ROSS STREET

Correction to "Fibrations in bicategories"

Cahiers de topologie et géométrie différentielle catégoriques, tome 28, nº 1 (1987), p. 53-56

<http://www.numdam.org/item?id=CTGDC_1987__28_1_53_0>

CAHIERS DE TOPOLOGIE
ET GÉOMÉTRIE DIFFÉRENTIELLE
CATÉGORIQUES

Vol. XXVIII-1 (1987)

CORRECTION TO "FIBRATIONS IN BICATEGORIES"

by Ross STREET

Given homomorphisms of bicategories J: $A \to$ *Cat* and S: $A \to K$, the bilimit {J, S} of S indexed by J can be constructed using biproducts, cotensor biproducts and biequalizers. However, the construction described in Section 1 (1.24), page 120, of the paper "Fibrations in bicategories" (these *Cahiers*, XXI-2, 1980, 111-160) is wrong. I am grateful to Max Kelly for noticing this error. He also recognized that {J,S} could be constructed if K admitted some further bilimits of a simple kind. In fact, no further bilimits are needed: I shall show that they can be constructed from those at hand.

Certain small categories **Iso, End, Aut,** T will be required. A functor from one of these into a category amounts to an isomorphism, endomorphism, automorphism, composable pair of isomorphisms, respectively, in the category. Notice that **Iso,** T are equivalent to 1 while **Aut** is not.

The *biequifier* of 2-cells

$$\theta,\ \phi\colon f \Rightarrow g : A \to B$$

in K is an arrow $h\colon X \to A$ which, for all objects X, induces an equivalence of categories between K(K, X) and the full subcategory of K(K, A) consisting of those $a\colon K \to A$ for which $\theta a = \phi a\colon fa \Rightarrow ga$. (If θ, ϕ are invertible this is the same as the *biequinverter* of θ, ϕ as used later (4.2) in the paper.) The *biidentifier* of an endo-2-cell

$$\psi\colon f \Rightarrow f\colon A \to B$$

is the biequifier of ψ and the identity of f. If ϕ is invertible, the biequifier of θ, ϕ is the biidentifier of $\phi^{-1}\theta$. So to construct biequifiers of invertible pairs it suffices to construct biidentifiers of auto-2-cells.

Consider an auto-2-cell $\psi\colon f \Rightarrow f\colon A \to B$. Let $\psi^{-}, f^{-}\colon A \to \{\mathbf{Aut}, B\}$ be the arrows corresponding to the automorphisms ψ, 1_f in $K(A, B)$. Let $h\colon H \to A$, $\sigma\colon \psi^{-}h \simeq f^{-}h$, be the biequalizer of ψ^{-}, f^{-}. Let $k\colon K \to A$, $\tau\colon fk \simeq fk$, be the biequalizer of f, f. Let $e\colon \{\mathbf{Aut}, B\} \to B$ be induced by the unique functor $1 \to \mathbf{Aut}$. There exist $l\colon H \to K$ and $v\colon kl \simeq h$ rendering $e\sigma$ isomorphic to τl by definition of K. Similarly, we obtain $d\colon A \to K$ and $kd \simeq 1_K$ rendering 1_f isomorphic to τd. Now form the bipullback

$$\begin{tikzcd} P \arrow[r, "v"] \arrow[d, "u"'] \arrow[dr, phantom, "="] & H \arrow[d, "l"] \\ A \arrow[r, "d"'] & K \end{tikzcd}$$

of d, l ; this is just the biequalizer of the two arrows from the biproduct of A, H into K which use d, l and the projections. I claim $u\colon P \to A$ is the biidentifier of $\psi\colon f \Rightarrow f$. Since bilimits are defined representably, we only need to check the construction in *Cat*. Then K can be taken to be the category of pairs (a, τ) where a is an object of A and $\tau\colon fa \simeq fa$ in B, while H can be taken to be the full subcategory of K consisting of the pairs (a, σ) with $\sigma.\psi a = \sigma$. Since σ is invertible, the last equation implies $\psi a = 1$. Also l is the inclusion and d takes a to $(a, 1_f a)$. With this we see that the objects of P are pairs (σ, ρ) where

$$\rho\colon a \simeq a' \text{ in A}, \quad (a, \sigma) \in \text{H and} \quad f\rho.\sigma = f\rho.$$

This last condition implies $\sigma = 1_{fa}$, and, since ψ is natural, $\psi a' = 1$. So P is equivalent to the full subcategory of A consisting of those a with $\psi a = 1$.

(The above construction with **Aut** replaced by **End** yields the biidentifier of any endo-2-cell ψ.)

