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p-ADIC TEICHMÜLLER SPACE FOR GENUS 2

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Over an algebraically closed complete nonarchimedean field k , like over the complex numbers, one has, for every integer $g \geq 2$, an analytic manifold \mathcal{T}_g and a group Ψ_g of analytic automorphisms of \mathcal{T}_g acting discontinuously on \mathcal{T}_g such that the quotient space is isomorphic to the space \mathcal{M}_g of Mumford curves of genus g . \mathcal{T}_g is called the p-adic Teichmüller space, Ψ_g the p-adic Teichmüller modular group (see [2]).

In this paper, we shall mainly consider the case $g = 2$. Here we have the result that \mathcal{T}_2 is a Stein domain. The proof relies on an effective algorithm to decide whether or not a given pair of hyperbolic transformations generates a Schottky group. It seems not very likely that a similar algorithm can be found for higher genus, although \mathcal{T}_g is probably a Stein domain for arbitrary g .

We begin with the study of treelike metric spaces, a generalization of the trees used in graph theory which possibly has some interest in itself.

In the second part of the paper, we construct for any ultrametric field a treelike metric space which for discrete fields coincides with the Bruhat-Tits-tree. Investigation of the action of hyperbolic linear transformations on this space is the main tool in proving that \mathcal{T}_2 is a Stein domain.

1. Treelike metric spaces.

Let (X, d) be a metric space. For $x, y \in X$ define the section $S(x, y)$ to be

$$S(x, y) := \{z \in X; d(x, y) = d(x, z) + d(y, z)\}.$$

Definition. - A metric space (X, d) is called treelike if, for any $x, y, z \in X$,

(T1) $S(x, y) \cap S(x, z) \cap S(y, z) \neq \emptyset$.

(T2) If $z \in S(x, y)$, then $S(x, z) \cup S(z, y) = S(x, y)$.

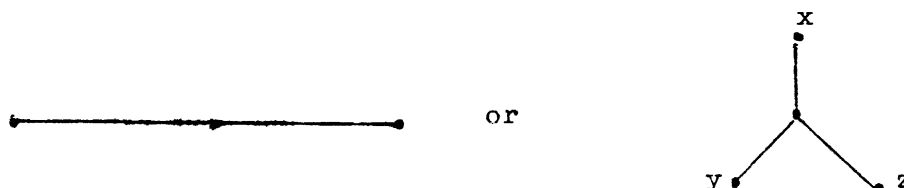
This definition is justified by the following property :

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Let G be a connected graph without loops and multiple edges, G_0 the set of vertices of G , and d the metric on G_0 defined by the minimal number of edges between two vertices. Then we have :

PROPOSITION 1. - (G_0, d) is treelike if, and only if, G is a tree.

Proof. - If G is a tree, for any $x, y \in G_0$, the section $S(x, y)$ consists of the unique simple path in G joining x and y , whence (T2). The first property results from the fact that the subtree of G spanned by x, y and z is iso-morphic to



Conversely suppose (G_0, d) is treelike, and assume there is a circle C of minimal length $n \geq 3$ in G .

If $n = 2m$ is even, let $x, y \in C$, such that $d(x, y) = m$ (this is possible due to the minimality of C). Since $m \geq 2$ there are $z_1 \neq z_2 \in C$, such that $d(z_1, x) = d(z_2, x)$ and $z_1 \neq y \neq z_2$. Obviously, $z_2 \notin S(x, z_1) \cup S(z_1, y)$.

If $n = 2m + 1$ is odd, choose $x, y \in C$ with $d(x, y) = 1$, and let $z \in C$ be the unique vertex such that $d(x, z) = d(y, z) = m$. Then $S(x, y) \cap S(x, z) \cap S(y, z) = \emptyset$ and the proposition is proved. A trivial example for a non discrete treelike metric space are the real numbers with the usual metric ; further, any subset of a treelike metric space is itself treelike.

Next we list some formal properties of the sections in a treelike metric space :

LEMMA 1. - Let (X, d) be a treelike metric space and $x, y, z \in X$. Then

(i) if $z \in S(x, y)$, then

$$S(x, y) = \{v \in S(x, y) ; d(x, y) \leq d(x, z)\}$$

and

$$S(x, z) \cap S(z, y) = \{z\},$$

(ii) there is $u \in X$ such that

$$S(x, y) \cap S(x, z) \cap S(y, z) = \{u\}$$

and

$$S(x, y) \cap S(x, z) = S(x, u).$$

Proof.

