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On \( p \)-Groups whose \( L \)-Automorphism Group is Transitive on the Atoms.

ANDREA LUCCHINI (*)

Introduction.

An isomorphic mapping of the subgroup lattice \( L(G) \) of a group \( G \) onto the subgroup lattice \( L(H) \) of a group \( H \) is called an \( L \)-isomorphism, or a projectivity, of \( G \) onto \( H \).

The study of finite groups whose \( L \)-automorphism group is transitive on the atoms of their subgroup lattice is introduced in [2]. In that paper the groups satisfying this property and whose order is divisible by two different prime numbers at least, are completely characterized.

For what concerns \( p \)-groups in view of the well known results by Shult (see [3]) on groups \( G \) such that \( \text{Aut} \ G \) is transitive on the minimal subgroups of \( G \), it is a natural question whether, in the case \( p \neq 2 \), a \( p \)-group whose group of autoprejectivities is transitive on the atoms is modular. An affirmative answer to this question was given [2] by assuming the more restrictive hypothesis that a cyclic subgroup of the \( L \)-automorphism group acts transitively on the atoms of the subgroup lattice.

The aim of the present work is to prove that such a result can be generalized assuming that the transitive subgroup of the \( L \)-automorphism group satisfies weaker properties rather than being cyclic. In particular the following results will be proved:

**Theorem A.** A finite \( p \)-group, \( p \neq 2 \), whose \( L \)-automorphism group contains a subgroup that is transitive on the atoms of the subgroup lattice and that has its order not divisible by \( p \), is modular.

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THEOREM B. A finite $p$-group, $p \neq 2$, whose $L$-automorphism group contains a soluble subgroup that is transitive on the atoms of the subgroup lattice is modular.

Notations.

We will indicate with $G$ a finite $p$-group, $p \neq 2$, of exponent $p^m$ and with $\Pi$ a subgroup of the $L$-automorphism group of $G$ that acts transitively on the minimal subgroups of $G$. In [2] it is proved that it is not restrictive to assume $G' \leq \Omega_1(G) \leq Z(G)$ and that the mapping $\pi$ from $\Omega_{m-1}(G)$ into $\Omega_1(G)$ defined by the formula $(a\Omega_{m-1}(G))^{\pi} = a^{p^{m-1}}$ is an isomorphism.

In this situation $\Omega_1(G)$ can be thought of as a $GF(p)$ algebra if we define, for every $x$ and $y$ in $\Omega_1(G)$, $\gamma(x, y) = [a, b]$ where $a$ and $b$ are two elements of $G$ such that $x = (a\Omega_{m-1}(G))^{\pi}$ and $y = (b\Omega_{m-1}(G))^{\pi}$. As it is remarked in [2] $G$ will be modular exactly when $\gamma(x, y) \in \in \langle x, y \rangle$ for every pair of elements $x, y$ in $\Omega_1(G)$.

We will indicate with $K$ the field $GF(p)$.

Let $\varphi$ be the homomorphism from $\Pi$ to the $L$-automorphism group of $\Omega_1(G)$ that maps an $L$-automorphism $\sigma$ of $G$ in its restriction to the subgroup lattice of $\Omega_1(G)$: it is $\Pi^{\varphi} \cong H/Z(GL(\Omega_1(G), K))$ where $H$ is a subgroup of $GL(\Omega_1(G), K)$. Obviously $H$ is transitive on the 1-dimensional subspaces of $\Omega_1(G)$ and for every $\alpha \in H$ the $L$-automorphism induced by $\alpha$ on $\Omega_1(G)$ coincides with the restriction to $\Omega_1(G)$ of a suitable $\sigma \in \Pi$.

Finally we will indicate with $n$ the dimension of $\Omega_1(G)$ as $K$-vector space: in the next section it will be proved that we can assume without loss of generality $n \geq 4$.

1. Proposition 1.1. A finite $p$-group, $p \neq 2$, generable by at most 3 elements and whose $L$-automorphism group is transitive on the atoms, is modular.

