RENDICONTI
del
SEMINARIO MATEMATICO
della
UNIVERSITÀ DI PADOVA

YANG KOK KIM

Groups preserving the cardinality of subsets product under permutations

Rendiconti del Seminario Matematico della Università di Padova, tome 95 (1996), p. 29-36

<http://www.numdam.org/item?id=RSMUP_1996__95__29_0>

REND. SEM. MAT. UNIV. PADOVA, Vol. 95 (1996)

Groups Preserving the Cardinality of Subsets Product under Permutations.

YANG KOK KIM (*)

ABSTRACT - A group G is said to preserve the cardinality of 2-element subsets product under permutations, or G is a $PC(2, n)$-group if either $G = 1$ or for each n-tuple $(S_1, \ldots, S_n)$ of 2-element subsets of G, there is a non-identity permutation σ in Σ_n such that $|S_1 S_2 \ldots S_n| = |S_{\sigma(1)} S_{\sigma(2)} \ldots S_{\sigma(n)}|$, where $|S|$ means the cardinality of a set S. Some characterizations of $PC(2, n)$-groups are presented here.

1. - Introduction.

Recently there has been much interest in the study of groups satisfying «finiteness conditions», for example, groups with various permutability conditions (see, for instance, [1,2] and [3]). A group G is called a PSP-group if there exists an integer $n > 1$ such that for each n-tuple $(H_1, \ldots, H_n)$ of subgroups of G, there is $\sigma(\neq 1) \in \Sigma_n$ such that the two complexes $H_1 H_2 \ldots H_n$ and $H_{\sigma(1)} H_{\sigma(2)} \ldots H_{\sigma(n)}$ are equal. It was shown in [5] that a finitely generated soluble PSP-group is finite-by-abelian. In this note, we consider a similar notion of permutable products, for 2-element subsets of G instead of subgroups of G.

NOTATIONS. For subsets $S, S_1, \ldots, S_n$ of a group G and an element g in G, $S_1 S_2 \ldots S_n = \{s_1 \ldots s_n;\ s_i \in S_i\}$, $S \cdot g = \{sg;\ s \in S\}$ and $g \cdot S = \{gs;\ s \in S\}$. Furthermore $|S|$ means the cardinality of a set S.

DEFINITION. For an integer $n > 1$, a group G is said to preserve the cardinality of 2-element subsets product under permutations, or G is a

(*) Indirizzo dell'A.: Department of Mathematics, Dongeui University, Pusan 614-714, Korea.

$PC(2, n)$-group if either $G = 1$ or for each n-tuple $(S_1, \ldots, S_n)$ of 2-element subsets of G, there is a permutation $\sigma(\neq 1)$ in Σ_n such that

$$(1.1) \qquad |S_1 S_2 \ldots S_n| = |S_{\sigma(1)} S_{\sigma(2)} \ldots S_{\sigma(n)}|.$$

Let $PC(2)$ be the class $\bigcup_{n>1} PC(2, n)$. We give a complete description of $PC(2, 2)$ and $PC(2, 3)$-groups and show that $PC(2)$-groups are center-by-finite exponent. As an immediate corollary, we note that $PC(2)$-groups are collapsing in the following sense. In [8], Semple and Shalev called a group G n-collapsing if for any set S of n-element in G, $|S^n| < n^n$ and G is collapsing if it is n-collapsing for some $n > 0$. They proved that for a finitely generated residually finite group G, it is collapsing if and only if it is nilpotent-by-finite.

As we see in the following remark, it makes sense to fix one side of 1.1.

2. - Remark.

