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ON THE POLES OF THE SCATTERING MATRIX
FOR TWO CONVEX OBSTACLES

by Mitsuru IKAWA

§1. Introduction.

Let $\mathcal{O}$ be a bounded open set in $\mathbb{R}^3$ with smooth boundary Γ . We set $\Omega = \mathbb{R}^3 - \overline{\mathcal{O}}$. Suppose that Ω is connected. Consider the following acoustic problem

$$(1.1) \quad \begin{cases} \square u(x,t) = \frac{\partial^2 u}{\partial t^2} - \Delta u = 0 & \text{in } \Omega \times (-\infty, \infty) \\ u(x,t) = 0 & \text{on } \Gamma \times (-\infty, \infty) \end{cases}$$

where $\Delta = \sum_{j=1}^3 \frac{\partial^2}{\partial x_j^2}$. Denote by $\mathcal{S}(z)$ the scattering matrix for this problem. About the definition of the scattering matrix, see for example Lax and Phillips[7,page 9]. The result I like to talk about is the following

Theorem 1. Let $\mathcal{O}_j$, $j=1,2$, be open and strictly convex sets in $\mathbb{R}^3$ with smooth boundary Γ_j , that is, the Gaussian curvature of Γ_j is positive everywhere on Γ_j . Suppose that $\overline{\mathcal{O}_1} \cap \overline{\mathcal{O}_2} = \emptyset$. Then the scattering matrix $\mathcal{S}(z)$ for

$$\mathcal{O} = \mathcal{O}_1 \cup \mathcal{O}_2$$

satisfies the following:

(1) There exist positive constant c_0 and c_1 such that $\mathcal{S}(z)$ is holomorphic in

$$\{z; \operatorname{Im} z \leq c_0 + c_1\} - \bigcup_{j=-\infty}^{\infty} D_j$$

where

$$D_j = \{z; |z - z_j| \leq C(1+|j|)^{-1/2}\},$$

$$z_j = ic_0 + \frac{\pi}{d} j, \quad d = \operatorname{dis}(\mathcal{O}_1, \mathcal{O}_2).$$

(2) For large $|j|$, every D_j contains exactly one pole of $\mathcal{S}(z)$.

(3) Denoting the pole in D_j by ζ_j we have an asymptotic expansion

$$(1.2) \quad \zeta_j \sim z_j + \beta_1 j^{-1} + \beta_2 j^{-2} + \dots \quad \text{for } |j| \rightarrow \infty$$

where $\beta_1, \beta_2, \dots$ are complex constants determined by $\mathcal{O}$.

(4) In D_j $\mathcal{S}(z)$ is of the form

$$(1.3) \quad \mathcal{S}(z)f = \frac{1}{z - \zeta_j} (f, \psi_j) m_j + \mathcal{H}_j(z)f$$

for all $f \in L^2(S^2)$, where $m_j, \psi_j \in L^2(S^2)$ such that $m_j \neq 0, \psi_j \neq 0$ and $(\cdot, \cdot)$ stands for the scalar product in $L^2(S^2)$, and $\mathcal{H}_j(z)$ is an $\mathcal{L}(L^2(S^2), L^2(S^2))^{1)}$ valued holomorphic function in D_j .

Concerning the existence of non-purely imaginary poles of $\mathcal{S}(z)$, Bardos, Guillot and Ralston[1] proved under the same assumption as ours the existence of an infinite number of the poles in

$$\{z; \operatorname{Im} z \leq \varepsilon \log(1+|z|)\}$$

for any $\varepsilon > 0$. This result is generalized by Petkov[11] and Petkov and Stojanov[12] to a case of many strictly convex obstacles. For non-strictly convex obstacles Ikawa[5] showed an example of two

1) We denote by $\mathcal{L}(E, F)$ the set of all linear bounded mappings from E into F .

convex obstacles whose scattering matrix has a sequence of the poles converging to the real axis. On the other hand Lebeau[9] considered the distribution of poles for one strictly convex obstacle.

§2. Reduction of the problem.