(i) is a straight forward application of (T2).

(ii) Let $S := S(x, y) \cap S(x, z)$. For $v \in S$, we have

$$d(y, z) \leq d(v, y) + d(v, z) = d(x, y) + d(x, z) - 2d(x, v).$$

Because of (T1) and (i) of this lemma, there exists a unique $u \in S$ such that $d(x, u) = \sup_{v \in S} d(x, v)$. This u is the only element of S such that $d(y, z) = d(u, y) + d(u, z)$, i. e. the only element of $S \cap S(z, y)$. The last identity follows from (i).

A subset Y of a treelike metric space (X, d) is called connected if $y_1, y_2 \in Y$ implies $S(y_1, y_2) \subset Y$. Note that the intersection of connected subsets of X is again connected.

For a connected subset $Y \subset X$, let

$$\text{diam}(Y) := \sup\{d(y_1, y_2) ; y_1, y_2 \in Y\}.$$

A ray $R \subset X$ is a connected subset with $\text{diam}(R) = \infty$ such that there exists a sequence $(x_i)_{i \geq 0}$ in R with $x_i \in S(x_0, x_{i+1})$ for all i and $\bigcup_{i=0}^{\infty} S(x_0, x_i) = R$. Two rays R, R' are called equivalent if $\text{diam}(R \cap R') = \infty$. (This is indeed an equivalence relation since $\text{diam}(R \cap R') = \infty$ implies that $R \cap R'$ is again a ray.) An equivalence class of rays in X is called an end of X .

An axis in X is a connected subset $A \subset X$ such that there are two rays R_1, R_2 in X with $R_1 \cup R_2 = A$ and $R_1 \cap R_2$ is a single point. Thus the axes in X are in 1-1 correspondence with those pairs (E_1, E_2) of ends of X for which $E_1 \neq E_2$.

The isometries of a treelike metric space can be characterized very much like the automorphisms of a tree (see [3]) because of the following observation :

If (X, d) is a treelike metric space and φ an isometry of (X, d) there always exists a treelike extension space (\tilde{X}, \tilde{d}) of (X, d) (i. e. there is a distance-preserving injection $X \hookrightarrow \tilde{X}$) and a continuation $\tilde{\varphi}$ of φ such that

$$\inf_{x \in \tilde{X}} \tilde{d}(x, \tilde{\varphi}(x)) = \inf_{x \in X} d(x, \varphi(x))$$

is attained in \tilde{X} .

LEMMA 2. - If φ is an isometry of (X, d) such that there is $y \in X$ with $d(y, \varphi(y)) = \inf_{x \in X} d(x, \varphi(x))$ then φ has exactly one of the following properties :

- (a) φ has a fixed point in X ,
- (b) there is $y \in X$ with $\varphi(y) \neq y$, $S(y, \varphi(y)) = \{y, \varphi(y)\} = \varphi(S(y, \varphi(y)))$,
- (c) there is an axis $A \subset X$ on which φ acts by nontrivial translation.

In (b) and (c) the pair $(y, \varphi(y))$ (resp. A) are unique. Of course (b) is impossible if X is everywhere dense, i. e. for any $x \neq y \in X$ exists $z \in S(x, y)$, $x \neq z \neq y$.

Proof. - If φ has neither property (a) nor (b), then it is easily checked that $\bigcup_{n=0}^{\infty} \varphi^n(S(y, \varphi(y)))$ and $\bigcup_{n=-1}^{\infty} \varphi^n(S(y, \varphi(y)))$ are rays defining an axis on which φ acts by translation by $d(y, \varphi(y)) > 0$.

2. Generalization of the Bruhat-Tits-tree.

For a field k with a non archimedean valuation $|\cdot|$, let

$$\mathfrak{K}(k) := \{B(a, r) ; a \in k ; r \in |k^*|\},$$

where $B(a, r) = \{z \in k ; |z - a| \leq r\}$. For $B_i = B(a_i, r_i) \in \mathfrak{K}(k)$, $i = 1, 2$, define

$$d(B_1, B_2) := \log \frac{r_{12}^2}{r_1 r_2}$$

where $r_{12} := \max\{|b_1 - b_2| ; b_1 \in B_1 ; b_2 \in B_2\}$.