A non-trivial group G has the following property. Let $n \geqslant 3$. For each n-tuple $(S_1, \ldots, S_n)$ of 2-element subsets of G, there exist distinct permutations $\sigma, \tau \in \Sigma_n$ such that the cardinalities of $S_{\sigma(1)} \ldots S_{\sigma(n)}$ and $S_{\tau(1)} \ldots S_{\tau(n)}$ are the same. Note $|S_1 S_2 \ldots S_n| \leqslant 2^n$. If $n \geqslant 4$, then $n! > 2^n$. So the number of permutations is strictly greater than the number of possible cardinalities of all permutable products. Hence there are two distinct permutations with the above property. Suppose $n = 3$. Let S_1, S_2 and S_3 be three given 2-element subsets of G. If $|S_{\sigma(1)} S_{\sigma(2)} S_{\sigma(3)}| \neq 2, 3$ for all $\sigma \in \Sigma_3$, we are already done. So we can assume $|S_{\sigma(1)} S_{\sigma(2)} S_{\sigma(3)}| = 2$ or 3 for some $\sigma \in \Sigma_3$. Write $S_1 = \{x_1, x_1 x\}$, $S_2 = \{y, y_1\}$ and $S_3 = \{z_1, z z_1\}$. Suppose $|S_1 S_2 S_3| = 2$. Then $|S_1 S_2| = |S_2 S_3| = 2$. Now by a simple calculation, we get that $|S_3 S_1 S_2|$ and $|S_2 S_3 S_1|$ are 2 or 4. Assume $|S_1 S_2 S_3| = 3$. Write $S_1' = \{1, x\}$ and $S_3' = \{1, z\}$. If $|S_1 S_2| = |S_1' S_2| = 2$, then we have $y = x y_1$ and $y_1 = xy$. Moreover $S_1' S_2 S_3' = \{y, y_1, yz, y_1 z\}$. Since $|S_1' S_2 S_3'| = 3$, we have $y = y_1 z$ or $y_1 = yz$. Notice that $y = y_1 z \Leftrightarrow x y_1 = y_1 z = xyz \Leftrightarrow y_1 = yz$. Hence $|S_1 S_2 S_3| = 2$, a contradiction. So $|S_1' S_2| = |\{y, y_1, xy, x y_1\}| = 3$. Without loss of generality, we can assume $y = x y_1$. Since $S_1' S_2 S_3' = S_1' S_2 \cup S_1' S_2 \cdot z$, there are two cases to examine.

Case (i). $y = xyz$, $y_1 = yz$ and $xy = y_1 z$.

Then $y = xy \cdot z = y_1 z \cdot z = y z^3$ and $y = xyz = x x y_1 z = x^3 y$. Thus $x^3 = z^3 = 1$. Note that $S_2 S_3 S_1 = S_2 S_3 \cdot x_1 \cup S_2 S_3 \cdot x_1 x$ and $S_2 S_3 =$

$= \{yz_1, yzz_1, yzzz_1\}$. Now suppose $|S_2 S_3 S_1| < 6$. Then at least one element in $S_2 S_3 \cdot x_1$ lies in $S_2 S_3 \cdot x_1 x$. Note that $yzz_1 x_1 = yz_1 x_1 x \Leftrightarrow$ $\Leftrightarrow zz_1 x_1 = z_1 x_1 x \Leftrightarrow yzzz_1 x_1 = yzz_1 x_1 x \Leftrightarrow yz_1 x_1 = yzzzz_1 x_1 = yzzz_1 x_1 x$ and $yzzz_1 x_1 = yz_1 x_1 x \Leftrightarrow yz_1 x_1 = yzz_1 x_1 x \Leftrightarrow yzz_1 x_1 = yzzz_1 x_1 x$. So that one element in $S_2 S_3 \cdot x_1$ lies in $S_2 S_3 \cdot x_1 x$ implies that the other two elements in $S_2 S_3 \cdot x_1$ belong to $S_2 S_3 \cdot x_1 x$. Hence $|S_2 S_3 S_1| = 6$ or 3. Similarly we can show $|S_3 S_1 S_2| = 6$ or 3.

Case (ii). $y = y_1 z$, $y_1 = xyz$ and $xy = yz$.

This case can be checked by the same argument as in case (i).

3. - Results.

Clearly $PC(2)$ contains all finite groups. So for a given n, it seems hard to characterize $PC(2, n)$-group. However in a very particular case, we have a complete result.

LEMMA 3.1. *Let G be a $PC(2, 2)$ or $PC(2, 3)$-group and $x, y \in G$. Then*

(i) *if $x^2 = 1$, then $x \in Z(G)$, the center of G;*

(ii) *if $[x, y] \neq 1$, then $x^y = x^{-1}$.*

PROOF. (i) If x has order 2 and $[x, y] \neq 1$, take $S_1 = \{1, x\}$, $S_2 = \{xy, y\}$ and $S_3 = \{1, y^{-1}xy\}$. Then $|S_1 S_2 S_3| \neq |S_{\sigma(1)} S_{\sigma(2)} S_{\sigma(3)}|$ for all $\sigma(\neq 1) \in \Sigma_3$ and $|S_1 S_2| \neq |S_2 S_1|$.

