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Correction to: Linear fractional transformations of continued fractions with bounded partial quotients

par Jeffrey C. LAGARIAS et Jeffrey O. SHALLIT

Résumé. Nous comblons un trou dans la démonstration d'un théorème de notre article cité dans le titre.

Abstract. We fill a gap in the proof of a theorem of our paper cited in the title.

The proof of Theorem 1.1 of our paper [1] has a gap, which we fill below. The gap is that in the subcase $c \neq 0$ and $qa - pc \neq 0$ the inequality (4.11) given in [1] does not follow from the inequality immediately preceding it, and may not be valid. We give here a corrected argument, which also clarifies the logic treating this subcase together with the subcase $c \neq 0$ and $qa - pc = 0$.

Theorem 1.1 asserts the inequality (4.1), which states that

$$L\left(\frac{a\theta + b}{c\theta + d}\right) \leq |\det(M)|L(\theta) + |c(c\theta + d)|. \tag{4.1}$$

It can be derived as follows. Define

$$\psi := \frac{a\theta + b}{c\theta + d}.$$

The case $c = 0$ is treated as in the paper. Suppose $c \neq 0$. We either have an infinite sequence of distinct approximations $x = q||q\psi||$ with

$$L(\psi) \geq \frac{1}{x} \geq L(\psi) - \epsilon,$$

with $\epsilon \to 0$, or else we have a single approximation with $L(\psi) = \frac{1}{x}$; we reduce the second case to the first by viewing it as an infinite sequence with the same value of q repeated over and over. By taking a suitable subsequence we may reduce to the case where either all of the approximations have $qa - pc = 0$ or all of them have $qa - pc \neq 0$, with $\epsilon \to 0$.

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In the first subcase, where all $qa - pc = 0$, the inequality (4.7) in [1] yields

$$L(\psi) - \epsilon \leq \frac{1}{x} \leq |c\theta + d| \leq |c(c\theta + d)|,$$

since $c \neq 0$. Letting $\epsilon \to 0$ yields (4.1).

In the second subcase, where all $qa - pc \neq 0$, we obtain as in [1] the inequality (4.10), which states that

$$|c\theta + d| \left|\frac{qa - pc}{q}\right| L(\theta) \geq \frac{1}{x} \geq L(\psi) - \epsilon. \tag{4.10}$$

We then continue by using the triangle inequality

$$\left|q(\frac{a}{c}) - p\right| \leq \left|q(\frac{a\theta + b}{c\theta + d}) - q(\frac{a}{c})\right| + \left|q(\frac{a\theta + b}{c\theta + d}) - p\right|.$$

Here the first term on the right side is equal to $q|\det(M)|\frac{1}{|c(c\theta+d)|}$ while the second term is $\frac{x}{q}$ where $x := q||q\psi||$, provided we pick the integer p properly, and we then have

$$\left|\frac{qa - pc}{c}\right| \leq q|\det(M)|\frac{1}{|c(c\theta + d)|} + \frac{x}{q}.$$

Multiplying this by $\frac{c}{q}$ and substituting it in (4.10) gives the inequality

$$L(\psi) - \epsilon \leq |\det(M)|L(\theta) + |c(c\theta + d)|\frac{xL(\theta)}{q^2}, \tag{4.11$'$}$$

which replaces (4.11). Now

$$x \leq \frac{1}{L(\psi) - \epsilon},$$

which when used in the inequality (4.11′) yields

$$L(\psi) - \epsilon \leq |\det(M)|L(\theta) + \frac{|c(c\theta - d)|}{q^2} \cdot \frac{L(\theta)}{L(\psi) - \epsilon}.$$

Now there are two cases. First, if $L(\theta) \geq L(\psi)$, then the inequality (4.1) holds trivially, since $|\det(M)| \geq 1$. Second, if $L(\theta) < L(\psi)$ then, letting $\epsilon \to 0$ in the preceding inequality, the ratio $\frac{L(\theta)}{L(\psi)-\epsilon}$ becomes ≤ 1 in the limit, and since $q \geq 1$, (4.1) follows. This completes the proof.

References

[1] J.C. LAGARIAS, J.O. SHALLIT, *Linear Fractional Transformations of Continued Fractions with Bounded Partial Quotients.* J. Théor. Nombres Bordeaux **9** (1997), 267–279.

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