Consider a boundary value problem with a parameter $\mu \in \mathbb{C}$

$$(2.1) \quad \begin{cases} (\mu^2 - \Delta)u(x) = 0 & \text{in } \Omega \\ u(x) = g(x) & \text{on } \Gamma \end{cases}$$

for $g \in C^\infty(\Gamma)$. For $\operatorname{Re} \mu > 0$ (2.1) has a solution u uniquely in $\bigcap_{m>0} H^m(\Omega)$. We denote the solution by $U(\mu)g$. Then $U(\mu)$ is holomorphic in $\operatorname{Re} \mu > 0$ as an $\mathcal{L}(C^\infty(\Gamma), C^\infty(\bar{\Omega}))$ -valued function. We shall show the following theorems on $U(\mu)$.

Theorem 2.1. (i) $U(\mu)$ is prolonged analytically as an $\mathcal{L}(C^\infty(\Gamma), C^\infty(\bar{\Omega}))$ -valued function into

$$\{\operatorname{Re} \mu \geq -c_0 - c_1\} - \bigcup_{j=-\infty}^{\infty} iD_j.$$

(ii) Set for $k \in \mathbb{R}$

$$G_k = \{\mu \in \mathbb{C}; |\mu + ik| \leq c_0 + c_1, \operatorname{Re} \mu \geq -c_0 - (\log(1 + |k|))^{-1}\}.$$

Then for large $|k|$, $U(\mu)$ is represented in $G_k \cap \{\operatorname{Re} \mu > 0\}$ as

$$(2.2) \quad U(\mu) = \frac{\beta(x, k, \mu)}{\mathcal{P}(\mu) - \gamma(k, \mu)} F(k, \mu) + V(k, \mu).$$

Here

(a) $\beta(\cdot, k, \mu)$ is $C^\infty(\bar{\Omega})$ -valued holomorphic function in G_k .

(b) $\mathcal{P}(\mu) = 1 - \lambda \bar{\lambda} e^{-2d\mu}$, $0 < \lambda$, $\bar{\lambda} < 1$.

(c) For any N positive integer

$$| \gamma(k, \mu) - \sum_{1 \leq \ell < N} \sum_{0 \leq h < N} \gamma_{\ell, h} k^{-\ell} (\mu + ik)^h | \leq C_N |k|^{-N}$$

where $\gamma_{\ell, h}$ are complex constants.

(d) $F(k, \mu)$ is an $\mathcal{L}(L^2(\Gamma), \mathbb{C})$ -valued holomorphic function in G_k .

(f) $V(k, \mu)$ is an $\mathcal{L}(C^\infty(\Gamma), C^\infty(\bar{\Omega}))$ -valued holomorphic function in G_k .

Corollary. $U(\mu)$ is prolonged analytically as $\mathcal{L}(C^\infty(\Gamma), C^\infty(\bar{\Omega}))$ valued function into

$$\bigcup_{|k|: \text{large}} (G_k - \{\mu; \mathcal{P}(\mu) - \gamma(k, \mu) = 0\}).$$

Theorem 2.2. Suppose that $\mu \in G_k$ and $\mathcal{P}(\mu) - \gamma(k, \mu) = 0$ for $|k|$ large. Then we have

$$\dim\{u; \mu\text{-outgoing solution of (2.1) for } g=0\} = 1.$$

Note that the zeros of $\mathcal{P}(\mu) = 0$ are $\{iz_j, j=0, \pm 1, \pm 2, \dots\}$ and

$$\left| \frac{d}{d\mu} (\mathcal{P}(\mu) - \gamma(k, \mu))_{\mu=iz_j} \right| \geq d - C|k|^{-1}.$$

By setting $k = -\frac{\pi}{d}j$ we have that $\mathcal{P}(\mu) - \gamma(k, \mu) = 0$ has only one zero in iD_j and it is simple. Denote it by $i\zeta_j$ and we see that ζ_j has an asymptotic expansion (1.2).

Theorem 1 is immediately derived from Theorems 2.1 and 2.2 if we recall the relationships between $\mathcal{S}(z)$ and $U(\mu)$ shown in Lax and Phillips[7], especially Theorem 5.1 of Chapter V, which says that $\mathcal{S}(z)$ has a pole at exactly those points z such that $\mu = iz$ is a pole of $U(\mu)$.

§3. Sketch of the proofs of Theorems 2.1 and 2.2.