(ii) Let G be a $PC(2, 3)$-group. For $S_1 = \{1, x\}$, $S_2 = \{y, x^{-1}y\}$ and $S_3 = \{1, y^{-1}xy\}$, there is a non-trivial $\sigma \in \Sigma_3$ such that $|S_1 S_2 S_3| = |S_{\sigma(1)} S_{\sigma(2)} S_{\sigma(3)}|$.

There are five cases to check. We consider one of them (the others are similar). Suppose $|S_1 S_2 S_3| = |S_3 S_1 S_2| \leq 4$. If $|S_1 S_2| = 2$, $x^2 = 1$ and so $x \in Z(G)$, a contradiction. Hence $|S_1 S_2| = |\{y, xy, x^{-1}y\}| = 3$. Note that $S_3 S_1 S_2 = S_1 S_2 \cup y^{-1}xy \cdot S_1 S_2$. So at least two elements in $y^{-1}xy \cdot S_1 S_2$ are in $S_1 S_2$. The non-trivial possible cases are (i) $y = y^{-1}xyxy$, (ii) $xy = y^{-1}xyx^{-1}y$, (iii) $x^{-1}y = y^{-1}xyy$ and (iv) $x^{-1}y = y^{-1}xyxy$. Moreover two of these relations should hold. Note that (i) or (iii) is equivalent to the relation we want. If (ii) and (iv) are true, then $y^{-1}xy = x^{-2} = x^2$. Since x^2 lies in the center of G, $y^{-1}xy = x^2$ gives a contradiction. If G is a $PC(2, 2)$-group, take $S_1 = \{1, x\}$ and $S_2 = \{xy, y\}$. We then get the same result by a simple calculation. ■

THEOREM 3.2. *G is a $PC(2, 2)$ or $PC(2, 3)$-group if and only if either G is abelian or the direct product of a quaternion group of order 8 and an elementary abelian 2-group.*

PROOF. Let G be a $PC(2, 2)$ or $PC(2, 3)$-group. Then by Lemma 3.1(ii), $x^y = x^{\pm 1}$, any x, y in G. So G is a Dedekind group and every element of odd order is in the centre of G. If G is not abelian, then G has no elements of odd order, otherwise, with x, y, z in G, $[x, y] \neq 1$, z of odd order, we get $(xz)^y = x^{-1}z \neq (xz)^{\pm 1}$. Now the result follows from the structure of Dedikind groups (see [6], p. 139).

For the converse, let $G = Q \times D$ where D is an elementary abelian 2-group and Q a quaternion group of order 8. First we show that G is in $PC(2, 3)$. Let A, B and C be three given 2-element subsets of G. Write $A = \{g_1, g_1 ax\}$, $B = \{by, cz\}$ and $C = \{g_2, dwg_2\}$, where $a, b, c, d \in Q$, $x, y, z, w \in D$ and $g_1, g_2 \in G$. Then $|ABC| = |A'BC'|$ and $|CAB| = |C''A'B|$, where $A' = \{1, ax\}$, $C' = \{1, dw\}$ and $C'' = \{1, d^{\varepsilon}w\}$. Note that in C'', $\varepsilon = 1$ if $g_2 g_1$ lies in the centeralizer of d, and $\varepsilon = -1$ if not.

Case (i). $|AB| = 4$.

Since $C' = \{1, dw\}$ and $C'' = \{1, d^{\varepsilon}w\}$, $A'BC' = A'B \cup A'B \cdot dw$ and $C''A'B = A'B \cup d^{\varepsilon}w \cdot A'B$. Note that if there is one element in $A'B \cdot dw$ which is in $A'B$, then there is one element in $d^{\varepsilon}w \cdot A'B$ which is in $A'B$. The converse is also true. For example, suppose that $by = abdxyw$. Then $by = abdxyw = d^{\eta}abxyw \Leftrightarrow by = d^{\varepsilon}abxyw$ if $\varepsilon = \eta$, and $d^{\varepsilon}by = abxyw \Leftrightarrow d^{\varepsilon}byw = abxy$ if not. This means $|A'BC'| = |C''A'B|$ and so $|ABC| = |CAB|$.

Case (ii). $|AB| = 3$.

This case can be checked by the same argument as in case (i).

Case (iii). $|AB| = 2$.

Since $|A'B| = |\{1, ax\}\{by, cz\}| = 2$, we have $b = ac$ and $c = ab$. So $c = ab = aac$ and $a^2 = 1$. Hence A' lies in the center of G. Thus $|A'BC'| = |BC'A'|$. Clearly $|BC'A'| = |BCA|$.

