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$\mathcal{F}$-constraint of the automorphism group of a finite group

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F-Constraint of the Automorphism Group
of a Finite Group.

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If \( \mathcal{F} \) is a homomorph and we denote \( \mathcal{F}' = \{ G | S^\mathcal{F} = S \forall S \leq G \} \), we say that a group \( G \) is \( \mathcal{F} \)-constrained when there is a maximal normal \( \mathcal{F} \)-subgroup \( \bar{M} \) of \( G = G/\mathcal{G}_{\mathcal{F}'} \), such that \( C_G(\bar{M}) \triangleleft \bar{M} \).

In ([7]), R. Laue proves that if \( G \) has no direct abelian factors and \( C_\mathcal{F}(F(G)) \triangleleft F(G) \) (i.e. \( G \) is Nilpotent-constrained) then \( C_{\text{Aut}G}(F(\text{Aut}G)) \triangleleft F(\text{Aut}G) \).

In ([10]) it is proved for a saturated Fitting formation and a group \( G \) verifying that:

i) \( G_{\mathcal{F}'} \triangleleft \Phi(G) \),

ii) \( G/G_{\mathcal{F}'} \) has no direct abelian factors,

iii) \( G \) is \( \mathcal{F} \)-constrained,

that \( \text{Aut}G \) is \( \mathcal{F} \)-constrained.

The purpose of this paper is mainly to prove the above result when \( \mathcal{F} \) is a homomorph closed for direct products (\( D_b \)-closed) and normal subgroups that is: i) saturated or ii) closed for central extensions.

All group considered are finite.

If \( \mathcal{F} \) is a homomorph then the class \( \mathcal{F}' \) is a s-closed (i.e. closed for subgroups )extensible Fitting formation and a group \( G \) is said \( \mathcal{F} \)-sepa-
rable if it possesses a normal series:

\[ G = G_0 \triangleright G_1 \triangleright \ldots \triangleright G_r = 1 \]

whose factor groups \( G_i/G_{i+1} \) are either \( \mathcal{F} \)-groups or \( \mathcal{F}' \)-groups.

If we do not state the contrary here on we shall suppose that \( \mathcal{F} \) is a \( n \)-closed (i.e. closed for normal subgroups) homomorph.

The class of \( \mathcal{F} \)-separable groups is an extensible Fitting formation that contains the solvable groups.

The product of all normal semisimple subgroups of a group \( G \) is again a semisimple normal subgroup of \( G \), and it is denoted by \( L(G) \).

It is called the semisimple radical of \( G \) ([6]).

We use the concepts of semisimple and perfect-quasisimple groups given by Gorenstein and Walter [6].

**Lemma 1** (s. [5] p. 127 or [9] p. III-32). For every group \( G \), we have:

\[ C_{\mathcal{F}}(F(G)L(G)) \leq F(G) \]

From the definition and properties of class \( \mathcal{F}' \) it follows that \( G \) is \( \mathcal{F} \)-constrained if and only if \( G/N \) is \( \mathcal{F} \)-constrained, when \( N \trianglelefteq G \).

**Lemma 2.** If \( \mathcal{F} \) is a saturated Fitting formation, the following are equivalent. 1) \( G \) is \( \mathcal{F} \)-constrained, 2) \( L(G) \in \mathcal{F} ([8]) \).

Our remainder notation is standard and it is based on Huppert’s book ([4]).

In the following we shall say that \( \mathcal{F} \) verifies:

\( A) \) If \( \{ Q, E_\phi, D_\phi, S_\phi \} \mathcal{F} = \mathcal{F} \);

\( B) \) If \( \{ Q, E_\phi, D_\phi, S_\phi \} \mathcal{F} = \mathcal{F} \), where \( E_\phi \mathcal{F} = \{ G|N; N \trianglelefteq Z(G), G/N \in \mathcal{F} \} \).

As a consequence of the following proposition we obtain that the \( \mathcal{F} \)-constraint is equivalent to the constraint with respect to a suitable saturated Fitting formation.

