

SÉMINAIRE D'ANALYSE FONCTIONNELLE ÉCOLE POLYTECHNIQUE

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Séminaire d'analyse fonctionnelle (Polytechnique) (1978-1979), exp. n° 21, p. 1-13

<http://www.numdam.org/item?id=SAF_1978-1979___A18_0>

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S E M I N A I R E
D ' A N A L Y S E F O N C T I O N N E L L E
1978-1979

A DISJOINTNESS PROPERTY OF ℓ_p^n SEQUENCES IN L_p

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In [1] L. E. Dor proved that a subspace of $L_1(0, 1)$ which is almost isometric to a $L_1(\mu)$ space is well complemented. The purpose of this note is to prove the analogous theorem for $1 < p < \infty$ thus solving a problem of Enflo and Rosenthal [2] and of Dor [1]. Since a detailed proof will appear shortly in [3], I'll try to give here a less formal and, hopefully, more intuitive proof.

Theorem 1: let $1 < p < \infty$. There exist a $\lambda_0 > 1$ and a function $\varphi(\lambda)$, defined for $1 < \lambda < \lambda_0$, such that $\varphi(\lambda) \rightarrow 1^+$ as $\lambda \rightarrow 1^+$ and if x_1, \dots, x_n are functions in $L_p(0, 1)$ which satisfy

$$\lambda^{-1} \left(\sum_{i=1}^n |a_i|^p \right)^{1/p} \leq \left\| \sum_{i=1}^n a_i x_i \right\| \leq \lambda \left(\sum_{i=1}^n |a_i|^p \right)^{1/p}$$

for all sequences a_1, \dots, a_n of scalars then $[x_i]_{i=1}^n$ is complemented in $L_p(0, 1)$ by means of a projection of norm at most $\varphi(\lambda)$.

It is well known that this implies that any $\mathfrak{L}_{p, \lambda}$ subspace of $L_p(0, 1)$ is complemented if λ is small enough (and the norm of the projection tends to 1 as $\lambda \rightarrow 1$). Also, a simple perturbation argument shows that Theorem 1 is a consequence of

Theorem 2: Let $1 < p < \infty$, $p \neq 2$. There exists a function $a(\epsilon)$ such that $a(\epsilon) \rightarrow 0$

as $\epsilon \rightarrow 0$ and, if x_1, \dots, x_n are functions in $L_p(0,1)$ which satisfy

$$(1-\epsilon) \left(\sum_{i=1}^n |a_i|^p \right)^{1/p} \leq \left\| \sum_{i=1}^n a_i x_i \right\| \leq (1+\epsilon) \left(\sum_{i=1}^n |a_i|^p \right)^{1/p}$$

for all scalars a_1, \dots, a_n , then there exist disjoint sets A_1, \dots, A_n of $[0,1]$ such that

$$\left\| \sum_{i=1}^n a_i (x_i - x_i|_{A_i}) \right\| \leq a(\epsilon) \left(\sum_{i=1}^n |a_i|^p \right)^{1/p}$$

for all scalars a_1, \dots, a_n .

Indeed, if Theorem 2 is true let P be a norm one projection from $L_p(0,1)$ onto $[x_i|_{A_i}]_{i=1}^n$. The conclusion of Theorem 2 ensures that $P|_{[x_i]_{i=1}^\infty}$ is an isomorphism provided ϵ is small enough (and $\|(P|_{[x_i]_{i=1}^n})^{-1}\| \rightarrow 1$ as $\epsilon \rightarrow 0$). So the desired projection is given by $(P|_{[x_i]_{i=1}^n})^{-1} P$.

Theorem 2 is a stronger version of the following theorem of Dor [1] :

Let $1 \leq p < \infty$, $p \neq 2$. There exists a function $a(\epsilon)$ such that $a(\epsilon) \rightarrow 0$ as $\epsilon \rightarrow 0$ and if x_1, \dots, x_n is a normalized sequence in $L_p(0,1)$ such that

$$(1-\epsilon) \left(\sum_{i=1}^n |a_i|^p \right)^{1/p} \leq \left\| \sum_{i=1}^n a_i x_i \right\| \leq (1+\epsilon) \left(\sum_{i=1}^n |a_i|^p \right)^{1/p}$$

for all scalars a_1, \dots, a_n , then there are disjoint sets A_1, \dots, A_n
of $[0,1]$ such that

$$\|x_i|_{A_i^c}\| \leq a(\epsilon), \quad i = 1, \dots, n. \quad (A_i^c \text{ is the complement of } A_i).$$

The case $p = 1$ easily implies the analogue of Theorem 2 for $p = 1$. As we we'll see the proof of Theorem 2 for $1 < p < \infty$ is also based on Dor's theorem, however the deduction of Theorem 2 from Dor's theorem is much more complicated in this case.