Proof. The case $G$ generable by two elements at most is discussed in [2], section 4. So let $G = \langle a, b, c \rangle$ with $a, b, c$ independent elements of order $p^m$, the exponent of $G$; in [2] it is proved that $G$ contains at least two independent and with maximal order elements, $x$ and $y$, such that the subgroup $\langle x, y \rangle$ is modular. Using this fact and changing eventually the operation on $G$ as it is described in [2], Lemma 3.2,
we may suppose \([a, b] = 1\); being the \(L\)-automorphism group of \(G\) transitive on the atoms, like \(\langle a^{s^m} \rangle\) and \(\langle b^{s^m} \rangle\), \(\langle (a^{s^m})^\ast \cdot (b^{s^m})^\ast \rangle\); if \(s = 0\) \(\langle a \rangle\) is normalized by a generator system and so it is in particular a Dedekind subgroup of \(G\): it follows that all cyclic subgroups of \(G\) are Dedekind subgroups and so \(G\) is modular. If \(s \neq 0\) we may assume \([a, c] = e^{s^{-1}}\); but \([b, c] = (a^{s^{-1}})^r (b^{s^{-1}})^r (e^{s^{-1}})^r\). \(\langle a^{s^{-1}} \rangle\) is contained in at least two different modular subgroups generated by two independent elements of order \(p^m\): the same must be true for every other element of \(\Omega_1(G)\), in particular for \((a^{s^{-1}} b^{-1})^m\): i.e. there must exists an element \(a^r b^s c\) with \([a^{s^{-1}} b^{-1}, a^r b^s c] \in \langle (a^{s^{-1}} b^{-1})^m, (a^r b^s c)^m \rangle\): but this is possible if and only if \(a^{s^{-1}} b^{-1}\) is normalized by \(a^r b^s c\) but then \(\langle a^{s^{-1}} b^{-1} \rangle\) is normal in \(G\) and again we deduce that \(G\) is modular. □

2. Before discussing the general case we need the following result.

**Lemma 2.1.** Let \(\alpha\) be an element of \(H\) that acts irreducibly on the \(K\)-vector space \(\Omega_1(G)\) and whose order is not divisible by \(p\): then \(G\) is modular or there exists a projectivity from \(G\) onto a group \(G_1\) with \(G_1 \leq \leq \Omega_1(G) \leq Z(G_1)\) and \(\Omega_1(G) = \Omega_1(G_1)\) and such that \(\alpha\) is an automorphism for the structure of algebra induced by \(G_1\) on \(\Omega_1(G_1)\).

**Proof.** Since \(p \nmid |z|\) and \(\alpha\) acts irreducibly on \(\Omega_1(G)\) there exists a \(K\)-isomorphism \(\varphi\) from \(\Omega_1(G)\) into the additive group of a finite extension \(K(\lambda)\) of \(K\) such that \(x^e = \lambda(x^e)\) for every \(x \in \Omega_1(G)\) (see [1], 1.7 p. 77); we translate on \(K(\lambda)\) the structure of algebra that we have defined on \(\Omega_1(G)\) by setting \(\beta(l, m) = \gamma(l^{-1}, m^{-1})e\) for every pair \((l, m)\) of elements in \(K(\lambda)\). By Lemma 4.1 of [2] it is \(\beta(\lambda a, \lambda b) = k_1(a, b) \lambda \beta(a, b) + k_2(a, b) \lambda a + k_3(a, b) \lambda b\) with \(k_i(a, b) \in K\) for every \(i, 1 \leq i \leq 3\). In [2], pp. 286-289, it is proved that if \(G\) is not modular then \(k_i(a, b)\) is independent of the choice of \(a\) and \(b\) unless there exists an element \(x \in K(\lambda)\) with \(\beta(x, y) \in \langle x, y \rangle \forall y \in K(\lambda)\); but we must exclude this last possibility: in fact, since the \(L\)-automorphisms act transitively on the atoms, this would imply that for every \(z \in K(\lambda)\) it is \(\beta(z, y) \in \langle z, y \rangle \forall y \in K(\lambda)\) and this is equivalent to saying that \(G\) is modular. Therefore the following relation holds:

\[
\beta(\lambda a, \lambda b) = k_1 \lambda \beta(a, b) + k_2(a, b) \lambda a + k_3(a, b) \lambda b.
\]
We consider in $K(\lambda)$ the element $\mu = k^{-1}\lambda$: since $K(\mu) = K(\lambda)$, $\mu$ also acts irreducibly and furthermore the following relation holds:

$$[\beta(\mu a, \mu b) - \mu \beta(a, b)]/\mu \in \langle a, b \rangle.$$  

The $K$-bilinear function $\delta$ from $K(\mu) \times K(\mu)$ into $K(\mu)$ defined by setting

$$\delta(a, b) = [\beta(\mu a, \mu b) - \mu \beta(a, b)]/\mu$$

satisfies the properties described in [2], Lemma 5.1. By these properties and since, $\mu$ acting irreducibly, every $n - 1$ dimensional subspace of $K(\mu)$ can be written in the form $\langle b\mu^{-1}, \ldots, b\mu^{-(n-2)} \rangle$ for a suitable $b \in K(\mu)$, we can repeat the construction described in [2] p. 191 in order to get a projectivity from $G$ onto a group $G_1$ such that for the bilinear function $\beta_1$ that represents on $K(\mu)$ the algebra structure induced by $G_1$, the equality

$$\beta_1(\mu a, \mu b) = \mu \beta_1(a, b)$$

holds for every $a$ and $b$. □

3. Our purpose is now to prove that for the subgroup $H$ of $GL(\Omega_1(G), K)$ defined above the following result holds.

**Proposition 3.1.** If $|H|$ is not divisible by $p$ then $G$ is modular.

**Proof.** The proof proceeds by a series of short steps.

a) It is useful first of all to recall the following result proved by Shult ([3], Th. 3): let $G$ a subgroup of $GL(n, p^s)$, and let $\Phi_n(x)$ denote the cyclotomic polynomial whose roots are the primitive $n$-th roots of units: if $\pi_0$ denotes the set of prime divisors of $\Phi_n(p)$ not dividing $n$ and $\pi(G)$ is the set of prime divisors of $|G|$, then one of the following holds:

i) $G$ contains a normal irreducible cyclic subgroup $C$ of index dividing $n$;

ii) $G$ is a central extension of $LF(2, 2n + 1)$ and $\pi = \{2n + 1\}$ or $\{n + 1, 2n + 1\}$;

iii) $\pi$ contains at most the single prime $n + 1$ where $(n + 1)^2 \nmid |G|$. 


Therefore we need the following remark: if \( x \) is an element of prime order \( p \), lying in \( GL(n, p^s) \), then \( x \) acts irreducibly on \( V(n, p^s) \) if and only if \( p \not\equiv 1 \pmod{p} \).

b) We apply the previous result to the group \( H \): since \( H \) is transitive on the 1-dimensional subspace of \( \Omega_1(G) \) \((p^n - 1)/(p - 1)\) divides \( |H| \), but then \( \Phi_n(p) \) also divides \( |H| \) and so it is \( \pi = \pi_0 \); therefore we may suppose \( \pi_0 \neq 0 \): in fact by [3], Lemma 9, it is \( \pi_0 = 0 \) if and only if \( n = 2 \) or \( p = 2 \). Therefore one of the following holds:

i) \( H \) contains a normal irreducible cyclic subgroup \( C \) of index dividing \( n \);

ii) \( \pi_0 = \{n + 1, 2n + 1\} \) or \( \pi_0 = \{2n + 1\} \) and \( H \) is a central extension of \( LF(2, 2n + 1) \);

iii) \( \pi_0 = \{n + 1\} \) and \((n + 1)^2\) does not divide \( |H| \).

c) If \( H \) contains an irreducible cyclic subgroup \( \langle \xi \rangle \) and we suppose, by absurd, that \( G \) is not modular, we may apply Lemma 2: so we may suppose that there exists a projectivity from \( G \) onto a group \( G_1 \) with \( \Omega_1(G) = \Omega_1(G_1) \) and \( G_1 \leq \Omega_1(G_1) \leq Z(G_1) \) and a \( K \)-isomorphism \( \phi \) from \( \Omega_1(G) \) onto \( K(\lambda) \) such that \( \lambda^{\phi(x)} = \lambda(x) \) for every \( x \) in \( \Omega_1(G_1) \) and, if we indicate with \( \beta \) the bilinear function induced on \( K(\lambda) \) by \( G_1 \) as described above, the relation \( \beta_i(\lambda a, \lambda b) = \lambda \beta_i(a, b) \) holds for every pair of elements \( a, b \) in \( K(\lambda) \).