3.1. Asymptotic solutions for oscillatory boundary data.

Let $a_j \in \Gamma_j$ be the points verifying

$$|a_1 - a_2| = \text{dis}(\sigma_1, \sigma_2).$$

Denote by $S_j(\delta)$ for $\delta > 0$ a connected component containing a_j of

$$S_j \cap \{x; \text{dis}(x, L) = \delta\}$$

where L is a straight line passing a_1 and a_2 , and denote by $\omega(\delta)$

a domain surrounded by $\{x; \text{dis}(x, L) = \delta\}$ and $S_j(\delta)$, $j=1, 2$. Let

$u_k(x)$ be a smooth function satisfying

$$u_k(x) = \begin{cases} 1 & \text{for } x \in S_1(k^{-\varepsilon}) \\ 0 & \text{for } x \notin S_1((1+\delta)k^{-\varepsilon}) \end{cases}$$

for some $\delta > 0$, $\varepsilon > 0$ small constants. Let $h(t) \in C^\infty(0, d/2)$

satisfying $h(t) \geq 0$ and $\int h(t) dt = 1$. Set

$$(3.1) \quad m(x, t; k) = e^{ik(\psi(x) - t)} w(x) h(t - j(x))$$

where $\psi \in C^\infty(S_1(\delta_0))$ is a real valued function satisfying some

conditions and $j(x)$ a fixed smooth function determined by σ .

We construct a sequence of functions of the form

$$(3.2) \quad u_q(x, t; k) = e^{ik(\varphi_q(x) - t)} \sum_{j=0}^N v_{j,q}(x, t; k) (ik)^{-j}.$$

(I) φ_q , $q=0, 1, \dots$ are determined successively by

$$\begin{cases} |\nabla \varphi_0| = 1 & \text{in } \omega(\delta) \\ \varphi_0 = \psi & \text{and } \partial \varphi_0 / \partial n > 0 \quad \text{on } S_1(\delta), \end{cases}$$

$$\begin{cases} |\nabla \varphi_1| = 1 & \text{on } \omega(\delta) \\ \varphi_1 = \varphi_0 \text{ and } \partial \varphi_1 / \partial n > 0 & \text{on } S_2(\delta), \end{cases}$$

$$\begin{cases} |\nabla \varphi_2| = 1 & \text{on } \omega(\delta) \\ \varphi_2 = \varphi_1 \text{ and } \partial \varphi_2 / \partial n > 0 & \text{on } S_1(\delta), \end{cases}$$

$$\vdots$$

(II) On amplitude functions.

Set

$$T_q = 2 \frac{\partial}{\partial t} + 2 \nabla \varphi_q \cdot \nabla + \Delta \varphi_q.$$

$v_{0,q}$, $q=0,1,2,\dots$ are defined successively by

$$\begin{cases} T_0 v_{0,0} = 0 & \text{in } \omega(\delta) \times \mathbb{R} \\ v_{0,0} = f(x,t) & \text{on } \Gamma_1 \times \mathbb{R} \end{cases}$$

where $f(x,t)=w(x)h(t-j(x))$, and for $p \geq 1$

$$\begin{cases} T_{2p-1} v_{0,2p-1} = 0 & \text{in } \omega(\delta) \times \mathbb{R} \\ v_{0,2p-1} = v_{0,2p-2} & \text{on } \Gamma_2 \times \mathbb{R}, \\ T_{2p} v_{0,2p} = 0 & \text{in } \omega(\delta) \times \mathbb{R} \\ v_{0,2p} = v_k(x) v_{0,2p-1} & \text{on } \Gamma_1 \times \mathbb{R}. \end{cases}$$

Next for $j \geq 1$, $v_{j,q}$, $q=0,1,2,\dots$ are defined successively for all $p \geq 0$ by

$$\begin{cases} T_{2p} v_{j,2p} = \square v_{j-1,2p} & \text{in } \omega(\delta) \times \mathbb{R} \\ v_{j,2p} = 0 & \text{on } \Gamma_1 \times \mathbb{R}, \\ T_{2p+1} v_{j,2p+1} = \square v_{j-1,2p+1} & \text{in } \omega(\delta) \times \mathbb{R} \\ v_{j,2p+1} = v_{j,2p} & \text{on } \Gamma_2 \times \mathbb{R}. \end{cases}$$

On the asymptotic behavior of $\varphi_q, v_{j,q}$ for $q \rightarrow \infty$, we have the following Lemmas.