PROPOSITION 2. - For any nonarchimedean valued field k , $(\mathfrak{K}(k), d)$ is a tree-like metric space.

Proof.

(i) Since $r_{12} \geq \max(r_1, r_2)$, for B_1 and B_2 as above we have $d(B_1, B_2) \geq 0$, and $d(B_1, B_2) = 0$ if, and only if, $r_{12} = r_1 = r_2$, i. e. $B_1 = B_2$. For any $B_1, B_2, B_3 \in \mathfrak{K}(k)$, we have $r_{12} \leq \max(r_{13}, r_{23})$ which implies

$$\frac{r_{12}^2}{r_1 r_2} \leq \frac{r_{13}^2}{r_1 r_3} \times \frac{r_{23}^2}{r_2 r_3}.$$

and thus proves the triangle inequality.

(ii) To prove (T1) note that for $B_1, B_2 \in \mathfrak{K}(k)$, we have

$$\begin{aligned} S(B_1, B_2) &= \{B_3 \in \mathfrak{K}(k) ; r_3 r_{12} = r_{13} r_{23}\} \\ &= \{B_3 \in \mathfrak{K}(k) ; r_{12} = \max(r_{13}, r_{23}) ; r_3 = \min(r_{13}, r_{23})\}. \end{aligned}$$

Thus if $B_3, B_4 \in S(B_1, B_2)$ and $r_{13} < \max(r_{14}, r_{24})$, then $r_{14} = r_{34}$, and $r_{12} = \max(r_{34}, r_{24}) = r_{23}$ and $r_4 = \min(r_{14}, r_{24}) = \min(r_{34}, r_{24})$, so $B_4 \in S(B_2, B_3)$.

If $B_1, B_2, B_3 \in \mathfrak{K}(k)$, choose the indices so that $r_{12} \leq \min(r_{13}, r_{23})$. Then it is easily verified that $B_4 := B(a_1, r_{12}) \in S(B_1, B_2) \cap S(B_1, B_3) \cap S(B_2, B_3)$ so (T2) also holds.

If the valuation of k is discrete, the Bruhat-Tits-tree for k can be reconstructed from $\mathfrak{K}(k)$ by letting the points of $\mathfrak{K}(k)$ be the vertices of a graph and by drawing edges between points on minimal distance.

If on the other hand k is algebraically closed $\mathcal{K}(k)$ is everywhere dense (but not complete).

An extension $k' : k$ of ultrametric fields gives a natural distance preserving embedding $\mathcal{K}(k) \hookrightarrow \mathcal{K}(k')$. Thus if k is algebraically closed, we may view $\mathcal{K}(k)$ as the direct limit of the Bruhat-Tits-trees for the discrete subfields of k .

For the completion \hat{k} of k , we always have $\mathcal{K}(\hat{k}) = \hat{\mathcal{K}}(k)$. If R is a ray in $\mathcal{K}(k)$, and B_0, B_1, B_2, \dots is a sequence of points on R with $d(B_0, B_n) \rightarrow \infty$ as $n \rightarrow \infty$ then either $r_n \rightarrow \infty$ or $r_{2n} = r_{2n+1}$ for $n \geq n_0$ and $r_n \rightarrow 0$. Therefore the ends of $\mathcal{K}(k)$ correspond to the points of $\tilde{P}^1(k)$.

$PGL_2(k)$ acts isometrically on $\mathcal{K}(k)$ if we make the following convention: if $\gamma^{-1}(\infty) \in B$ for a $B \in \mathcal{K}(k)$ and a $\gamma \in PGL_2(k)$, let γB be the affinoid hull (= geometric closure) of the "open" disk $\tilde{P}^1(k) - \gamma(B)$. This action commutes with field extensions.

$\gamma \in PGL_2(k)$ is hyperbolic if, and only if, it is of type (c) of lemma 2; the axis A_γ is determined by the fixed points of γ in $\tilde{P}^1(k)$, the shift v_γ on A_γ is given by $v_\gamma = -\log |t_\gamma|$, where t_γ is the multiplier of γ .