Similar argument can be applied to show that G is in $PC(2, 2)$. ■

THEOREM 3.3. *A $PC(2, n)$-group is center-by-(finite exponent $f(n)$).*

PROOF. We claim that there exists an integer k such that $[y^k, x] = 1$ for all $x, y \in G$. Let $x, y \in G$. We consider the n-tuple $(S_1, \dots, S_n)$ of 2-element subsets of G where $S_i = \{y, y^{1-i}xy^i\}$. Then $S_1 S_2 \dots S_n =$

$= \{y^n, xy^n, x^2y^n, \ldots, x^ny^n\}$, and $|S_1S_2 \ldots S_n| = \min(|x|, n+1)$. Since G is a $PC(2, n)$-group, there is a permutation $\sigma(\neq 1) \in \Sigma_n$ such that $|S_1S_2 \ldots S_n| = |S_{\sigma(1)}S_{\sigma(2)} \ldots S_{\sigma(n)}|$. Write $g(i, j) = S_{\sigma(i)}S_{\sigma(i+1)} \ldots S_{\sigma(j)}$ for $i \leqslant j$.

If $|g(n-i, l)|$ and $|g(l, j)|$ are strictly increasing functions of i, j for all l, then for an integer j such that $\sigma(j) + 1 \neq \sigma(j+1)$, $|S_{\sigma(j)}S_{\sigma(j+1)}| < 4$. Here $S_{\sigma(j)} = \{y, y^{1-\sigma(j)}xy^{\sigma(j)}\}$ and $S_{\sigma(j+1)} = \{y, y^{1-\sigma(j+1)}xy^{\sigma(j+1)}\}$. So we have a relation $x = x^{y^s}$ where $s(\neq 0)$ depends on σ and so on x, y. However note that there are only finitely many choices of s independent of x, y, say, $s_1, \ldots, s_m$. Let $k = \text{l.c.m.}\{s_i \colon i = 1, \ldots, m\}$. Then $[x, y^k] = 1$ for all x, y.

Suppose that $|g(n-i, l)|$ or $|g(l, j)|$ is not strictly increasing.

Case (i). $|x| > n + 1$.

Let $|g(l, j)| = |g(l, j+1)|$. Then $g(l, j+1) = g(l, j) \cdot y \cup g(l, j)x^{y^{\sigma(j+1)-1}} \cdot y$. So $g(l, j) = g(l, j)x^{y^r}$, where $r = \sigma(j+1) - 1$ and $y^{j-l+1}(x^{y^r})^h \in g(p, q)$, for any h. Since $|g(l, j)| \leqslant n + 1$, $|x| \leqslant n + 1$. This is a contradiction.

Case (ii). $|x| \leqslant n + 1$.

For $S_{\sigma(1)}S_{\sigma(2)} \ldots S_{\sigma(n)}$, let j be an integer such that $\sigma(j) + 1 \neq \sigma(j+1)$. Now we can assume that $|S_{\sigma(j)}S_{\sigma(j+1)}| = 4$. Then since $|S_1S_2 \ldots S_n| = |x|$, we can find p, q with $p \leqslant j < j + 1 \leqslant q$ such that $|g(p, q)| = |g(p, q+1)|$ or $|g(p-1, q)| = |g(p, q)|$. Let $|g(p, q)| = |g(p, q+1)|$ (the other case is similar). Then we have a relation $g(p, q) = g(p, q)x^{y^r}$, where $r = \sigma(q+1) - 1$. So $g(p, q) = g(p, q)(x^{y^r})^h$ for any h, and $g(p, q) = \{y^m, y^m(x^{y^r}), y^m(x^{y^r})^2, \ldots, y^m(x^{y^r})^{|x|-1}\}$, where $m = q - p + 1$. Thus for some integer t, we have relation $x^{y^{\sigma(j)-t}} = (x^{y^r})^a$ or $x^{y^{\sigma(j+1)-1-t}} = (x^{y^r})^b$ where $2 \leqslant a, b < |x|$. In any case we have $x^{y^s} = x^d$ for some $2 \leqslant d < |x|$. Since $|x| \leqslant n + 1$, $[y^k, x] = 1$ for some k. In every case our s and k depend on x, y. However there are still only finitely many choices of s and k that are independent of x, y. This completes the proof. ■

A group G is restrained if there is an integer n such that $\langle x \rangle^{\langle y \rangle}$ is generated by n elements for all $x, y \in G$. In [4], the following is proved.