We divide the proof into three parts. The first one is a reduction of the general case to the case where the x_i are exchangeable.

Fix n and $\epsilon > 0$. Given normalized x_1, \dots, x_n which satisfy the assumption of Theorem 2 we find disjoint sets $\{A_i\}_{i=1}^n$ as in Dor's theorem.

Let Π denote the set of all permutations of $(1, \dots, n)$, let $\{I_\pi\}_{\pi \in \Pi}$ be a collection of disjoint subintervals of $[0,1]$ each of length $1/n!$ and for $\pi \in \Pi$, let φ_π be the natural linear transformation of I_π onto $[0,1]$.

Fix $\{a_i\}_{i=1}^\infty$ such that $(\sum_{i=1}^n |a_i|^p)^{1/p} = n^{1/p}$ and define a sequence $\{f_i\}_{i=1}^n$

of $L_p(0,1)$ functions by:

$$f_i(t) = a_{\pi(i)} x_{\pi(i)}(\varphi_\pi(t)) \quad \text{for } 1 \leq i \leq n, \pi \in \Pi \text{ and } t \in I_\pi$$

in a similar manner we define $\{g_i\}_{i=1}^n$ and $\{h_i\}_{i=1}^n$ using $x_i|_{A_i}$ and $x_i - x_i|_{A_i}$, respectively, instead of x_i .

Lemma 1: (a)

$$(1-\epsilon)\left(\sum_{i=1}^n |b_i|^p\right)^{1/p} \leq \left\| \sum_{i=1}^n b_i f_i \right\| \leq (1+\epsilon)\left(\sum_{i=1}^n |b_i|^p\right)^{1/p}$$

for every sequence b_1, \dots, b_n of scalars.

(b) $f_i = h_i + g_i$, g_i and h_i are disjointly supported for each $i=1, \dots, n$. g_1, \dots, g_n are disjointly supported.

(c) $\|f_i\| = 1$, $\|h_i\| < a(\epsilon)$ and $\|g_i\| > (1-a(\epsilon)^p)^{1/p}$, $i = 1, \dots, n$.

(d) $\left\| \sum_{i=1}^n h_i \right\| = \left\| \sum_{i=1}^n a_i z_i \right\|$

(e) $\{(g_i, h_i)\}_{i=1}^n$ is an exchangeable sequence; i.e., the distribution of the sequence

$$(g_1, h_1, g_2, h_2, \dots, g_n, h_n)$$

is the same as the distribution of

$$(g_{\pi(1)}, h_{\pi(1)}, g_{\pi(2)}, h_{\pi(2)}, \dots, g_{\pi(n)}, h_{\pi(n)})$$

for any $\pi \in \Pi$.

The proof is very simple, we'll prove only (d) and (e),

$$\begin{aligned}
 \left\| \sum_{i=1}^n h_i \right\| &= \left(\sum_{\pi \in \Pi} \int_0^1 \left| \sum_{i=1}^n a_{\pi(i)} z_{\pi(i)} (\varphi_{\pi}(t)) \right|^p dt \right)^{1/p} \\
 &= \left(\frac{1}{n!} \sum_{\pi \in \Pi} \int_0^1 \left| \sum_{i=1}^n a_{\pi(i)} z_{\pi(i)} \right|^p dt \right)^{1/p} \\
 &= \left(\frac{1}{n!} \sum_{\pi \in \Pi} \int_0^1 \left| \sum_{i=1}^n a_i z_i \right|^p dt \right)^{1/p} \\
 &= \left\| \sum_{i=1}^n a_i z_i \right\|
 \end{aligned}$$

To prove (e) we notice that for any $\pi, \rho \in \Pi$

$$\text{dist} \left\{ (g_i, h_i) \Big|_{\mathbb{I}_{\pi}} \right\}_{i=1}^n = \text{dist} \left\{ (g_{\rho(i)}, h_{\rho(i)}) \Big|_{\mathbb{I}_{\pi \rho^{-1}}} \right\}_{i=1}^n$$

since both are equal to the distribution of

$$\left\{ (a_{\pi(i)} y_{\pi(i)}, a_{\pi(i)} z_{\pi(i)}) \right\}_{i=1}^n$$

with respect to $\frac{1}{n!}$ Lebesgue measure. □

Lemma 1 reduces the proof of Theorem 2 to showing

$$\left\| \sum_{i=1}^n h_i \right\|^p \leq b(\epsilon) \cdot n$$

for some function $b(\epsilon)$, depending on ϵ and p alone, such that $b(\epsilon) \rightarrow 0$ as $\epsilon \rightarrow 0$.