**Proposition 3.** If \( \bar{G} = G/G_\mathcal{F} \), and if \( \mathcal{F} \) verifies \( A \) or \( B \), the following are equivalent:

1) \( G \) is \( \mathcal{F} \)-constrained,
2) \( \bar{G} \) is (\( \mathcal{F} \)-separable)-constrained,
3) \( L(\bar{G}) \in \mathcal{F} \).
**Proof.** 1) $\Rightarrow$ 2) Obvious.

$2) \Rightarrow 3)$. By lemma 2, $L(\bar{G})$ is $\mathcal{F}$-separable, hence $L(\bar{G})/Z(L(\bar{G}))$ is $\mathcal{F}$-separable and direct product of non-abelian simple groups ([6]). Moreover, since $\mathcal{F}'$ is saturated \( \left( L(\bar{G})/Z(L(\bar{G})) \right)_{\mathcal{F}'} = 1 \), thus $L(\bar{G})/Z(L(\bar{G})) \in \mathcal{F}$, but since $Z(L(\bar{G})) = \Phi(L(\bar{G}))$, \( L(\bar{G}) \in \mathcal{F} \). Thus, since $\mathcal{G}_\mathcal{F} = 1$, both $L(\bar{G})$ and $F(\bar{G})$ are normal $\mathcal{F}$-subgroups of $\bar{G}$ and since $[L(\bar{G}), F(\bar{G})] = 1$, then $L(\bar{G})F(\bar{G})$ is a normal $\mathcal{F}$-subgroup of $\bar{G}$. Let $\bar{M}$ be a maximal normal $\mathcal{F}$-subgroup of $\bar{G}$ containing $L(\bar{G})F(\bar{G})$, by lemma 1 it follows:

\[
C_{\bar{G}}(\bar{M}) < C_{\bar{G}}(L(\bar{G})F(\bar{G})) < F(\bar{G}) < \bar{M}
\]

**Corollary.** If $G$ is $\mathcal{F}$-constrained group then $C_{\bar{G}}(\bar{M}) < \bar{M}$ for all maximal normal $\mathcal{F}$-subgroups of $\bar{G}$.

**Proof.** By proposition 3, $L(\bar{G}) \in \mathcal{F}$. Since $L(C_{\bar{G}}(\bar{M})) \leq L(\bar{G})$ then $L(C_{\bar{G}}(\bar{M})) \in \mathcal{F}$ hence $L(C_{\bar{G}}(\bar{M})) < \bar{M}$ by the maximality of $\bar{M}$ and so $L(C_{\bar{G}}(\bar{M})) = 1$, thus $C_{\bar{G}}(\bar{M})$ is a $\mathcal{N}$-constrained group. On the other hand $F(C_{\bar{G}}(\bar{M})) \in \mathcal{F}$ hence $F(C_{\bar{G}}(\bar{M})) < \bar{M}$.

Clearly:

\[
C_{\bar{G}}(\bar{M}) < C_{\bar{G}}(F(C_{\bar{G}}(\bar{M}))) \cap C_{\bar{G}}(\bar{M}) = C_{\bar{G}}(\bar{M}) \left( F(C_{\bar{G}}(\bar{M})) \right) < F(C_{\bar{G}}(\bar{M})) < \bar{M}
\]

**Remarks.** The following conditions, are not equivalent: i) $\bar{G}$ is ($\mathcal{F}$-separable)-constrained and ii) $G$ is ($\mathcal{F}$-separable)-constrained. In fact, in [6] it is proved that $G = C_{x}(\tau)$, where $X = SL(4,2^a), a > 1$, and $\tau$ is the central involution $I_4 + xE_{14}, x \neq 0$, is $2$-constrained, hence $G$ is ($2$-separable)-constrained or equivalently ($2'$-separable)-constrained, however $G$ is not $2'$-constrained. Thus $G$ is ($2'$-separable)-constrained but $G/O_4(G)$ is not ($2'$-separable)-constrained.

If $\mathcal{F}$ is a saturated Fitting formation it is known that:

i) The class of the $\mathcal{F}$-constrained groups is a Fitting class that contains the solvable groups ([8])

ii) $G$ is $\mathcal{F}$-constrained if and only if $\text{Inn} \ G$ is $\mathcal{F}$-constrained ([10]),

iii) The class of the $\mathcal{F}$-constrained groups is extensible ([10]).

As a consequence of the above proposition, the properties i), ii), iii) are still valid when $\mathcal{F}$ verifies $A$ or $B$. 

