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## On the Uniform Paracompactness.

UMBERTO MARCONI (\*)

### 0. Introduction.

Uniform paracompactness was defined by M. D. Rice in [R] (this concept was actually used in a previous paper by H.H. Corson [C]). In [H<sub>1</sub>], [F], Tamano's theorem on paracompactness has been given a uniform analogue.

Countable uniform paracompactness has been discussed in [H<sub>1</sub>], [H<sub>2</sub>]. In this work, we plan to discuss uniform  $\mu$ -paracompactness. In § 1 definitions and basic properties are given.

In § 2 uniform analogues of Morita's product theorems for  $\mu$ -paracompactness [M<sub>1</sub>] are obtained; finally, in § 3 countable uniform paracompactness is discussed, obtaining an analogue of Dowker's theorem.

### 1. Definitions and basic properties.

We will denote by  $uX$  a uniform space, by  $X$  the associated topological space, by  $fX$  the finest uniform space on the topological space  $X$ . Furthermore  $puX$  will denote the paracompact reflection of  $uX$  and  $p^\mu uX$  the coarsest uniform space for which the uniform maps from  $uX$  to metric spaces of density  $\mu$  are uniform.

A filter base  $\mathcal{F}$  of subsets of  $uX$  is said to be weakly Cauchy if for every uniform covering  $\mathcal{U}$  of  $uX$  there exists an element  $U \in \mathcal{U}$  such that  $U \cap F \neq \emptyset$  for every  $F \in \mathcal{F}$ .

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Let  $\mathcal{A}$  be a family of subsets of  $X$ ; denote by  $\mathcal{A}_r$  the family of all finite unions of elements of  $\mathcal{A}$ . A directed family is a family  $\mathcal{A}$  such that  $\mathcal{A} = \mathcal{A}_r$ . A family  $\mathcal{A}$  is said to be uniformly locally finite if there exists a uniform covering  $\mathcal{U}$  such that every  $U \in \mathcal{U}$  meets  $\mathcal{A}$  only for a finite number of elements of  $\mathcal{A}$ .

Let  $\mu$  be a cardinal number. Consider the following conditions on  $uX$ :

- 1) every weakly Cauchy filter base of cardinal  $\leq \mu$  has a cluster point;
- 2) every directed open covering  $\mathcal{A}$  of power  $\leq \mu$  is uniform;
- 3) every open covering  $\mathcal{A}$  of power  $\leq \mu$  has an open uniformly locally finite refinement.

We have the following obvious implications:  $1 \Leftrightarrow 2$  and  $3 \Rightarrow 2$ . Later we will prove that  $2 \Rightarrow 3$ .

**DEFINITION 1.** A uniform space  $uX$  is said to be uniformly  $\mu$ -paracompact if it satisfies the above condition 1.

In  $[M_1]$  a topological space  $X$  is said to be  $\mu$ -paracompact if every open covering of  $X$  of power  $\leq \mu$  has an open locally finite refinement.

**PROPOSITION 1.** *If  $uX$  is uniformly  $\mu$ -paracompact, then  $X$  is  $\mu$ -paracompact.*

**PROOF.** Let  $\mathcal{A}$  be an open covering of power  $\leq \mu$ ;  $\mathcal{A}_r$ , being a uniform covering, has a locally finite open refinement  $\mathcal{B}$ . For every  $B \in \mathcal{B}$ , consider a finite subset  $\mathcal{A}_B$  of  $\mathcal{A}$  such that  $B \subset \cup \mathcal{A}_B$ . Then the open covering

$$\{B \cap A : B \in \mathcal{B}, A \in \mathcal{A}_B\}$$

is a locally finite open refinement of  $\mathcal{A}$ .

I don't know if a  $T_{3\frac{1}{2}}$   $\mu$ -paracompact space is uniformly  $\mu$ -paracompact in the fine uniformity. This occurs if  $X$  is a normal space.

**PROPOSITION 2.** *A normal space  $X$  is  $\mu$ -paracompact if and only if  $fX$  is uniformly  $\mu$ -paracompact.*

**PROOF.** If  $X$  is a normal  $\mu$ -paracompact topological space then every open covering of power  $\leq \mu$  is normal in the sense of Tukey ( $[M_1]$  th. 1.1).

**REMARK 1.** There exist countably uniformly paracompact spaces that fail to be normal. Let  $X$  be a countably compact  $T_{3\frac{1}{2}}$  non normal space, for example  $\omega_1 \times (\omega_1 + 1)$ . If  $uX$  is a compatible uniform space,  $uX$  is countably uniformly paracompact.