Lemma 3.1. It holds that

$$|\varphi_{2p} - (\varphi_\infty + 2dp + d_0)|_m \leq c_m \alpha^{2p}$$

$$|\varphi_{2p+1} - (\tilde{\varphi}_\infty + (2p+1)d + d_0)|_m \leq c_m \alpha^{2p}$$

where $\varphi_\infty, \tilde{\varphi}_\infty$ are functions independent of ψ , and they verify

$$|\nabla \varphi_\infty| = 1 \quad \text{in } \omega(\delta) \quad \text{and} \quad \varphi(a_1) = 0,$$

$$|\nabla \tilde{\varphi}_\infty| = 1 \quad \text{in } \omega(\delta) \quad \text{and} \quad \tilde{\varphi}(a_2) = 0,$$

and d_0 is a constant depending on ψ , α is a positive constant < 1 .

Lemma 3.2. It holds that

$$\begin{aligned} |v_{j,2p}(x,t;k) - bw(A)(\lambda\tilde{\lambda})^p v_{j,\infty}(x,t-2pd-j(A)-d_\infty;k)|_m \\ \leq c_{j,m}(\alpha\lambda\tilde{\lambda})^{p_{M_{m+2j}}}, \end{aligned}$$

$$\begin{aligned} |v_{j,2p+1}(x,t;k) - bw(A)(\lambda\tilde{\lambda})^p \tilde{v}_{j,\infty}(x,t-2pd-j(A)-d_\infty;k)|_m \\ \leq c_{j,m}(\alpha\lambda\tilde{\lambda})^{p_{M_{m+2j}}}, \end{aligned}$$

where $\lambda, \tilde{\lambda}$ are constants determined by $\mathcal{O}$ such that $0 < \lambda, \tilde{\lambda} < 1$,

$$M_\ell = k^{\varepsilon_\ell} \sum_{|\beta| < \ell} \sup_{\Gamma_1 \times \mathbb{R}} |D_{x,t}^\beta f|,$$

$v_{j,\infty}$ and $\tilde{v}_{j,\infty}$ are functions of the form

$$\begin{aligned} v_{j,\infty}(x,t;k) &= \sum_{\ell=0}^{2j} a_{j,\ell}(x,k) h^{(\ell)}(t-j(x)), \\ \tilde{v}_{j,\infty}(x,t;k) &= \sum_{\ell=0}^{2j} \tilde{a}_{j,\ell}(x,k) h^{(\ell)}(t-\tilde{j}(x)), \end{aligned}$$

and b is a constant depending on ψ , A is a point in $S_1(\delta)$ depending on ψ .

Remark that we have

$$\square u_q = e^{ik(\mathcal{P}_q - t)} (ik)^{-N} \square v_{N,q}.$$

Next we construct by a usual method asymptotic solutions for

$$(3.3) \quad \begin{cases} \square u = 0 & \text{in } \omega \times \mathbb{R} \\ u = (1 - \chi_k(x)) u_{2p}(x, t; k) & \text{on } \Gamma_1 \times \mathbb{R} \end{cases}$$

Denote the asymptotic solution by u'_{2p} . Extend $\square(u_{2p} + u'_{2p})$ and $\square u_{2p+1}$ by a fixed manner into $\mathcal{O}$ so that these are smooth in $\mathbb{R}^3 \times \mathbb{R}$, and denote by u''_q the solution of

$$\begin{cases} \square u = -\square(u_q + u'_q) & \text{in } \mathbb{R}^3 \times \mathbb{R} \\ u = 0 & \text{for } t < 0 \end{cases}$$

where we set $u'_{2p+1} = 0$. By taking account of the continuity from u_q to u'_q and u''_q we have a Lemma of the type Lemma 3.2 on the convergence of u'_q and u''_q . Set