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\[\text{PROOF. 1) } \Rightarrow 2) \text{ Obvious.}\]

\[2) \Rightarrow 3). \text{ By lemma 2, } L(\bar{G}) \text{ is } \mathcal{F}-\text{separable, hence } L(\bar{G})/Z(L(\bar{G})) \text{ is } \mathcal{F}-\text{separable and direct product of non-abelian simple groups ([6])}. \text{ Moreover, since } \mathcal{F}' \text{ is saturated } \left( L(\bar{G})/Z(L(\bar{G})) \right)_{\mathcal{F}'} = 1, \text{ thus } L(\bar{G})/Z(L(\bar{G})) \in \mathcal{F}, \text{ but since } Z(L(\bar{G})) = \Phi(L(\bar{G})), \text{ } L(\bar{G}) \in \mathcal{F}. \text{ Thus, since } \mathcal{G}_\mathcal{F} = 1, \text{ both } L(\bar{G}) \text{ and } F(\bar{G}) \text{ are normal } \mathcal{F}-\text{subgroups of } \bar{G} \text{ and since } [L(\bar{G}), F(\bar{G})] = 1, \text{ then } L(\bar{G})F(\bar{G}) \text{ is a normal } \mathcal{F}-\text{subgroup of } \bar{G}. \text{ Let } \bar{M} \text{ be a maximal normal } \mathcal{F}-\text{subgroup of } \bar{G} \text{ containing } L(\bar{G})F(\bar{G}), \text{ by lemma 1 it follows:}
\]

\[C_{\bar{G}}(\bar{M}) < C_{\bar{G}}(L(\bar{G})F(\bar{G})) < F(\bar{G}) < \bar{M}\]

**Corollary.** If $G$ is $\mathcal{F}$-constrained group then $C_{\bar{G}}(\bar{M}) < \bar{M}$ for all maximal normal $\mathcal{F}$-subgroups of $\bar{G}$.

**Proof.** By proposition 3, $L(\bar{G}) \in \mathcal{F}$. Since $L(C_{\bar{G}}(\bar{M})) \leq L(\bar{G})$ then $L(C_{\bar{G}}(\bar{M})) \in \mathcal{F}$ hence $L(C_{\bar{G}}(\bar{M})) < \bar{M}$ by the maximality of $\bar{M}$ and so $L(C_{\bar{G}}(\bar{M})) = 1$, thus $C_{\bar{G}}(\bar{M})$ is a $\mathcal{N}$-constrained group. On the other hand $F(C_{\bar{G}}(\bar{M})) \in \mathcal{F}$ hence $F(C_{\bar{G}}(\bar{M})) < \bar{M}$.

Clearly:

\[C_{\bar{G}}(\bar{M}) < C_{\bar{G}}(F(C_{\bar{G}}(\bar{M}))) \cap C_{\bar{G}}(\bar{M}) = C_{\bar{G}}(\bar{M}) \left( F(C_{\bar{G}}(\bar{M})) \right) < F(C_{\bar{G}}(\bar{M})) < \bar{M}\]

**Remarks.** The following conditions, are not equivalent: i) $\bar{G}$ is ($\mathcal{F}$-separable)-constrained and ii) $G$ is ($\mathcal{F}$-separable)-constrained. In fact, in [6] it is proved that $G = C_{x}(\tau)$, where $X = SL(4,2^a), a > 1$, and $\tau$ is the central involution $I_4 + xE_{14}, x \neq 0$, is $2$-constrained, hence $G$ is ($2$-separable)-constrained or equivalently ($2'$-separable)-constrained, however $G$ is not $2'$-constrained. Thus $G$ is ($2'$-separable)-constrained but $G/O_4(G)$ is not ($2'$-separable)-constrained.

If $\mathcal{F}$ is a saturated Fitting formation it is known that:

i) The class of the $\mathcal{F}$-constrained groups is a Fitting class that contains the solvable groups ([8])

ii) $G$ is $\mathcal{F}$-constrained if and only if $\text{Inn} \ G$ is $\mathcal{F}$-constrained ([10]),

iii) The class of the $\mathcal{F}$-constrained groups is extensible ([10]).