LEMMA 3.4. *Let G be a finitely generated restrained group. If H is a normal subgroup of G such that G/H is cyclic, then H is finitely generated.*

Proof. For some $g \in G$, we can write G in the form $H\langle g\rangle$. Since G is finitely generated, there exist $h_1, h_2, \ldots, h_r$ in H such that $G = \langle h_1, h_2, \ldots, h_r, g\rangle$ and $H = \langle h_1, h_2, \ldots, h_r\rangle^G$. For each $i = 1, \ldots, r$, $\langle h_i^{\langle g\rangle}\rangle$ is finitely generated, say, $\langle h_i^{\langle g\rangle}\rangle = \langle h_{i1}, h_{i2}, \ldots, h_{id(i)}\rangle$. Now let $H_1 = \langle h_{il(i)};\ 1 \leq i \leq r,\ 1 \leq l(i) \leq d(i)\rangle$. Then clearly g lies in $N_G(H_1)$, the normalizer of H_1 in G and $\langle h_1, \ldots, h_r\rangle \leq H_1$. Hence $N_G(H_1) = G$. This means that $H_1 = H$ and H is finitely generated. ■

Now we mention some properties of $PC(2)$ as immediate consequences of Theorem 3.3. For closure properties, we follow notations in [7]. Consider the restricted direct product $G = \mathrm{Dr}\, A_n$, where A_n is the alternating group of degree $n > 4$. Then G is locally finite but has no center. Clearly the standard wreath product of two infinite cyclic groups is not center-by-finite exponent. Neither is a free product of two infinite cyclic groups.

Corollary 3.5. (i) *A $PC(2)$-group is collapsing.*

(ii) *A $PC(2)$-group is restrained.*

(iii) *The class of $PC(2)$-groups is not closed under any of the closure operations P, D, C, W, F, R, L.* ■

Questions. (i) For $G, H \in PC(2)$, is $G \times H$ in $PC(2)$?

(ii) Is $PC(2)$ quotient-closed?

Corollary 3.6. *A finitely generated soluble $PC(2)$-group G is center-by-finite.*

Proof. By Theorem 3.3, G is center-by-(finite exponent). And a finitely generated soluble group with finite exponent is finite. ■

Locally graded groups are those groups in which every finitely generated non-trivial subgroup has a finite non-trivial quotient.

Theorem 3.7. *If G is a finitely generated locally graded $PC(2)$-group, then G is center-by-finite.*

Proof. Let N be the finite residual of G. By Theorem 3.3 G is center-by-(finite exponent). Thus G/N is a finitely generated residually finite center-by-(finite exponent). It was shown in [11] that a finitely generated residually finite group of finite exponent is finite. Hence G/N is center-by-finite. G is restrained and so N is finitely generated by repeated applications of Lemma 3.4. Let $N \neq 1$. Since G is locally graded, N has a non-trivial finite factor group N/K. But then

$N/\mathrm{core}_G(K)$ is finite and $G/\mathrm{core}_G(K)$ is finite-by-(center-by-finite). This group is polycyclic-by-finite and so it is residually finite, contrary to the choice of N. ■

An element g of a group G is called an FC-element if it has only a finite number of conjugates in G. In particular if there is a positive integer m such that no element of G has more than m conjugates, then G is called a BFC-group. The subgroup of all FC-elements is called the FC-center.

THEOREM 3.8. *A finitely generated non-periodic PC(2)-group G is center-by-finite.*