The second step in the proof consists of the following inequality

Lemma 2: In the situation above

$$\left| \int_0^1 \left| \sum_{j=1}^n g_j + \sum_{k=1}^{\ell} h_k \right|^p - \frac{n}{\ell} \int_0^1 \left| \sum_{i=1}^{\ell} f_i \right|^p + \frac{n-\ell}{\ell} \int_0^1 \left| \sum_{k=1}^{\ell} h_k \right|^p \right| \leq \frac{n^2}{\ell} c(\epsilon)$$

for all $\ell = 1, \dots, n$ where $c(\epsilon)$ depends on p and ϵ alone and

$c(\epsilon) \rightarrow 0$ as $\epsilon \rightarrow 0$.

We postpone the proof and continue with the third step which is the deduction of Theorem 2 from Lemma 2. As we mentioned above we'll give a heuristic proof which we hope will give the idea behind the proof. A complete formal proof, which however looks quite mysterious, can be found in [3].

The first object is to show that any two partial sums of the h_i with the same number of terms are closed each to the other.

Lemma 3: Let M_1, M_2 be two subsets of $\{1, \dots, n\}$ of the same cardinality
then

$$\left\| \sum_{k \in M_1} h_k - \sum_{k \in M_2} h_k \right\|^p \leq d(\epsilon) \cdot n$$

where $d(\epsilon) \rightarrow 0$ as $\epsilon \rightarrow 0$ and depends on ϵ and p alone.

Proof: First notice that it is enough to prove the lemma for $M_1 \cap M_2 = \emptyset$, and then it is enough to consider $M_1 = \{1, \dots, \ell\}$, $M_2 = \{\ell+1, \dots, 2\ell\}$ for some $1 \leq \ell \leq \frac{n}{2}$. Now since $\{h_i\}_{i=1}^n$ is exchangeable $h_1 - h_{\ell+1}, h_2 - h_{\ell+2}, \dots, h_{\ell} - h_{2\ell}$ is a 1-unconditional basic sequence so

if $1 < p < 2$

$$\left\| \sum_{i=1}^{\ell} h_i - \sum_{i=\ell+1}^{2\ell} h_i \right\| \leq \|h_i - h_{\ell+1}\| \cdot \ell^{1/p} \leq 2\epsilon n^{1/p}.$$

If $p > 2$ the proof is more involved:

First notice that by Khinchine's inequality,

$$(1) \quad \left\| \sum_{i=1}^{\ell} h_i - \sum_{i=\ell+1}^{2\ell} h_i \right\| \leq K_p \left\| \left(\sum_{i=1}^{\ell} |h_i - h_{\ell+1}|^2 \right)^{1/2} \right\| \leq K_p^{2^{1/2}} \left\| \left(\sum_{i=1}^{2\ell} |h_i|^2 \right)^{1/2} \right\|$$

for some constant K_p depending only on p .

Now, let r_1, \dots, r_n be the first n Rademacher functions, then

$$\begin{aligned} 2\ell(1+\epsilon)^p &\geq \int_0^1 \int_0^1 \left| \sum_{i=1}^{2\ell} r_i(t) f_i(s) \right|^p dt ds \geq \int_0^1 \int_0^1 \left| \sum_{i=1}^{2\ell} r_i(t) f_i(s) \right|^2 dt \Big)^{p/2} ds \\ &= \int_0^1 \left(\sum_{i=1}^{2\ell} |f_i(s)|^2 \right)^{p/2} ds = \int_0^1 \left(\sum_{i=1}^{2\ell} |g_i(s)|^2 + \sum_{i=1}^{2\ell} |h_i(s)|^2 \right)^{p/2} ds \\ &\geq \int_0^1 \left(\sum_{i=1}^{2\ell} |g_i(s)|^2 \right)^{p/2} ds + \int_0^1 \left(\sum_{i=1}^{2\ell} |h_i(s)|^2 \right)^{p/2} ds \\ &= \int_0^1 \left| \sum_{i=1}^{2\ell} g_i(s) \right|^p ds + \int_0^1 \left(\sum_{i=1}^{2\ell} |h_i(s)|^2 \right)^{p/2} ds \\ &\geq (1-\epsilon^p) 2\ell + \left\| \left(\sum_{i=1}^{2\ell} |h_i|^2 \right)^{1/2} \right\|^p \end{aligned}$$

which together with (1) finishes the proof.

