}$ which is defined by $h(X) = X$ on the objects and by $h(f)(g) = gf$ on the morphisms $f \in \underline{K}$. Assume that we have $Y \in \underline{P}$, then there exists an assignment

$$h': \bar{\underline{K}}(X, Y) \to \underline{K}(X, Y) \quad \text{defined by} \quad h'(\hat{f}) = \hat{f}(1_Y).$$

All this can be accomplished in particular in the strong shape category $\bar{\underline{K}}$ (case 3 above) and we obtain

$$(1) \quad h'h(f) = f \qquad \text{and} \qquad (2) \quad hh'(\hat{f}) \approx \hat{f}$$

whenever the left expressions are defined. Here the $\approx$-sign refers to a homotopy relation in $\bar{\underline{K}}$, which is defined in complete analogy to ordinary homotopy by means of shape mappings $\bar{H} \in \bar{\underline{K}}(X \times [0, 1], Y)$.

In particular, we deduce from (2):

1.1. PROPOSITION. *To each* $\hat{f} \in \bar{\underline{K}}(X, Y)$, $Y \in \underline{P}$, *there exists a continuous* $f \in \underline{K}(X, Y)$ *with* $h(f) \approx \hat{f}$.

There is one important relation between Borsuk-Mardesič shape (1) and strong shape (3) which should be mentioned:

1.2. PROPOSITION [3]. *Let* $\underline{K} = Com$, $\underline{P}$ = *full subcategory of ANRs, then* $Sh\, X = Sh\, Y$ (*i. e.,* X *and* Y *are equivalent in the Borsuk shape category*) *iff they are homotopy equivalent in* $\underline{\bar{K}}_h$.

This result does not imply that the Borsuk shape category and the strong shape category are equivalent as categories!

The main objective of shape theory is to find a category $\underline{\bar{K}}$ in which one can do topology (in our case: algebraic topology, primarily homology theory) in the same way as in the category of CW-spaces but with a much richer supply of objects (including for example all compacta). So we need in our strong shape category a smash product $X \wedge Y$ for spaces (which could be accomplished quite easily) but also for shape mappings:

$$\hat{f} \wedge 1_Z : X \wedge Z \to Y \wedge Z .$$

The major obstacle in doing this is embodied in the fact that a continuous $g: Y \wedge Z \to P \in \underline{P}$ does not necessarily factors over a product

$$g_1 \wedge g_2 : Y \wedge Z \to P_1 \wedge P_2 ,$$

$P_i \in \underline{P}$. As a result we have to define a new kind of smash-product $X \bar{\wedge} Y$ even for spaces $X, Y \in \underline{K}$ which is not anymore a topological space in $\underline{K}$ (but which carries the structure of a special category).

This kind of smash-product is needed in $\underline{\bar{K}}$ for defining homology. In many special cases it turns out that this $\bar{\wedge}$-product coincides with the classical $\wedge$-product (cf. [4], Proposition 1.4).

2. HOMOLOGY.

Let $\underline{E} = \{ E_n, \sigma_n : \Sigma E_n \to E_{n+1} \}$ be a spectrum, i.e., a sequence of based spaces E_n, $n \in Z$, together with continuous mappings σ_n; Σ denotes reduced suspension. A CW-spectrum is one in which all the E_n are CW-complexes and all the σ_n are cellular inclusions. It is well-known that the last assumption is not essential and can always, up to homotopy, be achieved.

Let X be any based space: then we can define homology with coefficients in $\underline{E}$ by:

$$(1) \qquad \underline{E}_n(X) = \varinjlim_k \pi_{n+k}(E_k \wedge X), \quad n \in Z .$$

This functor is not very useful unless one agrees to impose some restrictions on $\underline{E}$ and on X. So, for a CW-spectrum $\underline{E}$ and a CW-complex X we evidently have:

2.1. PROPOSITION. *The natural transformation* $\omega : |S(X)| \to X$ $(S(\ ) =$ *singular complex,* $|...| =$ *geometric realization) induces an isomorphism:*

$$(2) \qquad \underline{E}_n(X) \approx \underline{E}_n(|S(X)|).$$

This factorization of $\underline{E}*$ over $S(\ )$ justifies calling $\underline{E}*$ «singular homology».

Of particular interest is the case $\underline{E} = \underline{K}(G)$, the Filenberg-Mac Lane spectrum to an abelian group G. In this case, the right hand side is simply ordinary singular homology $H_n(X;G)$.