As a consequence of the above proposition, the properties i), ii), iii) are still valid when $\mathcal{F}$ verifies $A$ or $B$. 

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LEMMA 4. Let $\mathcal{F}$ be a homomorph that verifies $A$ or $B$, then if $G$ is a group without direct abelian factors and $G_{\mathcal{F}'} = 1$, it follows $(\text{Aut } G)_{\mathcal{F}'} = 1$.

PROOF. Let $H$ be a normal $\mathcal{F}'$-subgroup of $\text{Aut}(G)$, then $H \cap \text{Inn}(G)$ is a normal $\mathcal{F}'$-subgroup of $\text{Inn}(G)$; as $(G/Z(G))_{\mathcal{F}'} = 1$, consequently $H \cap \text{Inn } G = 1$, thus $[H, \text{Inn } G] = 1$ and so $H < C_{\text{Aut } G}(\text{Inn } G) < F(\text{Aut } G)$ because $G$ has no direct abelian factors ([7]). Set $\pi = \text{car } \mathcal{F} = \{p | C_p \in \mathcal{F}\}$, since $F(G)$ is nilpotent and $G_{\mathcal{F}'} = 1$ it follows that $F(G)$ is a $\pi$-group and thus $F(\text{Aut } G)$ is a $\pi$-group ([7]). Since $H \in \mathcal{F}'$, $H$ is a $\pi'$-group and so $H = 1$.

COROLLARY 1. Let $\mathcal{F}$ be a homomorph that verifies $A$ or $B$, and $G$ a group such that $G/G_{\mathcal{F}'}$ has no direct abelian factors, then if $G$ is $\mathcal{F}$-constrained, $\text{Aut } (G/G_{\mathcal{F}'})$ is $\mathcal{F}$-constrained.

PROOF. As $G$ is $\mathcal{F}$-constrained group, so $G/G_{\mathcal{F}'}$ is $\mathcal{F}$-constrained. By proposition 3, $G/G_{\mathcal{F}'}$ is $(\mathcal{F}$-separable)$-constrained. We can apply (2.6) of [10] to obtain that $\text{Aut } (G/G_{\mathcal{F}'})$ is $(\mathcal{F}$-separable)$-constrained. By lemma 4 $(\text{Aut } (G/G_{\mathcal{F}'}))_{\mathcal{F}'} = 1$, hence by proposition 3, $\text{Aut } (G/G_{\mathcal{F}'})$ is $\mathcal{F}$-constrained.

COROLLARY 2. Let $\mathcal{F}$ be a homomorph verifying $B$, and $G$ a group without direct abelian factors. If $G$ is $\mathcal{F}$-constrained, then $\text{Aut } G$ is $\mathcal{F}$-constrained.

PROOF. If $\mathcal{F}$ verifies $B$, then $\mathcal{F}' = 1$ now we can apply corollary 1, to obtain the result.

However, in the case of saturated homomorph, it does not appear that the constraint of $\text{Aut } G$ can be obtained as an easy consequence of the constraint of $\text{Aut } (G/G_{\mathcal{F}'})$.

THEOREM 5. Let $\mathcal{F}$ be a homomorph verifying $A$, and $G$ a group that verifies:

i) $G_{\mathcal{F}'} < \Phi(G)$

ii) $G/G_{\mathcal{F}'}$ has no direct abelian factors

iii) $G$ is $\mathcal{F}$-constrained.

Then $\text{Aut } G$ is $\mathcal{F}$-constrained.
PROOF. We denote $\overline{\text{Aut } G} = \text{Aut } G/\langle \text{Aut } G \rangle_{\mathcal{F}}$. We have:

$$
\overline{\text{Inn } G} = \text{Inn } G/\langle \text{Aut } G \rangle_{\mathcal{F}}/\langle \text{Aut } G \rangle_{\mathcal{F}}.
$$