Let $\pi_\gamma : \mathcal{K}(k) \rightarrow A_\gamma$ denote the projection; this is meaningful also for ends of $\mathcal{K}(k)$ different from the fixed points of γ . γ defines an orientation $<_\gamma$ on A_γ such that $B <_\gamma \gamma B$ for all $B \in A$.

Let $\gamma_1, \gamma_2 \in PGL_2(k)$ be hyperbolic with mutually different fixed points x_1, x_{-1}, x_2, x_{-2} such that the translation of γ_i on $A_i := A_{\gamma_i}$ is towards x_i . Let

$$v_i := v_{\gamma_i}, \quad <_i := <_{\gamma_i}, \quad \pi_i := \pi_{\gamma_i};$$

let

$$B_{12} := \pi_1(x_{-2}), \quad B'_{12} := \pi_1(x_2) \quad \text{and} \quad d_{12} := d(B_{12}, B'_{12}).$$

Call γ_1, γ_2 parallel if $B_{12} <_1 B'_{12}$, otherwise antiparallel. Finally assume $v_1 \leq v_2$.

LEMMA 3. - With the above notations and assumptions, we have:

- (i) $\gamma_1 \gamma_2$ is not hyperbolic if γ_1, γ_2 are antiparallel, and $v_1 = v_2 \leq d_{12}$.
- (ii) $\gamma_1 \gamma_2$ is possibly not hyperbolic if γ_1, γ_2 are antiparallel and $v_1 = d_{12} < v_2$
- (iii) $\gamma_1 \gamma_2$ is hyperbolic in all other cases.

Proof. - In the first case, B_{12} is fixed point of $\gamma_1 \gamma_2$, in (ii) $\gamma_1 \gamma_2$ may have fixed points on $S(B_{12}, \gamma_2^{-1} B'_{12})$, in all other cases one easily sees that

$$S \cap \gamma_1 \gamma_2 S = \{\gamma_1 \gamma_2 B_{12}\} \text{ with } S = S(B_{12}, \gamma_1 \gamma_2 B_{12}) .$$

In view of the proof of lemma 2, this shows that $\gamma_1 \gamma_2$ is hyperbolic.

3. p-adic Teichmüller space \mathcal{T}_2

In this section, k is assumed to be algebraically closed and complete. We briefly recall from [2] the definition of the p-adic Teichmüller space \mathcal{T}_g , $g \geq 2$ an integer.

Let $G := PGL_2(k)$; for $\zeta = (\gamma_1, \dots, \gamma_g) \in G^g$, let $\Gamma(\zeta)$ be the subgroup of G generated by $\gamma_1, \dots, \gamma_g$. Then

$$\mathcal{T}_g := \tilde{\mathcal{T}}_g \text{ mod } G$$

where $\tilde{\mathcal{T}}_g := \{\zeta = (\gamma_1, \dots, \gamma_g) \in G^g; \Gamma(\zeta) \text{ is Schottky group of rank } g\}$, and G acts on $\tilde{\mathcal{T}}_g$ by componentwise conjugation. For the Teichmüller modular group and the connection with the space of Mumford curves, we refer to [2].

Recall that a subgroup $\Gamma \subset G$ is a Schottky group if, and only if, every element $\gamma \in \Gamma$, $\gamma \neq \text{id}$, is hyperbolic. As coordinates on $\tilde{\mathcal{T}}_g$, we use the multipliers t_i , the attracting and repelling fixed points x_i and x_{-i} of the hyperbolic transformation γ_i . For \mathcal{T}_g , we take the set of representatives normalized by the conditions $x_1 = 0$, $x_{-1} = \infty$, $x_2 = 1$.

In order to replace the condition that $\Gamma(\zeta)$ be a Schottky group by inequalities involving rational functions of the coordinates on \mathcal{T}_g , we introduce the following notations :

Let F_g be a nonabelian free group of rank g , e_1, \dots, e_g a fixed base of F_g , and $\alpha_\zeta : F_g \rightarrow \Gamma(\zeta)$, $e_i \rightarrow \gamma_i$, the canonical homomorphism for any $\zeta \in G^g$. If $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in GL_2(k)$, let $T(\gamma) := (a+d)^2/ad - bc$. Obviously $T(\lambda\gamma) = T(\gamma)$, so T is a rational function on G . $\gamma \in G$ is hyperbolic if, and only if, $|T(\gamma)| > 1$.