Another important example is furnished by the classifying spectrum BU (BO) of complex (real) K-theory. Here, $BU_n(X)$ is a very important group for a finite CW-space, while for an arbitrary space X, it does not deserve much interest.

The situation changes as soon as one tries to convey (1) into strong shape theory: Using the $\bar{\Lambda}$-product mentioned in Section 1, we can define shape homology with coefficients in $\underline{E}$ by

$$(3) \qquad \bar{\underline{E}}_n(X) = \varinjlim_k \bar{\pi}_{k+n}(E_k \bar{\Lambda} X),$$

where $\bar{\pi}_m(Y) = \bar{\underline{K}}_h(S^m, Y)$ denotes the shape homotopy groups. This furnishes a functor $\bar{\underline{E}}*: \bar{\underline{K}} \to Ab^{\mathbf{Z}}$ which is well-defined at least for any CW-spectrum. For $\underline{K} = Com$, we can say a little more:

2.3. THEOREM [4, 5]. *The functor* $\bar{\underline{E}}*: \bar{\underline{K}} \to Ab^{\mathbf{Z}}$ *has the following properties:*

1) $\hat{f}_0 \approx \hat{f}_1 \Rightarrow \bar{\underline{E}}*\hat{f}_0 = \bar{\underline{E}}*\hat{f}_1$.

2) There exists a natural isomorphism $\bar{\sigma}_n : \bar{\underline{E}}_n(X) \approx \bar{\underline{E}}_{n+1}(\Sigma X)$.

3) Let $i : A \subset X$ *be an inclusion,* $p : X \to X/A$ *the projection, then there exists an exact sequence:*

$$\bar{\underline{E}}_n(A) \xrightarrow{\ i*\ } \bar{\underline{E}}_n(X) \xrightarrow{\ p*\ } \bar{\underline{E}}_n(X/A).$$

Here we set e.g. $i* = \bar{\underline{E}}_n(h(i))$.

4) Let $i : A \subset X$ *be an inclusion,* A *contractible (in* $\bar{\underline{K}}$*); then the projection* $p : X \to X/A$ *induces an isomorphism* $\bar{\underline{E}}*(X) \approx \bar{\underline{E}}*(X/A)$.

These are the Eilenberg-Steenrod axioms for reduced homology. Axiom 4, for example, reflects the important fact that any inclusion in *Com* becomes a cofibration in $\overline{Com}$ (cf. [4]).

The question arises under what conditions the following axiom (Milnor's celebrated «cluster axiom») holds:

5) Let $X_i = (X_i, x_{i0}) \in Com$ be given, $i = 1, 2, \dots$, and

$$\overset{\infty}{\underset{i=1}{Cl}} X_i = \underset{\longleftarrow}{lim}\, X_1 \vee \dots \vee X_k$$

be the cluster (or strong wedge). Then the natural isomorphism

$$(4) \qquad \bar{\underline{E}}*(\overset{\infty}{\underset{i=1}{Cl}} X_i) = \prod_{i=1}^{\infty} \bar{\underline{E}}*(X_i)$$

becomes an isomorphism.

Since we can replace 1 by the following (weaker) axiom:

1') $f_0, f_1 \in Com, \ f_0 \approx f_1 \quad \Rightarrow \quad \bar{\underline{E}}*h\, f_0 = \bar{\underline{E}}*h\, f_1$,

we can speak about the Eilenberg-Steenrod axioms for $\bar{\underline{E}}*h : \underline{K} \to Ab^{Z}$.

In the case of the cluster axiom we will have to deal with the fact that sometimes (4) becomes only an isomorphism on the subcategory Com^f (consisting of all compacta of finite dimension), i.e., we require that all the X_i as well as the cluster $\overset{\infty}{\underset{i=1}{Cl}} X_i$ are spaces in Com^f. We will henceforth say that a spectrum $\underline{E}$ having this property *fulfills the cluster axiom on* Com^f. We have

2.4. THEOREM. *1) There exist CW-spectra $\underline{E}$ such that 5 does not hold (neither on Com nor on Com^f).*

2) For $\underline{E} = \underline{K}(G)$ (G an arbitrary abelian group) as well as for any suspension spectrum, the cluster axiom is valid.

3) The spectra BU and BO fulfill the cluster axiom on Com^f.

