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ON SPECIAL TYPES OF NONHOLONOMIC JETS

by *Ivan KOLAR*

Résumé. Le concept d'un type spécial de r -jets non-holonomes et de (k, r) -vitesses non-holonomes est discuté d'un point de vue général. Une attention particulière est porte à la composition de r -jets non-holonomes du même type. Les cas où le produit est préservé sont caractérisés en terme d'algèbres de Weil.

Let $\mathcal{M}f$ be the category of all manifolds and all smooth maps, $\mathcal{M}f_I$ be the category of all manifolds and local diffeomorphisms and $\mathcal{M}f_m$ be the category of m -dimensional manifolds and local diffeomorphisms. The constructions $J^r(M, N)$ of holonomic r -jets and $\tilde{J}^r(M, N)$ of non-holonomic r -jets between two manifolds M, N are bundle functor on the product category $\mathcal{M}f_I \times \mathcal{M}f$, provided for every smooth map $f : N \rightarrow \bar{N}$, every local diffeomorphism $g : M \rightarrow \bar{M}$ and every $Z \in \tilde{J}_x^r(M, N)_y$ we define

$$(1) \quad \tilde{J}^r(g, f)(Z) = (j_y^r f) \circ Z \circ (j_x^r g)^{-1},$$

where \circ denotes the composition of nonholonomic r -jets, [3], [6]. Further, $J^r(g, f)$ is the restriction and corestriction of $\tilde{J}^r(g, f)$ to the holonomic subbundles, [7]. Clearly, both functors J^r and \tilde{J}^r preserve products in the second factor, i.e.

$$(2) \quad \tilde{J}_x^r(M, N_1 \times N_2) = \tilde{J}_x^r(M, N_1) \times \tilde{J}_x^r(M, N_2), \quad x \in M,$$

and the same holds in the holonomic case. Many special types of non-holonomic r -jets were studied in differential geometry, [5], [9], the semi-holonomic r -jets, introduced already by C. Ehresmann, [3], are the best known example.

In [8], we described the bundle functors on $\mathcal{M}f_m \times \mathcal{M}f$ preserving products in the second factor by means of Weil algebras. Using this

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point of view, we introduced the general concept of r -jet functor E as a bundle functor on $\mathcal{M}f_m \times \mathcal{M}f$ satisfying $J^r \subset E \subset \tilde{J}^r$ and preserving products in the second factor, [5]. Then we deduced a complete characterization of E in terms of Weil algebras. However, the composition of nonholonomic r -jets cannot be included into this framework. That is why we discuss the composition problem in the present paper.

In Section 1 we summarize the basic properties of nonholonomic r -jets and nonholonomic (k, r) -velocities from the functorial point of view. In Section 2 we treat a general nonholonomic r -jet m -functor F_m on $\mathcal{M}f_m \times \mathcal{M}f$ and a general nonholonomic r -velocity m -functor V_m on $\mathcal{M}f$ without the assumption of preserving products. Such functors are said to be “weak”. A weak total r -jet functor is defined on the category $\mathcal{M}f_I \times \mathcal{M}f$ and has the composition property. In Proposition 1 we present a simple condition for a sequence (F_m) , $m \in \mathbb{N}$, of weak r -jet m -functors to form a weak total r -jet functor F . Section 3 is devoted to another characterization of a weak total r -jet functor in terms of a sequence (V_m) , $m \in \mathbb{N}$, of weak r -velocity m -functors. In Section 4 we add the condition of preserving products. The general theory of Weil bundles, [6], [7], yields directly a characterization of an r -jet m -functor F_m by means of a Weil algebra \mathbb{D}_m^F . (We remark that this approach is close to the synthetic differential geometry, [4].) In Proposition 4 we deduce a complete description of a total r -jet functor F in terms of a sequence (\mathbb{D}_m^F) , $m \in \mathbb{N}$, of Weil algebras and their homomorphisms.