For the proof of  $2 \Rightarrow 3$ , we need a lemma. (I am indebted to A. Hohti for a suggestion which led to this lemma).

**LEMMA 1.** *If a covering  $\mathcal{A}$  is locally finite, there exists an open covering  $\mathcal{B}$ , with  $|\mathcal{B}| \leq |\mathcal{A}|$ , such that every element of  $\mathcal{B}$  meets only a finite number of elements of  $\mathcal{A}$ .*

**PROOF.** Let  $\mathcal{U}$  be an open covering of  $X$  such that every member of  $\mathcal{U}$  meets only a finite number of elements of  $\mathcal{A}$ . For every finite subset  $\mathcal{F}$  of  $\mathcal{A}$  put  $V_{\mathcal{F}} = \cup \{V \in \mathcal{U} : V \cap A \neq \emptyset \text{ iff } A \in \mathcal{F}\}$ . Then  $\mathcal{B} = \{V_{\mathcal{F}} : \mathcal{F} \in \text{finite subsets of } \mathcal{A}\}$  satisfies the required properties.

**PROPOSITION 3.** *Condition 2  $\Rightarrow$  condition 3.*

**PROOF.** Let  $\mathcal{A}$  be an open covering of  $X$  of power  $\leq \mu$ . By proposition 1 and theorem 1.4 ch. VIII of [Du], it has an open locally finite refinement  $\mathcal{B}$ , with  $|\mathcal{B}| \leq |\mathcal{A}|$ . By lemma 1, there exists an open covering  $\mathcal{C}$ , with  $|\mathcal{C}| \leq \mu$ , such that every member of  $\mathcal{C}$  meets only a finite number of elements of  $\mathcal{B}$ . Since  $\mathcal{C}$ , is uniform,  $\mathcal{B}$  is uniformly locally finite.

**PROPOSITION 4.** *If  $uX$  is uniformly  $\mu$ -paracompact, every uniform covering of power  $\leq \mu$  belongs to  $p^\mu uX$  and  $p^\mu uX$  has a point finite base.*

**PROOF.** By [V] it suffices to prove that every uniform covering of power  $\leq \mu$  has a point finite uniform refinement. If  $\mathcal{U}$  is a uniform covering of power  $\leq \mu$ , it has a uniform open refinement  $\mathcal{V}$  of power  $\leq \mu$  (argue as in the proof of th. 1.4 ch. VIII of [Du]). By uniform  $\mu$ -paracompactness,  $\mathcal{V}$  has a uniformly locally finite refinement. Therefore, by [Sm] th. 4.5,  $\mathcal{V}$  has a locally finite uniform refinement.

From the above proposition we get the following

**COROLLARY 1.**  *$uX$  is uniformly  $\mu$ -paracompact if and only if every directed open covering  $\mathcal{A}$  of power  $\leq \mu$  belongs to  $p^\mu uX$ .*

## 2. Uniform products.

As for  $\mu$ -paracompactness ([M<sub>1</sub>] th. 2.1) uniform  $\mu$ -paracompactness is preserved under products by compact spaces. When considered as

uniform spaces, compact ( $T_2$ ) spaces are of course equipped with their unique admissible uniformity.

**THEOREM 1.** *If a uniform space  $uX$  is uniformly  $\mu$ -paracompact, and  $Y$  is compact, then  $uX \times Y$  is uniformly  $\mu$ -paracompact.*

**PROOF.** Let  $\mathcal{F}$  be a weakly Cauchy filter base of power  $\leq \mu$ . Since the first projection  $p_1: uX \times Y \rightarrow uX$  is a closed mapping, the filter

$$\mathcal{F}_1 = \{p_1(\overline{F}): F \in \mathcal{F}\}$$

is a weakly Cauchy filter base of closed sets, with  $|\mathcal{F}_1| \leq \mu$ . Therefore there exists a point  $p \in uX$  such that  $(\{p\} \times Y) \cap \overline{F} \neq \emptyset$  for every  $F \in \mathcal{F}$ . The compactness of  $Y$  ensures the existence of a point  $y \in Y$  such that  $(p, y) \in \overline{F}$  for every  $F \in \mathcal{F}$ .