Since $G$ is $\mathcal{F}$-constrained we know that $\text{Inn}(G)$ is $\mathcal{F}$-constrained and so $\overline{\text{Inn } G}$ is $\mathcal{F}$-constrained too. By proposition 3, $L(\overline{\text{Inn } G})$ is a characteristic $\mathcal{F}$-subgroup of $\overline{\text{Inn } G}$. Since $(\overline{\text{Inn } G})_{\mathcal{F}} = 1$, consequently $F(\overline{\text{Inn } G})$ is also a characteristic $\mathcal{F}$-subgroup of $\overline{\text{Inn } G}$. Since $[L(\overline{\text{Inn } G}), F(\overline{\text{Inn } G})] = 1$ and $\mathcal{F}$ is a $D$-closed homomorph, hence $L(\overline{\text{Inn } G}) F(\overline{\text{Inn } G})$ is a normal $\mathcal{F}$-subgroup of $\overline{\text{Aut } G}$. Let $\overline{M}$ be a maximal normal $\mathcal{F}$-subgroup of $\overline{\text{Aut } G}$ that contains $L(\overline{\text{Inn } G}) F(\overline{\text{Inn } G})$. Set $\overline{L} = C_{\overline{\text{Aut } G}}(\overline{M})$, since $\overline{L}$ centralizes $\overline{M}$, $\overline{L}$ will centralize $L(\overline{\text{Inn } G}) F(\overline{\text{Inn } G})$. Let $L$ be the subgroup of $\text{Aut } G$ such that $\overline{L} = L/\langle \text{Aut } G \rangle_{\mathcal{F}}$, and let $H$ be the normal subgroup of $G$ such that $L(\text{Inn } G) F(\overline{\text{Inn } G}) \simeq H/\langle \text{Aut } G \rangle_{\mathcal{F}}$. Then $L$ centralizes $H/\mathcal{G}_{\mathcal{F}} Z(G)$ and hence $L$ centralizes $G/\mathcal{G}_{\mathcal{F}} Z(G)$, too. Now since $H/\mathcal{G}_{\mathcal{F}} Z(G) = L(G/\mathcal{G}_{\mathcal{F}} Z(G)) F(G/\mathcal{G}_{\mathcal{F}} Z(G))$, by lemma 1 consequently:

$$
C_{\mathcal{G}/\mathcal{G}_{\mathcal{F}} Z(G)}(H/\mathcal{G}_{\mathcal{F}} Z(G)) < H/\mathcal{G}_{\mathcal{F}} Z(G),
$$

hence $C_{\mathcal{G}}(H/\mathcal{G}_{\mathcal{F}} Z(G)) < H$, and so $L$ centralizes $G/H$, therefore $L$ stabilizes the series:

$$
G/\mathcal{G}_{\mathcal{F}} Z(G) \triangleright H/\mathcal{G}_{\mathcal{F}} Z(G) > 1,
$$

thus $L/C_{L}(G/\mathcal{G}_{\mathcal{F}} Z(G))$ is nilpotent by a well-known P. Hall's result. We consider now $C_{\text{Aut } G}(G/\mathcal{G}_{\mathcal{F}} Z(G))$. This group induces an automorphism group of $G/\mathcal{G}_{\mathcal{F}}$, which is isomorphic to:

$$
(1)
C_{\text{Aut } G}(G/\mathcal{G}_{\mathcal{F}} Z(G))/C_{\text{Aut } G}(G/\mathcal{G}_{\mathcal{F}} Z(G))
$$

By ii) $G/\mathcal{G}_{\mathcal{F}}$ has no direct abelian factors, hence we know ([7]) that:

$$
C_{\text{Aut } (G/\mathcal{G}_{\mathcal{F}} Z(G))}(G/\mathcal{G}_{\mathcal{F}} Z(G))/G/\mathcal{G}_{\mathcal{F}} Z(G))
$$
is nilpotent and so:

\[ C_{\text{Aut}(G/G_{\mathcal{F}})}(G/G_{\mathcal{F}}'|Z(G)G_{\mathcal{F}}'/G_{\mathcal{F}}') \]

is nilpotent, too.