All manifolds and maps are assumed to be infinitely differentiable. Unless otherwise specified, we use the terminology and notation from the book [7].

1. Nonholonomic jets and velocities.

The r -th nonholonomic prolongation $\tilde{J}^r Y$ of a fibered manifold $Y \rightarrow M$ is defined by the iteration

$$\tilde{J}^r Y = J^1(\tilde{J}^{r-1} Y \rightarrow M),$$

$\tilde{J}^1 Y = J^1 Y$. The canonical inclusion $J^r Y \hookrightarrow \tilde{J}^r Y$ is determined by $j_{\underline{x}}^r s \mapsto j_x^1(u \mapsto j_u^{r-1} s)$ for every local section s of Y , $u \in M$. The space $\tilde{J}^r(M, N)$ of nonholonomic r -jets of M into N is the r -th nonholonomic prolongation of the product fibered manifold $M \times N \rightarrow M$. So we have $J^r(M, N) \subset \tilde{J}^r(M, N)$. Let Q be a third manifold. Ehresmann

introduced the composition of nonholonomic r -jets by the following induction, [3]. For $r = 1$, we have the composition of 1-jets. Write $\beta : \tilde{J}^{r-1}(M, N) \rightarrow N$ for the canonical projection. Let $X = j_x^1 s(u) \in \tilde{J}_x^r(M, N)_y$, $u \in M$, and $Z = j_y^1 \sigma \in \tilde{J}_y^r(N, Q)_z$, $y = \beta(s(x))$. Then

$$(3) \quad Z \circ X := j_x^1(\sigma(\beta(s(u)))) \circ s(u) \in \tilde{J}_x^r(M, Q)_z$$

with the composition of nonholonomic $(r-1)$ -jets on the right hand side. If X and Z are holonomic r -jets, then (3) coincides with the classical jet composition. Using induction, one deduces directly from (3) that the composition of nonholonomic r -jets is associative. We write

$$\tilde{L}_{m,n}^r = \tilde{J}_0^r(\mathbb{R}^m, \mathbb{R}^n)_0, \quad \tilde{L}^r = \bigcup_{m,n \in \mathbb{N}} \tilde{L}_{m,n}^r.$$

Hence \tilde{L}^r is the skeleton of the category of nonholonomic r -jets. The composition maps will be denoted by

$$(4) \quad \varkappa_{m,n,q} : \tilde{L}_{m,n}^r \times \tilde{L}_{n,q}^r \rightarrow \tilde{L}_{m,q}^r.$$

The functor \tilde{T}_k^r of nonholonomic (k, r) -velocities is defined by

$$(5) \quad \tilde{T}_k^r M = \tilde{J}_0^r(\mathbb{R}^k, M), \quad \tilde{T}_k^r f(Z) = (j_x^r f) \circ Z$$

for every manifold M , every map $f : M \rightarrow N$ and every $Z \in (\tilde{T}_k^r M)_x$. By (2), \tilde{T}_k^r is a product preserving bundle functor on $\mathcal{M}f$. Every $X \in \tilde{L}_{l,k}^r$ defines a natural transformation (denoted by the same symbol)

$$(6) \quad X : \tilde{T}_k^r \rightarrow \tilde{T}_l^r, \quad X_M(Z) = Z \circ X, \quad Z \in (\tilde{T}_k^r M)_x.$$

Indeed, $X_M(\tilde{T}_k^r f(Z)) = ((j_x^r f) \circ Z) \circ X = (j_x^r f) \circ (Z \circ X)$ by the associativity of the composition of nonholonomic r -jets.