As usual, let  $I$  denote the closed unit interval,  $D$  the discrete two-point space  $\{0, 1\}$ . It is well-known that a normal space  $X$  is  $\mu$ -paracompact if and only if  $X \times I^\mu$  is normal, or equivalently,  $X \times D^\mu$  is normal ([M<sub>2</sub>]; [D] for  $\mu = \omega$ ). We plan to give analogous characterizations of uniform  $\mu$ -paracompactness.

**DEFINITION.** *We say that a uniform space  $uX$  satisfies property  $P$  for a compact space  $Y$  if whenever  $A$  and  $B$  are disjoint closed sets of  $X \times Y$ , there exists a uniform covering  $\mathcal{C}$  of  $uX \times Y$  such that for every  $T \in \mathcal{C}$  the sets  $A \cap T$  and  $B \cap T$  are far (uniformly separated) in  $uX \times Y$ .*

**REMARK 2.** By compactness of  $Y$  the uniform covering  $\mathcal{C}$  of the above definition may be assumed of the form:

$$\mathcal{C} = \{U \times Y: U \in \mathcal{U}\}$$

for a suitable uniform covering  $\mathcal{U}$  of  $uX$ .

**THEOREM 2.** *Let  $uX$  be a normal uniform space. The following conditions are equivalent:*

- 1)  $uX$  is a uniformly  $\mu$ -paracompact space;
- 2)  $p^\mu uX$  satisfies property  $P$  for every compact space of weight  $\mu$ ;
- 3)  $uX$  satisfies property  $P$  for  $I^\mu$ ,
- 4)  $uX$  satisfies property  $P$  for  $D^\mu$ .

**PROOF.**  $1 \Rightarrow 2$ . Let  $Y$  be a compact space of weight  $\mu$ . Let  $\mathcal{B}$  be a directed basis of power  $\mu$  for the open sets of  $Y$ . Let  $\Delta = \{d_\alpha: \alpha \in \mu\}$  be a basis of power  $\leq \mu$  consisting of continuous pseudometrics of  $Y$ , such that every open covering of  $Y$  has  $d_\alpha$ -Lebesgue number 1, for some  $\alpha \in \mu$ . Denote by  $\Gamma$  the set of all triples  $(d_\alpha, H, K)$  with  $d_\alpha(H, K) > 1$ ,  $d_\alpha \in \Delta$ ,  $H, K \in \mathcal{B}$ ; of course  $|\Gamma| \leq \mu$ . Let  $A, B$  be closed and disjoint subsets of  $X \times Y$ . For every  $x \in X$  put

$$A[x] = \{y \in Y: (x, y) \in A\}, \quad B[x] = \{y \in Y: (x, y) \in B\}.$$

For every  $\gamma \in \Gamma$ , let

$$V_\gamma = \{x \in X: A[x] \subset H, B[x] \subset K, \text{ where } \gamma = (d_\alpha, H, K)\}.$$

From the compactness of  $Y$  follows that the family  $\mathcal{V} = \{V_\gamma: \gamma \in \Gamma\}$  is an open covering of  $uX$  of power at most  $\mu$ .

Therefore there exists a uniform covering  $\mathcal{U}$  of closed sets such that every  $U \in \mathcal{U}$  is contained in a finite union of elements of  $\mathcal{V}$ , that is  $U \subset \bigcup_{\gamma \in F_U} V_\gamma$  for a suitable finite subset  $F_U$  of  $\Gamma$ . For every  $U \in \mathcal{U}$  the open covering of  $U$

$$\{V_{\gamma_1} \cap U, \dots, V_{\gamma_n} \cap U: \gamma_i \in F_U\}$$

is induced by the finite open covering of  $X$ :

$$\mathcal{U}_U = \{X \setminus U, V_{\gamma_1}, \dots, V_{\gamma_n}: \gamma_i \in F_U\}.$$

By the normality of  $X$ ,  $\mathcal{U}_U$  is a uniform covering of the space  $pfX$ .

By corollary 1, covering  $\mathcal{U}$  may be taken belonging to  $p^\mu uX$ .  
Let

$$\mathcal{C} = \{U \times Y: U \in \mathcal{U}\}.$$

For every  $V_\gamma \in \mathcal{V}$ , there exist a pseudometric  $d_\gamma \in \Delta$  and two open sets of  $Y$ , say  $H_\gamma, K_\gamma$ , such that for every  $x \in V_\gamma$  we have  $A[x] \subset H_\gamma$ ,  $B[x] \subset K_\gamma$  and  $d_\gamma(H_\gamma, K_\gamma) \geq 1$ . If  $F_U = \{\gamma_1, \dots, \gamma_n\}$  let  $d_U = d_{\gamma_1} \vee \dots \vee d_{\gamma_n}$ . Let  $\sigma_U$  be an admissible pseudometric of  $pfX$  such that the covering  $\mathcal{U}_U$  has  $\sigma_U$ -Lebesgue number 1. Let  $T \in \mathcal{C}$ ,  $T = U \times Y$  for some  $U \in \mathcal{U}$ .