$$r_q = u_q + u'_q + u''_q$$

and we have

Lemma 3.3. It holds that

$$\begin{aligned} & |r_{2p}(x, t; k) - bw(A) e^{-ik(j(A) + d_\infty)} e^{ikd_0} (\lambda \tilde{\lambda})^p \\ & \quad \cdot r_\infty(x, t - 2pd - j(A) - d_\infty; k)|_m \leq C_m (\alpha \lambda \tilde{\lambda})^p k^{m+1} \\ & |r_{2p+1}(x, t; k) - bw(A) e^{-ik(j(A) + d_\infty)} e^{ikd_0} (\lambda \tilde{\lambda})^p \\ & \quad \cdot \tilde{r}_\infty(x, t - 2pd - j(A) - d_\infty; k)|_m \leq C_m (\alpha \lambda \tilde{\lambda})^p k^{m+1}, \end{aligned}$$

where $r_\infty, \tilde{r}_\infty$ are functions independent of ψ and w .

Set

$$r(x, t; k) = \sum_{q=0}^{\infty} (-1)^q r_q(x, t; k).$$

Evidently it holds that

$$\square r = 0 \quad \text{in} \quad \Omega \times \mathbb{R}.$$

We consider the Laplace transformation of r in t , that is,

$$(3.4) \quad \hat{r}(x, \mu; k) = \int e^{-\mu t} r(x, t; k) dt.$$

We have from Lemma 3.3 the following

Proposition 3.4. Let $\operatorname{Re} \mu > 0$. Then (3.4) converges and we have a representation of $\hat{r}(x, \mu; k)$

$$(3.5) \quad \begin{aligned} \hat{r}(x, \mu; k) \\ = bw(A) e^{-(j(A)+d_{\infty})(\mu+ik)} e^{ikd_0} \mathcal{P}(\mu)^{-1} r_{\infty}(x, \mu; k) + s(x, \mu; k), \end{aligned}$$

where $\hat{r}_{\infty}(x, \mu; k)$ is an entire function in μ independent of ψ and w , and $\hat{s}(x, \mu; k)$ is holomorphic in $\operatorname{Re} \mu > -c_0 - c_1$. Moreover we have on Γ_1

$$\begin{aligned} \hat{r}(x, \mu; k) &= e^{-(\mu+ik)j(x)} e^{ik\psi(x)} w(x) \hat{h}(\mu+ik) \\ &= e^{ikd_0} bw(A) e^{-(j(A)+d_{\infty})(\mu+ik)} \{v_k(x) \sum_{1 \leq j \leq N} \sum_{0 \leq h \leq N} a_{j,h}(x) (ik)^{-j} \\ &\quad x(\mu+ik)^h \hat{h}(\mu+ik) + a_1(x, k, \mu)\} \mathcal{P}(\mu)^{-1} + e_1(x, \mu; k), \end{aligned}$$

and on Γ_2

$$\hat{r}(x, \mu; k) = e^{ikd_0} \frac{1}{\mathcal{P}(\mu)} bw(A) e^{-(j(A)+d_{\infty})(\mu+ik)} a_2(x, k, \mu) + e_2(x, k, \mu),$$

where $a_{j,h}(x)$ are smooth functions on $S_1(\delta)$, a_1 and a_2 are entire functions independent of ψ and w having an estimate

$$\sup_{x \in \Gamma_j} |a_j(x, k, \mu)| \leq C_{N,R} |k|^{-N}$$

and e_1 and e_2 are holomorphic in $\operatorname{Re} \mu > -c_0 - c_1$ and satisfy

$$|e_1(x, k, \mu)| \leq \begin{cases} |k|^{-1} & \text{on } S_1((1+\delta)|k|^{-1}) \\ |k|^{-N} & \text{on } \Gamma_1 \setminus S_1((1+\delta)|k|^{-1}), \end{cases}$$

$$|e_2(x, k, \mu)| \leq |k|^{-N} \quad \text{on } \Gamma_2.$$

3.2. Reduction to an integral equation on Γ_1 .

Suppose that Γ_1 is represented as $x(\sigma) = (\sigma_1, \sigma_2, x_3(\sigma_1, \sigma_2))$ near a_1 . Let $g(x) \in C_0^\infty(S_1(\delta_0))$. Then