Replacing $\tilde{J}_0^r(\mathbb{R}^k, M)$ by $J_0^r(\mathbb{R}^k, M)$, we obtain the classical functor T_k^r of holonomic (k, r) -velocities, [2], [7]. Even T_k^r preserves products. In the holonomic case, all natural transformations $T_k^r \rightarrow T_l^r$ are of the form (6) with $X \in L_{l,k}^r = J_0^r(\mathbb{R}^l, \mathbb{R}^k)_0$. However, in the nonholonomic case there are further natural transformations beside (6). The simplest example is the case of the functor \tilde{T}_1^2 , which is naturally isomorphic to the iterated tangent functor TT . Using the general procedure for finding the homomorphisms of Weil algebras, one evaluates easily all natural transformations $TT \rightarrow TT$, [6]. Then one sees directly that not all of them are of the form (6).

A fundamental result reads that every product preserving bundle functor F on $\mathcal{M}f$ is a Weil functor T^A , whose Weil algebra is $A = F\mathbb{R}$, [6], [7]. The Weil algebra of T_k^r is $\mathbb{D}_k^r = J_0^r(\mathbb{R}^k, \mathbb{R})$. Using translations on \mathbb{R}^k , one identifies \tilde{T}_k^r with $T_k^1 \circ \dots \circ T_k^1$. Hence the Weil algebra $\tilde{\mathbb{D}}_k^r = \tilde{J}_0^r(\mathbb{R}^k, \mathbb{R})$ is identified with the tensor product

$$\tilde{\mathbb{D}}_k^r = \mathbb{D}_k^1 \underbrace{\otimes \dots \otimes}_{r\text{-times}} \mathbb{D}_k^1.$$

The canonical projection $\mathbb{D}_k^1 \rightarrow \mathbb{R}$ combined with the identities on the other factors defines r projections

$$\pi_i : \tilde{\mathbb{D}}_k^r \rightarrow \mathbb{D}_k^1 \underbrace{\otimes \dots \otimes}_{(r-1)\text{-times}} \mathbb{D}_k^1 = \tilde{\mathbb{D}}_k^{r-1}, \quad i = 1, \dots, r.$$

The Weil algebra $\bar{\mathbb{D}}_k^r = \bar{J}_0^r(\mathbb{R}^k, \mathbb{R})$ corresponding to the semiholonomic case is the subalgebra of all elements $Z \in \tilde{\mathbb{D}}_k^r$ satisfying $\pi_i(Z) = \pi_j(Z)$ for all $i, j = 1, \dots, r$.

2. Weak velocity and jet functors.

For a bundle functor E on $\mathcal{M}f$, the condition $T_m^r \subset E \subset \tilde{T}_m^r$ means

$$J_0^r(\mathbb{R}^m, N)_x \subset (EN)_x \subset \tilde{J}_0^r(\mathbb{R}^m, N)_x, \quad x \in N,$$

$$Ef(Z) = (j_x^r f) \circ Z, \quad Z \in (EN)_x$$

for every manifold N and every map $f : N \rightarrow \bar{N}$.

Let G_m^r be the r -th jet group in dimension m .

Definition 1. A weak r -velocity m -functor V_m is a bundle functor on $\mathcal{M}f$ such that $T_m^r \subset V_m \subset \tilde{T}_m^r$ and

$$(7) \quad Z \in (V_m N)_x, \quad X \in G_m^r \quad \text{implies} \quad Z \circ X \in (V_m N)_x.$$

Hence we have an action of G_m^r on each $(V_m N)_x$. In particular, for every m -dimensional manifold M we can construct the associated fiber bundle $P^r M[V_m N]$, where $P^r M$ is the r -th order frame bundle of M .

For a bundle functor E on $\mathcal{M}f_m \times \mathcal{M}f$, the condition $J^r \subset E \subset \tilde{J}^r$ means

$$J_x^r(M, N)_y \subset E_x(M, N)_y \subset \tilde{J}_x^r(M, N)_y, \quad x \in M, \quad y \in N,$$

$$E(g, f)(Z) = (j_y^r f) \circ Z \circ (j_x^r g)^{-1} \quad Z \in E_x(M, N)_y,$$

for every map $f : N \rightarrow \overline{N}$ and every local diffeomorphism $g : M \rightarrow \overline{M}$.