The next bilimit required is the *descent object* Desc(X) of a truncated bicosimplicial diagram

$$X\colon \quad \begin{tikzcd} X_0 \arrow[r, shift left=2, "\delta_0"] \arrow[r, shift right=2, "\delta_1"'] & X_1 \arrow[l, "\iota"'] \arrow[r, shift left=3, "\delta_0"] \arrow[r, "\delta_1"] \arrow[r, shift right=3, "\delta_2"'] & X_2 \end{tikzcd}$$

$$\sigma_{ij} : \delta_i\delta_{j-1} \simeq \delta_j\delta_i \quad \text{for} \quad i < j, \qquad n_i : 1 \simeq \iota\delta_i$$

in a bicategory K. When K is Cat, the category Desc(X) has objects pairs (x, θ) where x is an object of X_0 and θ: $\delta_0 x \simeq \delta_1 x$ in X_1 such that

$$\iota\theta = \mu_1\mu_0^{-1}, \quad \sigma_{12}.\delta_1\theta.\sigma_{01} = \delta_2\theta.\sigma_{02}.\delta_0\theta,$$

and has arrows χ: $(x, \theta) \to (x', \theta')$ where χ: $x \to x'$ is an arrow of X_0 such that $Q'.\delta_0\chi = \delta_1\chi.\theta$. For a general K, the descent object of X consists of an object D, an arrow h: D $\to$ X_0 and an invertible 2-cell ω: $\delta_0 h \simeq \delta_1 h$ inducing an equivalence between K(K, D) and Desc(K(K,X)). Notice that X can be regarded as a homomorphism from an appropriate category A into K and, if we take J: $A \to Cat$ to be the functor amounting to the diagram

$$\begin{tikzcd} 1 \arrow[r, shift left=2] \arrow[r, shift right=2] & \mathrm{Iso} \arrow[l] \arrow[r, shift left=2] \arrow[r] \arrow[r, shift right=2] & T \end{tikzcd}$$

the bilimit {J, X} is equivalent to Desc(X).

The descent object can be constructed using biequalizers and biidentifiers of auto-2-cells. First, take the biequalizer

$$h\colon H \to X_0, \qquad \theta\colon \delta_0 h \simeq \delta_1 h,$$

of δ_0, δ_1, then the biequifier k: K $\to$ H of the two invertible 2-cells

$$\begin{tikzcd} \iota\delta_0 h \arrow[rr, "\iota\theta"] \arrow[dr, "\mu_0^{-1}h"'] & & \iota\delta_1 h \\ & h \arrow[ur, "\mu_1 h"'] & \end{tikzcd}$$

and then, the biequifier m: M $\to$ L of the two invertible 2-cells

$$\begin{tikzcd} & \delta_1\delta_0 hk \arrow[r, "\delta_1\theta k"] & \delta_1\delta_1 hk \arrow[dr, "\sigma_{12}hk"] & \\ \delta_0\delta_0 hk \arrow[ur, "\sigma_{01}hk"] \arrow[dr, "\delta_0\theta k"'] & & & \delta_2\delta_1 hk \\ & \delta_0\delta_1 hk \arrow[r, "\sigma_{02}hk"'] & \delta_2\delta_0 h \arrow[ur, "\delta_2\theta k"'] & \end{tikzcd}$$

Then L, hkm, θkm form Desc(X).

The bilimit {J,S} can be obtained as the descent object Desc(X) where

$$X_0 = \Pi\Pi_A \{JA, SA\}, \qquad X_1 = \Pi\Pi_{A,B} \{A(A, B)\times JA, SB\},$$

$$X_2 = \Pi\Pi_{A,B,C}\ \{A(B,\ C)\times A(A,\ B)\times JA,\ SC\}.$$

In (1.25) it was stated that indexed pseudo-limits in a 2-category could be constructed from cotensor products, products and equalizers. This is certainly true since pseudo-limits are particular indexed limits and all indexed limits can be so constructed (see [14], using the *Bibliography* of the paper). The proof outlined in (1.25) was a modification of (1.24). Using the corrected (1.24), we can squeeze out more from the method. Many naturally occuring 2-categories have *iso-inserters*. The iso-inserter of the diagram *f, g:* $A \rightrightarrows B$ is its *limit* (not pseudo-limit) indexed by the diagram

$$1 \rightrightarrows \mathrm{Iso} \qquad \text{in } \mathit{Cat}.$$

An iso-inserter is a biequalizer but not conversely. The *strict descent object* of a truncated simplicial object X (this time μ_i, σ_{ij} are identities) is defined as for the descent object except that we insist on an isomorphism between $K(K,\ D)$ and $\mathrm{Desc}(K(K,\ X))$, not merely an equivalence. It can be constructed using an iso-inserter and identifiers of auto-2-cells (the latter are defined as were biidentifiers except that we ask for an isomorphism in the representation property). Then psdlim(J, S) is the strict descent object for X as before with biproducts and cotensor biproducts replaced by their "non-bi" versions. However, it does not seem possible to construct identifiers of auto-2-cells using a "non-bi" version of the construction of biidentifiers. The object P we are led to does support an idempotent whose splitting gives the identifier; but this is already true of the iso-inverter of ψ^-, $f^\cdot$. So *products, cotensor products, iso-inserters, and, either identifiers of auto-2-cells or splittings of idempotents, imply all indexed pseudo-limits.*

I would like to stress that I am currently using the word "weighted" in preference to "indexed" in this context.

Finally, there is a typographical error in (4.2) on page 140. The functors between 1 and Iso should have their directions reversed.

Department of Mathematics
Macquarie University
N.S.W. 2109
AUSTRALIA