$$\begin{aligned} g(x(\sigma)) &= (2\pi)^{-2} \int e^{ik\sigma \cdot \xi} \hat{g}(k\xi) k^2 d\xi \\ &= (2\pi)^{-2} w(x(\sigma)) \int e^{ik\sigma \cdot \xi} \hat{g}(k\xi) k^2 d\xi \end{aligned}$$

where $w(x) \in C_0^\infty(S_1(2\delta_0))$ such that $w(x)=1$ on $S_1(\delta_0)$, and

$$\hat{g}(\xi) = \int e^{-i\sigma \cdot \xi} g(x(\sigma)) d\sigma.$$

If we define $\tilde{U}_1(k, \mu)$ an operator from $L^2(S_1(\delta_0))$ into $C^\infty(\bar{\Omega})$ by

$$(\tilde{U}_1(k, \mu)g)(x) = (2\pi)^{-2} \int u(x, \xi; k, \mu) \hat{g}(k\xi) k^2 d\xi$$

where $u(x, \xi; k, \mu)$ denotes $\hat{r}(x, \mu, k)/\hat{h}(\mu + ik)$ constructed for $\psi(x(\sigma)) = \sigma \cdot \xi$. Then we have from Proposition 3.4

Proposition 3.5. $\tilde{U}_1(k, \mu)$ is of the form

$$(3.6) \quad \tilde{U}_1(k, \mu)g = \frac{\hat{r}_\infty(x, \mu; k)}{\rho(\mu)} F_0(k, \mu)g + S(k, \mu)g$$

where

$$\begin{aligned} (3.7) \quad F_0(k, \mu)g &= (2\pi)^{-2} \int b(\xi) w(A(\xi)) e^{-(j(A(\xi)) + d_\infty(\xi))(\mu + ik)} \\ &\quad \cdot e^{ikd_0(\xi)} \hat{g}(k\xi) k^2 d\xi, \end{aligned}$$

$S(k, \mu)$ is $\mathcal{L}(L^2(S_1(\delta_0)), C^\infty(\bar{\Omega}))$ -valued holomorphic function in $\operatorname{Re} \mu > -c_0 - c_1$. Moreover it holds that

$$(3.8) \quad (\mu^2 - \Delta) \tilde{U}_1 g = 0 \quad \text{in } \Omega,$$

$$(3.9) \quad \tilde{U}_1 g = g - \frac{\alpha(x, k, \mu)}{\mathcal{P}(\mu)} F_0(k, \mu) g - E(k, \mu) g \quad \text{on } \Gamma_1$$

$$(3.10) \quad \tilde{U}_1 g = \frac{\tilde{\alpha}(x, k, \mu)}{\mathcal{P}(\mu)} F_0(k, \mu) g + \tilde{E}(k, \mu) g \quad \text{on } \Gamma_2,$$

$$(3.11) \quad |\alpha(x, k, \mu) F_0(k, \mu) g| \leq C |k|^{-\varepsilon} \|g\|_{L^2(\Gamma_1)}$$

$$(3.12) \quad \|E(k, \mu) g\|_{L^2(\Gamma_1)} \leq C |k|^{-\varepsilon} \|g\|_{L^2(\Gamma_1)},$$

$$(3.13) \quad |\tilde{\alpha}(x, k, \mu) F_0(k, \mu) g| \leq C |k|^{-N} \|g\|_{L^2(\Gamma_1)},$$

$$(3.14) \quad \|E(k, \mu) g\|_{L^2(\Gamma_2)} \leq C |k|^{-N} \|g\|_{L^2(\Gamma_1)}$$

Note that the solution $U_2 h$ of

$$\begin{cases} (\mu^2 - \Delta) u = 0 & \text{in } \mathbb{R}^3 - \bar{\mathcal{O}}_2 \\ u = h & \text{on } \Gamma_2 \end{cases}$$

is continued into $\{\mu; \operatorname{Re} \mu \geq -a \log(|\mu| + 1)\}$ for some $a > 0$. Then

$$U_1(k, \mu) g = \tilde{U}_1(k, \mu) g - U_2(\mu) (\tilde{U}_1(k, \mu) g|_{\Gamma_2})$$

is also of the form (3.6) and satisfies (3.8), (3.9), (3.11) and (3.12), and

$$(3.10)' \quad U_1(k, \mu) g = 0 \quad \text{on } \Gamma_2.$$

Remark. We can extend the definition of $U_1(k, \mu)$ for any $f \in L^2(\Gamma_1)$ by using the argument in §8 of [2]. Hereafter we denote by U_1 the extended one.