Definition 2. A weak r -jet m -functor F_m is a bundle functor on $\mathcal{M}f_m \times \mathcal{M}f$ satisfying $J^r \subset F_m \subset \tilde{J}^r$.

Given V_m , we define a bundle functor jV_m on $\mathcal{M}f_m \times \mathcal{M}f$ by

$$jV_m(M, N) = P^r M[V_m N], \quad jV_m(g, f) = P^r g[V_m f],$$

where $P^r g : P^r M \rightarrow P^r \overline{M}$ is the principal bundle morphism induced by g . According to (6) and (7), $V_m f$ is a G_m^r -equivariant map, so that $P^r g[V_m f]$ is a well defined morphism of associated bundles.

Given F_m , we define a bundle functor vF_m on $\mathcal{M}f$ by

$$vF_m(N) = (F_m)_0(\mathbb{R}^m, N), \quad (vF_m)f(Z) = F_m(\text{id}_{\mathbb{R}^m}, f)(Z),$$

$Z \in vF_m(N)$. For $X = j_0^r \gamma \in G_m^r$, we have

$$Z \circ X = F_m(\gamma^{-1}, \text{id}_N)(Z),$$

so that vF_m is a weak r -velocity m -functor. Thus, we have established a bijection between the weak r -velocity m -functors and weak r -jet m -functors.

Since $\mathcal{M}f_I \times \mathcal{M}f$ is the union of $\mathcal{M}f_m \times \mathcal{M}f$ for all $m \in \mathbb{N}$, every bundle functor F on $\mathcal{M}f_I \times \mathcal{M}f$ is a sequence $F = (F_m)$ of bundle functors on $\mathcal{M}f_m \times \mathcal{M}f$.

Definition 3. A weak total r -jet functor is a sequence $F = (F_m)$ of weak r -jet m -functors with the composition property: if $Z_1 \in F_x(M, N)_y$ and $Z_2 \in F_y(N, Q)_z$, then $Z_2 \circ Z_1 \in F_x(M, Q)_z$.

If we write

$$L_{m,n}^F = F_0(\mathbb{R}^m, \mathbb{R}^n)_0, \quad L^F = \bigcup_{m,n \in \mathbb{N}} L_{m,n}^F,$$

then L^F is a category satisfying $L^r \subset L^F \subset \tilde{L}^r$. The elements of

$$J^F(M, N) := F_m(M, N), \quad m = \dim M,$$

will be called F -jets of M into N . Every element $Z \in J_x^F(M, N)_y$ is of the form

$$Z = v \circ X \circ u^{-1}, \quad u \in P_x^r M, \quad v \in P_y^r N, \quad X \in L_{m,n}^F,$$

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$n = \dim N$. Given another $\bar{Z} \in J_y^F(N, Q)_z$, $\bar{Z} = w \circ \bar{X} \circ v^{-1}$, $w \in P_z^r Q$, $\bar{X} \in L_{n,q}^F$, we have

$$\bar{Z} \circ Z = w \circ (\bar{X} \circ X) \circ u^{-1}.$$

This defines the category of F -jets and L^F is its skeleton.

Conversely, if we have a subcategory $L^r \subset \mathcal{L} \subset \tilde{L}^r$, we construct a weak total r -jet functor \mathcal{F} as follows. The jet composition yields an action of $G_m^r \times G_n^r$ on $\mathcal{L}_{m,n}$ and we define $\mathcal{F}(M, N)$ to be the associated fiber bundle

$$\mathcal{F}(M, N) = (P^r M \times P^r N)[\mathcal{L}_{m,n}],$$

$m = \dim M$, $n = \dim N$. Given a map $f : N \rightarrow \bar{N}$, a local diffeomorphism $g : M \rightarrow \bar{M}$ and $Z \in F_x(M, N)_y$, we can write $Z = v \circ X \circ u^{-1}$, $j_y^r f = \bar{v} \circ X_2 \circ v^{-1}$, $j_x^r g = \bar{u} \circ X_1 \circ u^{-1}$, $\bar{v} \in P_{f(y)}^r \bar{N}$, $\bar{u} \in P_{g(x)}^r \bar{M}$, $X_2 \in \mathcal{L}_{n,\bar{n}}$, $X_1 \in \mathcal{L}_{m,m}$. Then we set

$$\mathcal{F}(g, f)(Z) = \bar{v} \circ (X_2 \circ X \circ X_1^{-1}) \circ \bar{u}^{-1}.$$

