ELKEDAGMAR HEINRICH

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ON THE CLASS GROUP OF AFFINOID SPACES

by Elkedagmar HEINRICH (*)
[Universität Bochum]

The field k is supposed to be complete with respect to a non-archimedean valuation and to be stable (i. e. $|k^*| = \sqrt{|k^*|}$, and for every finite field extension ℓ of k one has $[\ell : k] = [\bar{\ell} : \bar{k}]$). By $\overset{\bullet}{k}$ and $\bar{k}$, we denote the valuation ring and the residue field of k .

The affinoid k-algebra A is supposed to be reduced and to be provided with its spectral norm $\|.\|$. Furthermore we assume that $\|.\|$ takes its value in $|k^*|$. By $\bar{A}$, we denote the affine $\bar{k}$-algebra $\mathring{A} \otimes \bar{k}$, where $\mathring{A} := \{x \in A ; \|x\| \leq 1\}$. The group of isomorphism classes of projective, rank 1 , A-modules is called the class group $Cl(A)$ of A .

THEOREM.

(i) There is a natural injective map

$$\alpha : Cl(\bar{A}) \hookrightarrow Cl(A) .$$

(ii) If $\bar{A}$ is regular, α is also surjective.

Definition of α .

(a) $\beta : Cl(\mathring{A}) \to Cl(\bar{A})$

$$\beta([M]) := [M \otimes_{\mathring{A}} \bar{A}]$$

It is easy to show that β is a bijective homomorphism.

(b) $\gamma : Cl(\mathring{A}) \to Cl(A)$

$$\gamma([M]) := [M \otimes_{\mathring{A}} A] .$$

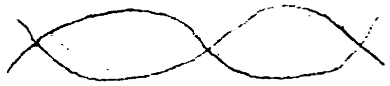
The homomorphism γ is injective, and we define

$$\alpha := \gamma \circ \beta^{-1} .$$

Several examples show that in general α is not surjective.

(*) Elkedagmar HEINRICH, Institut für Mathematik, Universität Bochum, Postfach 102148, D-4630 BOCHUM 1 (Allemagne fédérale).

These examples are constructed geometrically : every 1-dimensional, normal, connected affinoid space $\gamma = Sp(A)$ is an affinoid subspace of a non-singular complete curve. We consider special affinoid subspaces of Tak-curves, of an elliptic curve with good reduction, of a Mumford-curve of genus 2 with stable reduction

 .

We also consider the product of two of these subspaces with the A-dimensional unit disk.

A detailed paper "Über die Picardgruppen affinoider Algebren", by M. van der PUT and E. HEINRICH is to appear.