PROOF. Let $G = \langle x_1, x_2, \ldots, x_r \rangle$ be a $PC(2, n)$-group and let z be an element of infinite order in $Z(G)$, the center of G. For $w \in G$, let Ny be a right coset of N, the normalizer of $\langle x \rangle$ where $x = wz$ if w has finite order, and $x = w$ if not. Suppose that y is reduced and $l(y) = m \geq n$, where $l(y)$ denotes the length of the shortest word for y. Write $S = \{x_i^{\pm 1}: i = 1, \ldots, r\}$ and $y = y_1 y_2 \ldots y_m$ where $y_i \in S$. Now we consider an n-tuple $(S_1, \ldots, S_n)$ of 2-element subsets of G where $S_i = \{y_i, x^{\pi_{i-1}} y_i\}$, $\pi_0 = 1$, $\pi_j = y_1 y_2 \ldots y_j$. Since G is a $PC(2, n)$-group, there is $\sigma (\neq 1) \in \Sigma_n$ such that $|S_1 S_2 \ldots S_n| = |S_{\sigma(1)} S_{\sigma(2)} \ldots S_{\sigma(n)}|$. Write $g(i, j) = S_{\sigma(i)} S_{\sigma(i+1)} \ldots S_{\sigma(j)}$ for $i \leq j$. Since x is of infinite order, $|g(n-i, l)|$ and $|g(l, j)|$ are strictly increasing functions of i, j for all l. Let j be an integer for which $\sigma(j) + 1 \neq \sigma(j+1)$. Note that $S_1 S_2 \ldots S_n = \{y_1 y_2 \cdots y_n, x y_1 y_2 \cdots y_n, x^2 y_1 y_2 \cdots y_n, \ldots, x^n y_1 y_2 \cdots y_n\}$, and $|S_1 S_2 \ldots S_n| = n + 1$. Hence $|S_{\sigma(j)} S_{\sigma(j+1)}| < 4$. Since $S_{\sigma(j)} = \{y_{\sigma(j)}, x^{\pi_{\sigma(j)-1}} y_{\sigma(j)}\}$ and $S_{\sigma(j+1)} = \{y_{\sigma(j+1)}, x^{\pi_{\sigma(j+1)-1}} y_{\sigma(j+1)}\}$, we get $x^{\pi_{\sigma(j)}} = x^{\pi_{\sigma(j+1)-1}}$, or $(x^{-1})^{\pi_{\sigma(j)}} = x^{\pi_{\sigma(j+1)-1}}$. Hence $\pi_{\sigma(j)} \pi_{\sigma(j+1)-1}^{-1}$ lies in N. So $N\pi_{\sigma(j)} = N\pi_{\sigma(j+1)-1}$. By the repeated applications of the above argument, we can assume that $Ny = Ny'$, where $l(y') < n$. Hence N has finite index in G and so does $C(wz) = C(w)$. In fact there is an integer m such that $|G : C(w)| < m$ for all $w \in G$. Hence G is a BFC-group. Since G is finitely generated, it is center-by-finite. ■

COROLLARY 3.9. A torsion-free $PC(2)$-group is abelian. ■

Acknowledgement. I would like to thank Dr. Chun for calling my attention to this problem, and Dr. Rhemtulla for helpful conversations.

REFERENCES

[1] R. D. BLYTH, *Rewriting products of group elements I*, J. Algebra, **116** (1988), pp. 506-521.
[2] R. D. BLYTH - A. H. RHEMTULLA, *Rewritable products in FC-by-finite groups*, Canad. J. Math., **41**, No. 2 (1989), pp. 369-384.
[3] M. CURZIO - P. LONGOBARDI - M. MAJ - D. J. S. ROBINSON, *A permutational property of groups*, Arch. Math. (Basel), **44** (1985), pp. 385-389.
[4] Y. K. KIM - A. H. RHEMTULLA, *Weak maximality condition and polycyclic groups*, in *Proceedings of the American Mathematical Society*, Vol. 123, No. 3, March 1995, pp. 711-714.
[5] A. H. RHEMTULLA - A. R. WEISS, *Groups with permutable subgroup products*, in *Group Theory: Proceedings of the 1987 Singapore Conference*, edited by K. N. Cheng and Y.K. Leong, de Gruyter (1989), pp. 485-495.
[6] D. J. S. ROBINSON, *A Course in the Theory of Groups*, Springer-Verlag, New York (1980).
[7] D. J. S. ROBINSON, *Finiteness Conditions and Generalized Soluble Groups*, Part I, Springer-Verlag, New York (1972).
[8] J. F. SEMPLE - A. SHALEV, *Combinatorial conditions in residually finite groups I*, J. Algebra, **157** (1993), pp. 43-50.
[9] M. J. TOMKINSON, *FC-groups*, Research Notes in Mathematics, **96**, Pitman Advanced Publishing Program, Boston (1984).
[10] J. S. WILSON, *Two-generator conditions for residually finite groups*, Bull. London Math. Soc., **23** (1991), pp. 239-248.
[11] E. I. ZELMANOV, *On some problems of group theory and Lie algebras*, Math. USSR Sbornik, **66** (1990), No. 1, pp. 159-168.

Manoscritto pervenuto in redazione il 15 settembre 1994
e, in forma rivisionata, il 5 dicembre 1994.