3.3. Representation of $U(\mu)$.

Lemma 3.6. Let H and E be linear operators with $\|H\|, \|E\| < 1/2$.

Then we have

$$(I - H - E)^{-1} = I + \mathcal{C}_1 + \mathcal{C}_2,$$

where

$$\mathcal{C}_1 = \mathcal{H} + \mathcal{H}E + \mathcal{H}E\mathcal{H} + \mathcal{H}E\mathcal{H}E + \dots,$$

$$\mathcal{C}_2 = \mathcal{E} + \mathcal{E}\mathcal{H} + \mathcal{E}\mathcal{H}E + \mathcal{E}\mathcal{H}E\mathcal{H} + \dots,$$

$$\mathcal{E} = E + E^2 + E^3 + \dots,$$

$$\mathcal{H} = H + H^2 + H^3 + \dots.$$

Pose

$$H(k, \mu)g = \frac{\alpha(x, k, \mu)}{\mathcal{P}(\mu)} F_0(k, \mu)g.$$

An application of the above lemma gives

$$(3.15) \quad (I - H - E)^{-1} = (I + \mathcal{E}) + \frac{(I + \mathcal{E})\alpha}{\mathcal{P}(\mu) - \gamma} F_0(I + \mathcal{E})$$

where

$$\gamma(k, \mu) = F_0(k, \mu) ((I + \mathcal{E}(k, \mu))\alpha(\cdot, k, \mu)).$$

Evidently we have in $\text{Re } \mu > 0$

$$(3.16) \quad U(\mu) = U_1(k, \mu) (I - H(k, \mu) - E(k, \mu))^{-1}.$$

Then a substitution of (3.15) into (3.16) gives

$$U(\mu) = \frac{r_\infty(x, k, \mu)}{\mathcal{P}(\mu) - \gamma(k, \mu)} F_0(k, \mu) (I + \mathcal{E}(k, \mu)) + S(k, \mu) (I + \mathcal{E}(k, \mu)).$$

By posing

$$F(k, \mu) = F_0(k, \mu) (I + \mathcal{E}(k, \mu)),$$

$$V(k, \mu) = S(k, \mu) (I + \mathcal{E}(k, \mu)),$$

we have a representation (2.2).

References

- [1] C.Bardos, J.C.Guillot and J.Ralston, La relation de Poisson pour l'équation des ondes dans un ouvert non borné. Application à la théorie de la diffusion, Comm. Partial Diff. Equ., 7(1982), 905-958.
- [2] M.Ikawa, Decay of solutions of the wave equation in the exterior of two convex obstacles, Osaka J.Math., 19(1982), 459-509.
- [3] ———, On the poles of the scattering matrix for two strictly convex obstacles, J.Math.Kyoto Univ., 23(1983), 127-194.
- [4] ———, On the poles of the scattering matrix for two strictly convex obstacles: Addendum, J.Math.Kyoto Univ., 23(1983), 795-802.
- [5] ———, Trapping obstacles with a sequence of poles of the scattering matrix converging to the real axis, to appear in Osaka J.Math.
- [6] ———, Precise informations on the poles of the scattering matrix for two strictly convex obstacles, in preparation.
- [7] P.D.Lax and R.S.Phillips, Scattering theory, Academic Press, New York, (1967).
- [8] ———, A logarithmic bound on the location of the scattering matrix, Arch.Rat.Mech.and Anal., 40(1971), 268-280.
- [9] G.Lebeau, to appear.
- [10] R.Melrose, Polynomial bound on the distribution of poles in scattering by obstacles, Journées "Equations aux dérivées partielles", Soc.Math.France, (1984).

- [11] V.M.Petkov, Propriétés génériques des rayons réfléchissants et applications aux problèmes spectraux, Séminaire Bony-Sjöstrand-Meyer, 1984-1985, Exposé n°XII.
- [12] V.M.Petkov and L.Stojanov, Periods of multiple reflecting geodesics and inverse spectral problems, preprint.