Let  $(x_1, y_1) \in A \cap T$  and  $(x_2, y_2) \in B \cap T$ . If  $\sigma_U(x_1, x_2) \leq 1$  there exists some  $\gamma_i \in F_U$  such that  $x_1, x_2 \in V_{\gamma_i}$  and therefore  $y_1 \in A[x_1] \subset H_{\gamma_i}$ ,

$y_2 \in B[x_2] \subset K_{\gamma_1}$ . Therefore  $d_U(y_1, y_2) \geq d_{\gamma_1}(y_1, y_2) \geq d_{\gamma_1}(H, K) \geq 1$ . Thus  $(\sigma_U \times d_U)((x_1, y_1), (x_2, y_2)) \geq 1$  and therefore  $A \cap T$  and  $B \cap T$  are separated by a uniform covering of  $pfX \times Y$ .

2  $\Rightarrow$  3. Obvious.

3  $\Rightarrow$  4. Obvious, because  $D^\mu$  is a closed subspace of  $I^\mu$ .

Before proving that 4)  $\Rightarrow$  1) we need the following:

**LEMMA 2.** *Let  $B$  a subset of  $X \times Y$  and  $Y_0$  a subset of  $Y$ . If  $B$  and  $X \times Y_0$  are separated by the covering  $\mathcal{U} \times \mathcal{V}$ , then they are separated by the covering  $\{X\} \times \mathcal{V}$ .*

**PROOF.** Let  $\mathcal{U} \times \mathcal{V} = \{U_\alpha \times V_\beta : U_\alpha \in \mathcal{U}, V_\beta \in \mathcal{V}\}$ . Thus

$$\text{St}(X \times Y_0, \mathcal{U} \times \mathcal{V}) = X \times \text{St}(Y_0, \mathcal{V}) = \text{St}(X \times Y_0, \{X\} \times \mathcal{V}),$$

$$\text{St}(B, \{X\} \times \mathcal{V}) = X \times (\cup \{V_\beta : B \cap (X \times V_\beta) \neq \emptyset\}).$$

If the stars meet, there exist  $V_\beta, V_{\beta'} \in \mathcal{V}$ ,  $V_\beta \cap V_{\beta'} \neq \emptyset$ , such that  $V_{\beta'} \cap Y_0 \neq \emptyset$  and  $(X \times V_\beta) \cap B \neq \emptyset$ .

Let  $y \in V_\beta \cap V_{\beta'}$  and let  $x \in X$  such that  $(\{x\} \times V_\beta) \cap B \neq \emptyset$ . Then

$$(x, y) \in \text{St}(B, \mathcal{U} \times \mathcal{V}) \cap \text{St}(X \times Y_0, \mathcal{U} \times \mathcal{V}),$$

against the hypothesis.

**PROOF OF 4  $\Rightarrow$  1.** For every  $\alpha \in \mu$ , let  $p_\alpha: D^\mu \rightarrow D$  the projection on the  $\alpha$ -th coordinate.

Let  $\mathbf{0}$  be the point of null coordinates.

If  $\mathcal{A} = \{A_\alpha : \alpha \in \mu\}$  is an open covering of  $uX$  of power  $\mu$ , the open set

$$\Omega = \bigcup_{\alpha \in \mu} A_\alpha \times p_\alpha^{-1}(\mathbf{0})$$

is a neighborhood of  $X_0 = X \times \{\mathbf{0}\}$ .