Thus, we have proved

Proposition 1. *A sequence $F = (F_m)$ of weak r -jet m -functors form a weak total r -jet functor, if and only if the values of the restriction of $\mathfrak{A}_{m,n,q}$ to $L_{m,n}^F \times L_{n,q}^F$ lie in $L_{m,q}^F$. \square*

In this case we write

$$(8) \quad \mathfrak{A}_{m,n,q}^F : L_{m,n}^F \times L_{n,q}^F \rightarrow L_{m,q}^F.$$

Remark. In differential geometry, the velocity and jet functors are constructed by iterating the jet prolongation procedure in various ways (in the case of all nonholonomic r -jets we iterate the construction of 1-jets in each step) and by adding natural conditions similar to the well known case of semiholonomic jets. None of these constructions yields a functor that is weak in the proper sense, i.e. not product preserving. However, the weakness assumption is necessary for a systematic approach to the subject.

3. Another characterization of weak total r -jet functors.

Consider a sequence $V = (V_m)$ of weak r -velocity m -functors.

Definition 4. The sequence $V = (V_m)$ is said to be admissible, if for every manifold N , every $x \in N$ and every $k \in \mathbb{N}$,

$$Z \in (V_m N)_x \text{ and } X \in L_{k,m}^r \text{ implies } Z \circ X \in (V_k N)_x.$$

Clearly, if $F = (F_m)$ is a weak total r -jet functor, then (νF_m) is an admissible sequence.

For an admissible sequence V , we define $L_{k,m}^V$ as the set of all $X \in \tilde{L}_{k,m}^r$ such that $Z \circ X \in (V_k N)_x$ for every manifold N , every $x \in N$ and every $Z \in (V_m N)_x$.

Lemma. *If V is an admissible sequence, then*

$$X_1 \in L_{m,n}^V \text{ and } X_2 \in L_{n,q}^V \text{ implies } X_2 \circ X_1 \in L_{m,q}^V.$$

Proof. We have $W \circ X_1 \in (V_m N)_x$ for all $W \in (V_n N)_x$ and $Z \circ X_2 \in (V_n N)_x$ for all $Z \in (V_q N)_x$. Hence $Z \circ (X_2 \circ X_1) \in (V_m N)_x$. \square

If we write $V_{m,n} = (V_m \mathbb{R}^n)_0$, then $L_{m,n}^V \subset V_{m,n}$. Indeed, for $X \in L_{m,n}^V$ we take $Z = j_0^r \text{id}_{\mathbb{R}^n} \in V_n(\mathbb{R}^n)_0$. Then $Z \circ X = X \in V_m(\mathbb{R}^n)_0$.

Assume that each $L_{m,n}^V$ is a submanifold of $V_{m,n}$. Clearly, the r -th jet group G_n^r in dimension n satisfies $G_n^r \subset L_{n,n}^V$. Hence the jet composition defines an action of G_n^r on $L_{m,n}^V$ and we construct the associated fiber bundle

$$\mathcal{V}_m N = P^r N [L_{m,n}^V].$$

For a map $f : N \rightarrow Q$, we define $\mathcal{V}_m f : \mathcal{V}_m N \rightarrow \mathcal{V}_m Q$ as follows. We write $j_x^r f = w \circ Z \circ u^{-1}$, $u \in P_x^r N$, $Z \in L_{n,q}^r$, $w \in P_{f(x)}^r Q$. For an equivalence class $\{u, X\} \in \mathcal{V}_m N$, we set

$$\mathcal{V}_m f(\{u, X\}) = \{w, Z \circ X\}.$$

Then each \mathcal{V}_m is a weak r -velocity m -functor. In the same way as in Section 2, we deduce

Proposition 2. *The sequence $(j\mathcal{V}_m)$ is a weak total r -jet functor.*

Since $L_{m,n}^V \subset V_{m,n}$ we have $\mathcal{V}_m N \subset V_m N$ for every manifold N .