By [1], Lemma 1.8, \( n \) elements \( \zeta_{ij} \ (i, j = 1, \ldots, n) \) in \( K(\lambda) \) are uniquely determined with \( \zeta_{ij} = \zeta_{ij} + \zeta_{ij} \) for every \( i \) and \( j \) and such that \( \beta_i(a, b) = \sum_{i,j=1}^{n} \zeta_{ij} a^{i-1} b^{j-1} \) for every pair \( (a, b) \) of elements in \( K(\lambda) \).

Since \( \beta_i(\lambda a, \lambda b) = \lambda \beta_i(a, b) \) we have \( \zeta_{ij} \lambda^{(i-1)s} = \zeta_{ij} \lambda \) (see [1] p. 86). But then either \( \xi_{ij} = 0 \) for every \( i, j \) or there exist two integers \( i \) and \( j \) with \( 1 \leq i < j \leq n \) such that \( \lambda^{(i-1)s} \equiv 1 \pmod{\lambda} \). If \( \xi_{ij} = 0 \) for every \( i, j \) then \( G_1 \) is abelian and so \( G \) is modular since there exists a projectivity of \( G \) onto \( G_1 \).

So we can conclude that if \( G \) is not modular but \( H \) contains a normal irreducible cyclic subgroup \( C \) then there exist two integers \( r \) and \( s \) with \( 0 \leq r < s \leq n - 1 \) and \( p^r + p^s \equiv 1 \pmod{|C|} \), furthermore it must be \( r > 0 \) since \( p \nmid |C| \).

d) \( n \) is not a prime number: in fact by [3], p. 646, this happens if and only if \( H \) is a cyclic group, while in [1] it is proved that a finite \( p \)-group with a cyclic subgroup of the \( L \)-automorphism group transitive on the atoms is modular.
e) Cases (ii) and (iii) of step (b) don't hold. The proof that (ii) does not hold is the same as the one in [3], p. 649, step (n). For what concerns (iii), in [3] p. 650 step (o) it is shown that it can hold only in three cases:

1) \( n = 4, \ p = 3; \)
2) \( n = 6, \ p = 3; \)
3) \( n = 6, \ p = 5. \)

It is furthermore proved that in all these cases \( H \) contains an irreducible cyclic subgroup whose order is divisible by 20 in the first case, by 91 in the second, by 217 in the third. By step (c) (1) holds if and only if there exist two integers \( r \) and \( s \) with \( 1 \leq r < s \leq 3 \) and such that \( 3^r + 3^s \equiv 1 \mod 20 \) while (2) and (3) imply that there exist \( r \) and \( s \) with \( 1 \leq r < s \leq 5 \) and \( 3^r + 3^s \equiv 1 \mod 91 \) or \( 5^r + 5^s \equiv 1 \mod 217 \) respectively. It is easy to verify that all these congruences are impossible.

f) We need now an auxiliary numerical result:

**Lemma.** For every prime number \( p, \ p \neq 2, \) and every integer number \( m \) with \( m \geq 12, \) the following inequality holds:

\[
2 p^{m/3} m \leq \frac{p^m - 1}{p - 1} \cdot \frac{1}{m}.
\]

**Proof.** It is \( 2m \leq 3^{m/3 - 1} \) for every natural number \( m \geq 12: \) the same inequality obviously holds if we substitute 3 with any other prime number \( p \) different from 2: so it is \( 2m \leq p^{m/3 - 1}, \) from which we deduce \( 2mp^{m/3} \leq p^{m-1} \leq (p^m - 1)/(p - 1). \)

g) Let us suppose that case (i) of step (b) holds, i.e. that \( H \) contains a normal irreducible subgroup \( C \) of index dividing \( n; \) since \( |H| \) is divisible by \( (p^n - 1)/(p - 1) \) the inequality \( (1/n)(p^n - 1)/(p - 1) \leq \leq |C| \) holds.