Let $\underline{P}_0 \subset Com$ denote the full subcategory of all (based) finite CW-spaces. The importance of 2.4, 2 and 3, lies in the validity of the following uniqueness theorem:

2.5. THEOREM. *Let $\underline{E}$ be a CW-spectrum such that the cluster axiom holds (resp. holds in Com^f); then any extension of $\underline{E}*: \underline{P}_0 \to Ab^{Z}$ over Com (resp. over Com^f) which fulfills the Eilenberg-Steenrod axioms 1-4 and the cluster axiom 5 (resp. on Com^f) is naturally isomorphic to*

$$\bar{\underline{E}}*h: Com \to Ab^{\mathbf{Z}} \quad (resp. \quad \bar{\underline{E}}*h: Com^f \to Ab^{\mathbf{Z}}).$$

As an immediate corollary we obtain:

2.6. COROLLARY. *Let* $\underline{E}$ *be as in Theorem 2.5; then any extension of* $\underline{E}*: \underline{P}_0 \to Ab^{\mathbf{Z}}$ *over Com (over* Com^f*) which fulfills the Eilenberg-Steenrod axioms 1-4 and the cluster axiom 5 (resp. the cluster axiom on* Com^f*) allows an extension over* $\overline{Com}$ *(resp. over* $\overline{Com^f}$*).*

We will return to this point very soon.

3. HOMOLOGY AND SINGULAR COMPLEX.

In Proposition 2.1 we took notice of the fact that $\underline{E}_n(\ )$ factors over the singular complex. In $\bar{\underline{K}}$ (now we take for $\underline{K}$ the category Top_0 and for $\underline{P}$ the full subcategory of all ANEs (for metric spaces), we have also a kind of singular complex (cf. [2, 4]):

$$\bar{S}: \bar{K} \to \mathcal{S}_E \quad (= \text{category of Kan-complexes}),$$
$$\bar{S}(X)_n = \{ \bar{\sigma}^n \in \bar{\underline{K}}(\Delta^n, X) \},$$

with face and degeneracy defined in the classical manner. Moreover there exists a natural transformation of functors $\bar{\omega}: |\bar{S}(\ )| \to 1_{\bar{K}}$ which induces a weak homotopy equivalence in $\bar{\underline{K}}$ (cf. [2]). Here a weak homotopy equivalence is evidently a shape mapping $\hat{f} \in \bar{\underline{K}}(X, Y)$ having the property that $\bar{\pi}*(\hat{f})$ becomes an isomorphism.

We are trying to detect those CW-spectra $\underline{E}$ which have (at least for suitably chosen categories K) the following property:

W) Every weak homotopy equivalence $\hat{f}$ induces an isomorphism $\bar{\underline{E}}*(\hat{f})$.

For this purpose we need a special class of spectra:

A *cs-spectrum* (= compact skeleton spectrum) $\underline{E} = \{E_n\}$ is a CW-spectrum having the following properties:

a) Every m-skeleton $(E_n)^m$ for all $m = 0, 1, \ldots$ and all $n \in \mathbf{Z}$ is compact.

b) There exists a n_0 such that all E_n, $n \geq n_0$, are simply connected.

Then we have:

3.1. THEOREM. *Let $\underline{E}$ be a cs-spectrum and:*

a) let $\hat{f} \in \overline{Com}(X, Y)$ be a weak homotopy equivalence for connected X, Y; then $\bar{\underline{E}}_*(\hat{f})$ is an isomorphism.

b) Let $X \in Com$ be a continuum; then we have an isomorphism

$$\bar{\underline{E}}_*(|\bar{S}(X)|) \approx \bar{\underline{E}}_*(X). \tag{1}$$

In other words: $\bar{\underline{E}}_*(X)$ turns out to be singular homology in the strong shape category.

REMARKS. 1) Condition b for a cs-spectrum is not essential, because for each $\underline{E}$ there always exists a spectrum $\underline{E}'$ such that b holds as well as a natural isomorphism $\bar{\underline{E}}_* \approx \bar{\underline{E}}'_*$.

2) 3.1 a ensures that, at least on the category of continua, a Whitehead axiom holds for $\bar{\underline{E}}_*$ ($\underline{E}$ being a cs-spectrum). One cannot expect this being true for Com or even for Top_0. However there exists an extension $\tilde{\underline{E}}_*$ of $\bar{\underline{E}}_*$ for any spectrum $\underline{E}$ such that (W) holds (cf. [5]).