Therefore there exists a uniform covering  $\mathcal{U}$  of  $uX$  such that for every  $U \in \mathcal{U}$  the sets  $X_0 \cap (U \times Y)$  and  $(X \times Y \setminus \Omega) \cap (U \times Y)$  are far in  $pfX \times Y$ . By lemma 2 there exists an open covering  $\mathcal{U}_\sigma$  of  $D^\mu$  such that

$$\text{St}(U \times \{\mathbf{0}\}, U \times \mathcal{U}_\sigma) \subset \bigcup_{\alpha \in \mu} A_\alpha \times p_\alpha^{-1}(\mathbf{0}).$$

Let  $F$  be a finite subset of  $\mu$  such that

$$\bigcap_{\alpha \in F} p_\alpha^{-1}(0) \subset \text{St}(0, \mathcal{U}_\sigma).$$

Let  $y \in D^\mu$  such that  $p_\alpha(y) = 0$  exactly for  $\alpha \in F$ . If  $x \in U$ , then  $(x, y) \in \bigcup_{\alpha \in F} A_\alpha \times p_\alpha^{-1}(0)$  and therefore  $x \in \bigcup_{\alpha \in F} A_\alpha$ .

Then the directed covering  $\mathcal{A}_r$  is uniform and the proof is complete.

From the proof of the above theorem we can deduce the following result.

**COROLLARY 2.** *A normal uniform space  $uX$  is uniformly  $\mu$ -paracompact if and only if for a suitable (and thus for every) compact space  $Y$  of weight  $\mu$  and for a suitable (and thus for every) compact subspace  $Y_0$  of  $Y$  the following condition is satisfied: for every closed subspace  $K$  of  $X \times Y$  disjoint from  $X_0 = X \times Y_0$ , there exists an open covering of the form  $\mathcal{W} = \{U_\alpha \times V_\beta^\alpha\}$ , where  $\{U_\alpha\}$  is a uniform covering of  $uX$  and, for each  $\alpha$ ,  $\{V_\beta^\alpha\}$  is a uniform covering of  $Y$ , such that*

$$K \cap \text{St}(X_0, \mathcal{W}) = \emptyset.$$

Covering  $\{U_\alpha\}$  may be taken belonging to  $p^\mu uX$ .

**PROOF.** Sufficiency is proved in the same way as the implication  $4 \Rightarrow 1$  of theorem 2.

Necessity: the same theorem ensures the existence of a uniform covering  $\mathcal{U}$  of  $p^\mu uX$  and, for every  $U \in \mathcal{U}$ , of an open covering  $\mathcal{U}_\sigma$  of  $Y$  such that

$$K \cap \text{St}(U \times Y_0, U \times \mathcal{U}_\sigma) = \emptyset$$

for every  $U \in \mathcal{U}$ .

We claim that  $K \cap \text{St}(X_0, \mathcal{W}) = \emptyset$ , where  $\mathcal{W} = \{U \times V : U \in \mathcal{U}, V \in \mathcal{U}_\sigma\}$ . In fact, if  $(x, y) \in \text{St}(X_0, \mathcal{W})$ , there exist  $U \in \mathcal{U}$  and  $V \in \mathcal{U}_\sigma$  such that  $V \cap Y_0 \neq \emptyset$  and  $(x, y) \in U \times V$ .

Therefore  $(x, y) \in \text{St}(U \times Y_0, U \times \mathcal{U}_\sigma)$  and so  $(x, y) \notin K$ .

Recall that if  $uX$  and  $vY$  are two uniform spaces, the semiuniform product  $uX * vY$  is the uniform space whose uniform coverings are those coverings which are normal with respect to the coverings considered in the above corollary 2.



If  $p^\mu uX$  admits a point finite basis, the coverings of this form,  $\{U_\alpha \times V_\beta^\alpha\}$ , are a basis for the uniform coverings of  $p^\mu uX * vY$  (see [F]). Therefore, by proposition 4, theorem 2 may be stated in a much nicer form, which generalizes results found in [H<sub>1</sub>], [F].

**THEOREM 3.** *Let  $uX$  be a normal uniform space. The following conditions are equivalent:*

- 1)  $uX$  is uniformly  $\mu$ -paracompact;
- 2) For every compact space  $Y$  of weight  $\mu$  and for every closed subspace  $Y_0 \subset Y$ , whenever  $A$  and  $X \times Y_0$  are disjoint closed subsets of  $X \times Y$ , they are uniformly separated in  $uX * Y$ ,
- 3) If  $A$  a closed subspace of  $X \times I^\mu(X \times D^\mu)$  disjoint from  $X \times \{0\}$ ,  $A$  and  $X \times \{0\}$  are uniformly separated in  $uX * I^\mu(uX * D^\mu)$ .