Therefore if \( G \) is not modular there exist two integers \( r \) and \( s \) with \( 0 < r < s < n \) and such that \( p^r + p^s \equiv 1 \mod |C|: \) it is obviously also \( p^{r+k} + p^{s+k} \equiv p^k \mod |C| \) for every \( k \in \mathbb{Z}; \) furthermore being \( C \) an irreducible subgroup of \( H, \) \( p \) has order \( n \mod |C| \) and so the exponents in the last relation may be thought reduced mod \( n, \) as we will do from now on.

One side of the congruence \( p^{r+k} + p^{s+k} \equiv p^k \mod |C| \) must represent
an integer exceeding \(|C|\); if \(n \geq 12\) by the numerical lemma proved in the previous step it is \(|C| \geq (p^n - 1/p - 1)(1/n) \geq 2p^{2n/3}\) and, for every \(k \in \mathbb{Z}\), one of the two numbers \(p^{r+k} + p^{s+k}\) and \(p^k\) exceeds \(|C|\): it follows that the reduction modulo \(n\) of one of the numbers \(r+k\), \(s+k\), \(k\) must be greater than \((2/3)n\); this implies that it is \(r = n/3\) and \(s = 2n/3\), but then from \(p^{n/3} + p^{2n/3} = 1\) mod \(|C|\) it follows that \(p^{n/3} + p^{2n/3} > |C| \geq p^{2n/3}2\), a contradiction.

By this last remark and keeping in mind the result contained in step (d) the only possibilities for \(n\) are \(n = 4, 6, 8, 9\) or \(10\). We must now discuss separately all these cases.

\(h)\) Suppose \(n = 4\): it must be \(p^r + p^s = 1\) mod \(|C|\) with \(1 \leq r < s \leq 3\). We distinguish three possibilities:

1) \(r = 1\) \(s = 3\): it is \(p + p^3 \equiv 1\) mod \(|C|\); by multiplying this congruence through by \(p\) we get \(p^2 + 1 \equiv p\) mod \(|C|\); but then it must be \(p^2 + 1 > |C|\), an absurdity since \(|C| \geq (1 + p + p^2 + p^3)/4 = (1 + p)(1 + p^3)/4 \geq 1 + p^2\).

2) \(r = 2\) \(s = 3\): multiplying through by \(p^2\) the congruence \(p^2 + p^3 \equiv 1\) mod \(|C|\) we get \(1 + p \equiv p^2\) mod \(|C|\) that is impossible since both \(1 + p\) and \(p^2\) are less than \(|C|\).

3) \(r = 1\) \(s = 2\): it is \(p + p^2 \equiv 1\) mod \(|C|\); multiplying through by \(p\) and \(p^2\) we get the congruences \(p^2 + p^3 \equiv p\) and \(1 + p^3 \equiv p^2\) mod \(|C|\). Subtracting these two congruences the one from the other and comparing with the first we deduce \(p^2 - p \equiv p\) mod \(|C|\): again both sides are less than \(|C|\) and so also (3) produces a contradiction.

\(i)\) Suppose \(n = 6\). It is \(p^r + p^s = 1\) mod \(|C|\) with \(1 \leq r < s \leq 5\): since \(|C| > p^2 + p^3\) it must be \(s \geq 4\). We distinguish between four different possible cases:

1) \(r = 1\) \(s = 4\): it is \(p + p^4 \equiv 1\) mod \(|C|\); multiplying through by \(p^2\) we get \(1 + p^3 \equiv p^2\) mod \(|C|\), a contradiction.

2) \(r = 2\) \(s = 4\). \(p^2 + p^4 \equiv 1\) mod \(|C|\) and multiplying through by \(p^2\) we get \(1 + p^4 \equiv p^2\) mod \(|C|\): subtracting the two congruences we deduce \(2(p^2 - 1) \equiv 0\) mod \(|C|\), a contradiction.