Since the subgroup of $\text{Aut}(G/G_{\mathcal{F}})$ isomorphic to (1) is contained in (2), we deduce that (1) is nilpotent. Now, since $C_{\text{Aut}G}(G/\Phi(G))$ is nilpotent ([11]) and by (1) $C_{\text{Aut}G}(G/G_{\mathcal{F}})$ is nilpotent too. Consequently $C_{\text{Aut}G}(G/G_{\mathcal{F}}',Z(G))$ is solvable and so $L$ and hence $\bar{L}$ are solvable. Now $F(\bar{L}) \in \mathcal{F}$, hence $F(\bar{L})M \in \mathcal{F}$, as $M$ is a maximal normal $\mathcal{F}$-subgroup of $\text{Aut}G$, then $F(\bar{L}) < \bar{M}$, therefore:

\[ \bar{L} = C_{\text{Aut}G}(\bar{M}) < C_{\text{Aut}G}(F(\bar{L})) \cap \bar{L} = C_{\text{Aut}G}(F(\bar{L})) < F(\bar{L}) < \bar{M}. \]

Next, we give some counterexamples which prove that the conditions imposed in theorem 5, are not superfluous.

1) The assumption $G_{\mathcal{F}}' < \Phi(G)$ is necessary. It is enough to take $G = C_3 \times C_3 \times C_3 \times \bar{Q}$ where $\bar{Q}$ is the quaternion group of order 8. The $\text{Aut}G = G\text{L}(3,3) \times S_4$. If $\mathcal{F}$ is the class of 2-groups, then $G_{\mathcal{F}}' = C_3 \times C_3 \times C_3$ is not a subgroup of $\Phi(G) = Z(Q)$. Make note that $G$ is 2-constrained because it is solvable, now, if $\text{Aut}G$ is 2-constrained, $G\text{L}(3,3)$ would be also 2-constrained and hence his normal subgroup $S\text{L}(3,3)$ would be also 2-constrained, but $S\text{L}(3,3)$ is a simple group that is neither a 2-group nor a 2'-group and therefore it is not 2-constrained group.

2) The condition that $G/G_{\mathcal{F}}$ has no direct abelian factors is equally necessary. We can consider $G = C_3 \times C_3 \times C_3$ and as $\mathcal{F}$ the class of nilpotent groups, then $G$ is $\mathcal{N}$-constrained but $\text{Aut}G$ is not $\mathcal{N}$-constrained, because $G\text{L}(3,3)$ is not $\mathcal{N}$-constrained by a similar argument, to the above one.

Finally, we give some examples of classes verifying $A$ or $B$ that are not saturated Fitting formations

a) The class of the supersolvable groups is a formation that verifies $A$ and $B$ but it is not a Fitting class.

b) Let $\mathcal{F} = \mathcal{N} \mathcal{A} \cap S_\pi$ be, where $\mathcal{A}$ is the class of abelian groups and $S_\pi$ is the class of $\pi$-groups, $\pi$ is a set of prime numbers. This class is a $n$-closed saturated formation but it is neither a Fitting class nor
$E_2$-closed. Of course if we take $\pi = \{2\}$ and $G = C_5 \times Q$ where $Q$ is the quaternion group of order 8, then $G/Z(G) \cong C_2 \times C_2 \in \mathcal{F}$ but $G \notin \mathcal{F}$. On the other hand if we consider:

$$\pi = \{2, 5\}, \quad G = [C_5 \times C_5] \cdot Q_8,$$

$$C = [C_5 \times C_2] \langle a \rangle \quad \text{and} \quad D = [C_5 \times C_2] \langle b \rangle$$

where $Q = \langle a, b | a^4 = 1, a^2 = b^2, b^{-1} ab = a^{-1} \rangle$ then $C, D \in \mathcal{F}$ but $G \notin \mathcal{F}$ because $G'$ is not nilpotent.

c) The class of solvable groups with absolute arithmetic 3-rank a $3'$-number is a not saturated Fitting formation [3]; but it is an $E_2$-closed class, because the 3-chief factors under $Z(G)$ have order 3.

d) Following Cossey, we consider inside solvable groups, the class $\mathcal{X} = S_p, S'p, S_p$ where $S_p, S'p$ are the class of $p'$-groups and $p$-groups respectively. We denote by $c_p(G)$ the least common multiple of the absolute degrees of the complemented $p$-chief factors. Thus the class:

$$\mathcal{Y} = \{G \in \mathcal{X} | c_p(G) \text{ is coprime to } p\}$$

is both a Fitting class and a Schunck class but it is not a formation[1]. This class verifies $A$ and also satisfies $B$ by a similar argument to that of $c$).

BIBLIOGRAPHY