Therefore

$$\mathcal{T}_g = \{(t_1, \dots, t_g; x_{-2}, x_3, x_{-3}, \dots, x_g, x_{-g}) \in k^{3g-3};$$

$$0 < |t_i| < 1; i = 1, \dots, g; |T(\alpha(w))| > 1 \text{ for all } w \in F_g; w \neq 1\} .$$

Note that since γ_i can be represented by the matrix

$$\begin{pmatrix} x_i - t_i x_{-i} & (t_i - 1) x_i x_{-i} \\ 1 - t_i & t_i x_i - x_{-i} \end{pmatrix} \quad (i \geq 2),$$

$T(\alpha(w))$ is indeed rational in the t_i, x_i and x_{-i} .

Every Schottky group $\Gamma \subset G$ has a Schottky base $\gamma_1, \dots, \gamma_g$, i. e. there are $B_1, B'_1, \dots, B_g, B'_g \in X(k)$ such that $\gamma_i B_i = B'_i$ and there is an $x \in \mathbb{P}^1(k)$

such that $\pi_i(x) \in S(B_i, B'_i)$, $i = 1, \dots, g$. One sees immediately that hyperbolic transformations $\gamma_1, \dots, \gamma_g$ form a Schottky base if, and only if, for $i = 1, \dots, g$,

$$d(\pi_i(x_j), \pi_i(x_k)) < v_i \text{ for all } j, k \neq \pm i.$$

If $g = 2$, this reduces to the following description of the space \mathcal{B}_2 of Schottky bases of rank 2 (cf. [1], § 2) :

$$\mathcal{B}_2 = \{(t_1, t_2, y) \in k^3; y \neq 1; d_{12} < v_i; i = 1, 2\}$$

(where $d_{12} = d(\pi_1(x_2), \pi_1(x_{-2}))$ as in lemma 3).

The following lemma is crucial in the proof of the main result :

LEMMA 4. - Let $\zeta = (\gamma_1, \gamma_2) \in \mathcal{B}_2$ be a normed Schottky base and

$$v := \min\{v_\gamma; \gamma \in \{\gamma_1, \gamma_2, \gamma_1 \gamma_2, \gamma_1 \gamma_2^{-1}\}\}.$$

Then $v_\gamma \geq v$ for any $\gamma \in \Gamma(\zeta)$.

Proof. - By replacing if necessary γ_1 or γ_2 by $\gamma_1 \gamma_2$ or $\gamma_1 \gamma_2^{-1}$ or by taking inverses, we may assume that γ_1, γ_2 are antiparallel and that $2d_{12} \leq v_1 \leq v_2$, so $v = v_1$. As conjugation doesn't change the multiplier we only have to consider elements of the form

$$\gamma = \gamma_1^{v_1} \gamma_2^{\mu_1} \dots \gamma_1^{v_r} \gamma_2^{\mu_r}, \quad v_i, \mu_i \in \mathbb{Z} \setminus \{0\}, \quad r \geq 1.$$

By induction on r , one easily verifies

$$(i) \quad \pi_1(\gamma B_{12}) \notin S(\gamma_1 B'_{12}, \gamma_1^{-1} B_{12}) \setminus \{\gamma_1 B'_{12}, \gamma_1^{-1} B_{12}\},$$

$$(ii) \quad d(\gamma B_{12}, A_1) \geq v_2 - d_{12} \geq v_1 - d_{12},$$

$$(iii) \quad B_{12} \in A_\gamma$$

which shows that

$$v_\gamma = d(B_{12}, \gamma B_{12}) \geq v_1 - d_{12} + v_1 - d_{12} \geq v_1.$$

THEOREM. - \mathcal{C}_2 is a Stein domain.

More precisely : Let $\epsilon \in |k^*|$, $0 < |\epsilon| < 1$. Then the following affinoid domains $\mathcal{C}_2^{(n)} \subset k^3$, $n \geq 1$, exhaust \mathcal{C}_2 :