3) \(r = 3\) \(s = 4\). Multiplying through by \(p^3\) the congruence \(p^3 + p^4 \equiv 1\) mod \(|C|\) we get \(1 + p \equiv p^3\) mod \(|C|\) that is impossible since both sides are less than \(|C|\).
4) $s = 5$. It is $p^r + p^s \equiv 1 \mod |C|$, from which it follows $p^{r+1} + 1 \equiv p \mod |C|$: it must be $r \geq 3$; so it is $p^{s-r} + p^s \equiv 1 \mod |C|$ with $1 \leq i \leq 2$; multiplying through by $p^{i+1}$ we deduce $1 + p^i \equiv p^{i+1} \mod |C|$, a contradiction.

$j)$ Suppose $n = 8$. There exist two integers $r$ and $s$ with $1 \leq r < s \leq 7$ such that $p^r + p^s \equiv 1 \mod |C|$: in particular $p^r + p^s$ must exceed $|C|$: since

$$|C| \geq (1 + p + \ldots + p^7)/8 > (p^5 + p^6 + p^7)/8 \geq p^5$$

and $p^3 + p^4 \leq p^5$ it is $s \geq 5$. There are the following possible cases:

1) $s = 5$. From $p^r + p^s \equiv 1 \mod |C|$, multiplying through by $p^3$, we deduce $p^{r+3} + 1 \equiv p^3 \mod |C|$: it must be $p^{r+3} + 1 > |C|$, from which it follows $r = 5 - i$ with $1 \leq i \leq 2$. So the congruence $p^{s-r} + p^s \equiv 1 \mod |C|$ holds, and we get $1 + p^i \equiv p^{i+1}$, an impossibility, since both the sides are less than $|C|$.

2) $s \geq 6$, $p^{r+k} + p^{s+k} \equiv p^k \mod |C|$ holds for every $k \in \mathbb{Z}$: in particular choosing $k = 8 - s$ we deduce $p^{r+8} + 1 \equiv p^{s-8} \mod |C|$, that can be verified only if $s - r \leq 2$: multiplying through by $p^r$ the congruence $p^r + p^s \equiv 1 \mod |C|$ we deduce $1 + p^{s-r} \equiv p^{s-r} \mod |C|$, again a contradiction since both the sides are less than $|C|$.

$k)$ Suppose $n = 9$. Using arguments quite similar to the previous ones and remarking that $|C| \geq (1 + p + \ldots + p^5)/9 > (1 + p + \ldots + p^8)/9 > p^5 + p^8$ we deduce that, for every $k \in \mathbb{Z}$, one of the numbers $s + k$, $r + k$, $k$, reduced modulo 9, must be $\geq 7$. If $k = 2$ it is $0 \leq s + 2 \leq 1$ and so $r + 2 \geq 7$, i.e. $r \geq 5$. But then, for $k = 4$, we get $0 \leq r + 4 \leq 2$ and $2 \leq s + 4 \leq 3$, a contradiction.

$l)$ Suppose $n = 10$. The argument is again the same: since $|C| \geq (1 + p + \ldots + p^8)/10 > p^8 + p^7$, for every $k \in \mathbb{Z}$ one of the numbers $s + k$, $r + k$, $k$, reduced modulo 10, is $\geq 8$: from this fact, choosing $k = 0$ we deduce $8 \leq s \leq 9$. Therefore, for $k = 2$, being $0 \leq s + 2 \leq 1$, we get $r \geq 6$ and at last, for $k = 4$, we have a contradiction since $0 \leq r + 4 \leq 2$ and $2 \leq s + 4 \leq 3$.

This complete the proof of Proposition 3.1. □

Since if $|H|$ is not divisible by $p$ $|H|$ also is not divisible by $p$, from this proposition we get immediately the proof of Theorem A.
We conclude with the proof of Theorem B: let $\Pi$ be a soluble subgroup of the $L$-automorphism group of $G$ that is transitive on the atoms of the subgroup lattice of $G$ and let $\Pi^*$ be a $p'$-Hall subgroup of $\Pi$: since the set of the atoms has cardinality $(p^n - 1)/(p - 1)$, a number not divisible by $p$, $\Pi^*$ also is transitive on the atoms and so we can apply the previous theorem. □

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