**PROOF.** It suffices to prove  $1 \Rightarrow 2$ . By corollary 2, there exists a covering  $\mathcal{W} = \{U_\alpha \times V_\beta^\alpha\}$ , where

- $\{U_\alpha\}$  is a uniform covering of  $p^\mu uX$  and, for each  $\alpha$ ,
- $\{V_\beta^\alpha\}$  is a uniform covering of  $Y$ , such that

$$A \cap \text{St}(X \times Y_0, \mathcal{W}) = \emptyset.$$

If  $\mathcal{S}$  is a uniform star refinement of  $\mathcal{W}$ , we have  $\text{St}(A, \mathcal{S}) \cap \text{St}(X \times Y_0, \mathcal{S}) = \emptyset$ .

### 3. Countable uniform paracompactness.

The characterization of uniform countable paracompactness has a form which is more expressive than the general case. Equivalence  $1 \Leftrightarrow 2$  of the following theorem is a uniform analogue of Dowker's theorem in [D].

**THEOREM 4.** *Let  $uX$  be a normal uniform space. The following conditions are equivalent:*

- 1)  $uX$  is countably uniformly paracompact;
- 2) For every closed subset  $B$  of  $uX * I$  disjoint from  $X_0 = X \times \{0\}$ ,  $B$  and  $X_0$  are uniformly separated in  $uX * I$ .

3)  $X$  is countably paracompact and for every zero set  $Z$  of  $\beta X$  disjoint from  $X$  there exists a uniform covering  $\mathcal{U}$  of  $uX$  such that  $Z \cap \text{cl}_{\beta X} U = \emptyset$  for every  $U \in \mathcal{U}$ .

PROOF.  $1 \Rightarrow 2$ . Follows from theorem 3.  $2 \Rightarrow 1$ . Let  $f: D^\omega \rightarrow I$  be the map  $f(t) = \sum_{i=0}^{\infty} (t_i/2^{i+1})$ , where  $t = (t_i)_{i \in \omega}$ .

Consider the map  $\tilde{f}: X \times D^\omega \rightarrow X \times I$  so defined:  $\tilde{f}(x, t) = (x, f(t))$ .  $\tilde{f}$  is continuous and closed and furthermore if  $A$  is a closed subset of  $X \times D^\omega$  disjoint from  $X_0 = X \times \{0\}$ ,  $\tilde{f}(A)$  is disjoint from  $\tilde{f}(X_0) = X \times \{0\}$ .

Since  $\tilde{f}(A)$  and  $\tilde{f}(X_0)$  are separated in  $uX * I$ ,  $A$  and  $X \times \{0\}$  are separated in  $uX * D^\omega$ . The conclusion follows from the implication  $3 \Rightarrow 1$  of theorem 3.

$1 \Rightarrow 3$ . Obviously  $X$  is countably paracompact (proposition 1).

Let  $Z$  be a zero set of  $\beta X \setminus X$ .  $Z = Z(f)$  for some  $f \in C(\beta X)$ ,  $f \geq 0$ .

Let  $A_n = \{x \in X: f(x) > 1/(n+1)\}$ . The countable open covering of  $uX$ ,  $\mathcal{A} = \{A_n: n \in \omega\}$  is uniform because  $\mathcal{A} = \mathcal{A}_\gamma$ . Furthermore, for every  $n \in \omega$ ,  $Z \cap \text{cl}_{\beta X} A_n = \emptyset$ .

$3 \Rightarrow 1$ . Let  $\mathcal{A} = \{A_n: n \in \omega\}$  be a directed open covering of  $uX$ . From the countable paracompactness of the normal space  $X$ , there exists a countable open covering of cozero sets,  $\{\text{coz}(f_n): n \in \omega\}$  where each  $f_n$  is continuous and bounded, such that  $\text{coz}(f_n) \subset A_n$  for every  $n \in \omega$ ; we may also assume that  $\text{coz}(f_n) \subset \text{coz}(f_{n+1})$ , for every  $n \in \omega$ . Let  $Z = \bigcap_{n \in \omega} Z(f_n^\beta)$ , where  $f_n^\beta$  denotes the extension of  $f_n$  to  $\beta X$ . There exists a uniform covering  $\mathcal{U}$  of  $uX$  such that  $Z \cap \text{cl}_{\beta X} (U) = \emptyset$  for every  $U \in \mathcal{U}$ . Therefore, by the compactness of  $\beta X$ , for every  $U \in \mathcal{U}$  there exists an index  $n_U \in \omega$  such that  $Z(f_{n_U}) \cap U = \emptyset$ ; thus  $\mathcal{A}$  is uniform.

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