4. RELATIONS TO OTHER HOMOLOGY THEORIES.

In [7] J. Milnor proved that the so-called Steenrod-Sitnikov homology $H_*^S(X, A; G)$, defined on the category of compact metric pairs and for an arbitrary abelian coefficient group G, is, up to a natural isomorphism, determined by the properties 1-4 in Theorem 2.3 together with the cluster axiom 5.

Differently from our approach, he uses these axioms for pairs (not for based spaces as we did), but the transition from a homology for based spaces to a homology which is defined on a category of pairs and vice-versa is standard and does not need any further explanation.

In particular we deduce from [7] that $H_*^S(X; G)$, $X = (X, x_0) \in Com$, fulfills the Eilenberg-Steenrod axioms as well as the cluster axiom. From Theorem 2.5 we obtain the following assertion, which implies Milnor's original uniqueness Theorem:

4.1. THEOREM. *Steenrod-Sitnikov homology (in its reduced form, defined on the category Com) with coefficients in an abelian group G is naturally isomorphic to $\overline{\underline{K}(G)}_*(\;)$ (more precisely to $\overline{\underline{K}(G)}_{*h}(\;)$).*

For G being finitely generated, we can realize $\underline{K}(G)$ as a cs-spectrum, hence we have:

4.2. THEOREM. *Let X be a continuum, G finitely generated; then we have a natural isomorphism:*

$$\overline{\underline{K}(G)}_*(X) \approx H_*(|\bar{S}(X)|; G).$$

The case $G = Z$ has already been settled in [2], Theorem 7.7, but by using different methods.

As our second application, we deduce from Theorem 2.5 that every functor $\tilde{H}_*: Com^f \to Ab^Z$ is, up to an isomorphism, determined by the Eilenberg-Steenrod axioms, the cluster axiom and the fact that $\tilde{H}_*$ is isomorphic to BU_* on $\underline{P}_0$ (= category of compact CW-spaces).

In [6] the authors introduced a kind of homology theory $\mathfrak{S}_*$ on Com by purely functional analytic methods, which plays an important role in analysis. Moreover they were able to verify that $\mathfrak{S}_*$ fulfills the Eilenberg-Steenrod axioms, the cluster axiom and, in addition, that $\mathfrak{S}_*$ is isomorphic to BU_* on P_0. As a consequence, we have due to Theorem 2.4:

4.3. THEOREM. *The isomorphism $a: \mathfrak{S}_* \approx BU_*$ on $\underline{P}_0$ allows a unique extension $\bar{a}: \mathfrak{S}_* \approx \overline{BU}_*h$ over Com^f.*

REMARK. One should observe that the excision axiom for $\mathfrak{A}_{CM}$ (= category of compact pairs) which is obtained by translating property 4 in 2.3 is in fact the strong excision of [7].

5. COHOMOLOGY.

Although we are dealing in this talk exclusively with homology, a few words on cohomology are in order:

Let $\underline{E}$ be a CW-spectrum, $X = (X, x_0)$ any space, then we can define cohomology with coefficients in E :

$$(1) \qquad \underline{E}^n(X) = \varinjlim_k [\Sigma^k X, E_{n+k}], \quad n \in Z.$$

Because all spaces E_n are by assumption CW-spaces, we deduce from Proposition 1.1 that there is no need to distinguish between $\underline{E}^n(X)$ and $\bar{\underline{E}}^n(X)$ (the latter defined in $\bar{\underline{K}}_h$). A well-known theorem (cf. [8], also for further references) asserts that, under mild restrictions on the coverings involved, $\underline{K}(G)^*(\ )$ is nothing else than Čech cohomology.

This changes immediately whenever one agrees to deal with spectra $\underline{E}$ which are not CW-spectra (cf. [4]). Furthermore, it should be kept in mind that (1) in general is not identical with the cohomology in the sense of Boardman (defined in the Boardman category [1]):

$$(2) \qquad {}_B\underline{E}^n(X) = \{\underline{X}, \Sigma^{-n}\underline{E}\},$$

where $\underline{X} = \{\Sigma^k X\}$ is either a suspension spectrum (whenever X is a CW-space itself) or a CW-substitute for such a spectrum, and where the $\{\ ..,..\}$ brackets are denoting the morphisms in the Boardman category.

While (1) is Čech cohomology with coefficients in $\underline{E}$, (2) is what one should call singular cohomology with coefficients in $\underline{E}$.

The relations between homology (in the sense of Section 2 (3)) and (1) are treated elsewhere (cf. [4]).

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