$$\begin{aligned} \mathcal{C}_2^{(n)} &:= \{(t_1, t_2, y) \in k^3; \\ &\epsilon^n \leq |t_i| \leq \epsilon^{1/n}, \quad i = 1, 2, \\ &\epsilon^n \leq |y| \leq \epsilon^{-n}, \quad \epsilon^n \leq |1 - y|, \\ &|t_1 t_2| \leq \epsilon^{1/n} |y|^\alpha, \quad \alpha = \pm 1, \\ &|T(\gamma_i \gamma_j^\nu)| \geq \epsilon^{-1/n}, \quad i, j = 1, 2, \quad i \neq j, \quad \nu = \pm 1, \dots, \pm n^2, \\ &|T(\gamma_i (\gamma_i \gamma_j^\nu)^\mu)| \geq \epsilon^{-1/n}, \quad i, j, l = 1, 2, \\ &\quad i \neq j, \quad \nu = \pm 1, \dots, \pm n^2, \quad \mu = \pm 1, \dots, \pm (n^2 - 1)\} \end{aligned}$$

Proof. - The conditions can be rephrased in terms of v_i, d_{12} , etc. :

- (1) $n \cdot \epsilon' \geq v_i \geq \epsilon'/n, \quad i = 1, 2$ (where $\epsilon' := -\log \epsilon$),
- (2) $d_{12} \leq n\epsilon', \quad d(A_1, A_2) \leq n\epsilon',$
- (3) $v_1 + v_2 \leq \epsilon'/n \cdot d_{12},$
- (4) $v_\gamma \leq \epsilon'/n$ for the γ listed above.

Now we divide the proof into several steps :

1° $\mathcal{C}_2 \subset \bigcup_{n=1}^\infty \mathcal{C}_2^{(n)}$: let $\zeta = (\gamma_1, \gamma_2) \in \mathcal{C}_2$. (1), (2) are obviously satisfied for large n .

Lemma 3 shows that $v_1 + v_2 < d_{12}$ is necessary to ensure that $\gamma_1 \gamma_2 \gamma_1^{-1} \gamma_2^{-1}$ and $\gamma_1 \gamma_2^{-1} \gamma_1^{-1} \gamma_2$ are hyperbolic, and (4) results from lemma 4.

2° $\mathcal{C}_2^{(n)} \subset \mathcal{C}_2$: Let $\zeta = (\gamma_1, \gamma_2) \in \mathcal{C}_2^{(n)}$. We may again assume $v_1 \leq v_2$ and γ_1, γ_2 antiparallel. Then $d_{12} \leq n^2 v_1$ because of (1), (2), and $d_{12} < 2v_2$ because of (3). Let $m \in \{0, \dots, n^2\}$ such that $mv_1 \leq d_{12} < (m+1)v_1$.

We consider the following cases :

(a) $mv_1 \leq d_{12} < v_2$. - Here $\gamma_2' := \gamma_2 \gamma_1^m$ is hyperbolic by lemma 3 (resp. by condition (4)), and $v_2' := v_{\gamma_2'} \leq v_2 - mv_1$, with equality if $mv_1 < d_{12}$.

$$d_{12}' := d(\pi_1(x_1'), \pi_1(x_{-2}')) = d_{12} - mv_1 < \max(v_1, v_2'),$$

so $(\gamma_1, \gamma_2') \in \mathcal{C}_2$.

(b) $mv_1 \leq v_2 \leq d_{12}$. - Again $\gamma_1' := \gamma_1 \gamma_2^m$ is hyperbolic because of condition (4), and $v_1' \geq \epsilon'/n$. Therefore $d_{12}' = d_{12} - mv_1 < v_1 \leq n\epsilon' \leq n^2 v_1'$, so γ_1', γ_1 lead to case (a) with an $m' \leq n^2 - 1$; the γ listed in condition (4) are now precisely those needed to show that $\gamma_1', \gamma_1(\gamma_1')^{m'}$ is a Schottky base.

(c) $v_2 < mv_1 \leq d_{12}$. - There similar to case (b) $\gamma_1' = \gamma_2 \gamma_1^m$ and γ_2 lead to case (a) with an $m' \leq n^2 - 1$, so that $\gamma_1', \gamma_2(\gamma_1')^{m'}$ is a Schottky base.

$\mathfrak{P} \mathfrak{C}_2^{(n)} \subset \mathfrak{C}_2^{(n+1)}$: Inspection of the construction in $\mathfrak{C}_2^{(n)}$ shows that any $\zeta \in \mathfrak{C}_2^{(n)}$ has a Schottky base satisfying the condition of lemma 4 with $v \geq \varepsilon^{1/n}$. So an application of this lemma concludes the proof of the theorem.

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