Definition 5. An admissible sequence $V = (V_m)$ is said to be distinguished, if $L_{m,n}^V = V_{m,n}$ for all $m, n \in \mathbb{N}$.

Applying Proposition 1, we obtain

Proposition 3. *A sequence (F_m) of weak r -jet m -functors form a weak total r -jet functor, if and only if the sequence (vF_m) is distinguished.*

Example. Consider the sequence (\overline{T}_m^r) , $m \in \mathbb{N}$ of semiholonomic m -dimensional r -velocity functors, [9]. Using the formula for the composition of semiholonomic r -jets, [1], one deduces easily that (\overline{T}_m^r) is a distinguished sequence.

4. The product preserving cases.

In the product preserving case, we have a simple description of both velocities and jet functors in terms of Weil algebras.

Definition 6. If a weak r -velocity m -functor V_m on $\mathcal{M}f$ preserves products, then V_m is said to be an r -velocity m -functor.

According to the general theory, [6]. [7], V_m is characterized by a Weil algebra \mathbb{D}^{V_m} satisfying

$$\mathbb{D}_m^r \subset \mathbb{D}^{V_m} \subset \tilde{\mathbb{D}}_m^r.$$

In the jet case, our ideas from [5] and Section 3 can be formulated as follows.

Definition 7. An r -jet m -functor F_m on $\mathcal{M}f_m \times \mathcal{M}f$ or a total r -jet functor F is a weak r -jet m -functor or a weak total r -jet functor that preserves products in the second factor.

This definition yields directly that F_m is an r -jet m -functor, if and only if vF_m is an r -velocity m -functor.

Consider a sequence $F = (F_m)$ of r -jet m -functors. We write

$$\mathbb{D}^{vF_m} =: \mathbb{D}_m^F = \mathbb{R} \times N_m^F,$$

where N_m^F is the nilpotent part of \mathbb{D}_m^F . Since each F_m preserves products, we have $(N_m^F)^n \subset \tilde{L}_{m,n}^r$. Hence Proposition 3 implies that F is a total r -jet functor, if and only if (4) restricts and corestricts to

$$(9) \quad \chi_{m,n,q}^F : (N_m^F)^n \times (N_n^F)^q \rightarrow (N_m^F)^q.$$

An n -tuple X of elements of N_m^F belongs to $\tilde{L}_{m,n}^r$. By Section 1, X determines a natural transformation $\tilde{T}_n^r \rightarrow \tilde{T}_m^r$, so an algebra homomorphism (denoted by the same symbol)

$$X : \tilde{\mathbb{D}}_n^r \rightarrow \tilde{\mathbb{D}}_m^r.$$

Thus, (9) can be reformulated as follows.

Proposition 4. *The total r -jet functors are in bijection with the sequences (\mathbb{D}_m^F) , $m \in \mathbb{N}$, of Weil algebras such that*

- (i) $\mathbb{D}_m^r \subset \mathbb{D}_m^F \subset \tilde{\mathbb{D}}_m^r$ for all $m \in \mathbb{N}$,
- (ii) each algebra homomorphism $X : \tilde{\mathbb{D}}_n^r \rightarrow \tilde{\mathbb{D}}_m^r$ determined by $X \in (N_m^F)^n \subset \tilde{L}_{m,n}^r$ maps \mathbb{D}_n^F into \mathbb{D}_m^F .

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