We are going to use Lemma 2 only for $\ell = n/2$ and $\ell = n/4$ (assuming

for simplicity that n is divisible by 4), for these values $\frac{n^2}{\ell} < 4 \cdot n$

also, using Lemma 3, we can write the conclusion of Lemma 2 as

$$(2) \quad \int_0^1 \left| \sum_{j=1}^n g_j + \frac{\ell}{n} \sum_{k=1}^n h_k \right|^p + \frac{n-\ell}{\ell} \int_0^1 \left| \frac{\ell}{n} \sum_{k=1}^n h_k \right|^p - n \leq n \cdot d(\epsilon)$$

$\ell = \frac{n}{2}$ or $\ell = \frac{n}{4}$, $d(\epsilon) \rightarrow 0$ as $\epsilon \rightarrow 0$ and depends on ϵ and p alone.

$$\text{Put} \quad g = \sum_{i=1}^n g_i, \quad h = \sum_{i=1}^n h_i.$$

From Clarkson's inequality we get, for $1 < p < 2$,

$$\|g + \frac{1}{2}h\|^p + \|\frac{1}{2}h\|^p \leq \frac{1}{2}(\|g + h\|^p + \|g\|^p) \leq (1 + \frac{\epsilon}{2})n$$

so (2) with $\ell = \frac{n}{2}$ says that we have an almost equality in Clarkson's inequality. The same thing holds for $p > 2$. Recall that equality in Clarkson's inequality holds if and only if the two functions are disjoint. This suggests that $g + \frac{1}{2}h$ and $\frac{1}{2}h$ are almost disjoint, that is, there exist two disjoint sets A and B such that $A \cup B = [0, 1]$ and

$$(3) \quad \|h|_A\|^p < e(\epsilon) \cdot n, \quad \|(g + \frac{1}{2}h)|_B\|^p < e(\epsilon) \cdot n$$

for some $e(\epsilon)$ with the same properties as the previous functions. (This can be proved using the proof of Proposition 2.1 in [1]).

Using (3) we can write (2) for $\ell = \frac{n}{4}$ in the form (\approx means that the difference between the two sides is of the form $e(\epsilon) \cdot n$ for an appropriate $e(\epsilon)$).

$$n \approx \int_0^1 |g + \frac{1}{4}h|^p + 3 \int_0^1 |\frac{1}{4}h|^p$$

$$\begin{aligned}
&\approx \int_A |g|^p + \int_B |g + \frac{1}{4}h|^p + 3 \int_B |\frac{1}{4}h|^p \\
&\approx \int_A |g|^p + \int_B |\frac{1}{2}g|^p + (2^p - 1) \int_B |\frac{1}{2}g|^p + (4-2^p) \int_B |\frac{1}{4}h|^p \\
&\approx \int_A |g|^p + \int_B |g|^p + \frac{4-2^p}{4^p} \int_0^1 |h|^p \\
&\approx n + \frac{4-2^p}{4^p} \|h\|^p
\end{aligned}$$

and this means that

$$\|h\|^p \leq b(\epsilon) \cdot n$$

We return now to the proof of Lemma 2. We first need another lemma.

Denote the support of g_i by B_i , $i = 1, \dots, n$. By (e) of Lemma 1, whenever M_1 and M_2 are two subsets of $\{1, \dots, n\}$ of the same cardinality and $1 \leq i, j \leq n$ satisfy either $i \in M_1$ and $j \in M_2$ or $i \notin M_1$ and $j \notin M_2$

$$\int_{B_i} |\sum_{k \in M_1} h_k|^p = \int_{B_j} |\sum_{k \in M_2} h_k|^p$$

and

$$\int_{B_i} |g_i + \sum_{k \in M_1} h_k|^p = \int_{B_j} |g_j + \sum_{k \in M_2} h_k|^p.$$

Indeed in each of these two cases there exists $\pi \in \Pi$ such that $\pi(M_1) = M_2$ and $\pi(i) = j$, so,

$$\text{dist}(g_i, \sum_{k \in M_1} h_k) = \text{dist}(g_j, \sum_{k \in M_1} h_{\pi(k)}) = \text{dist}(g_j, \sum_{k \in M_2} h_k).$$

The next lemma asserts that, up to a certain error, the same is true without any restrictions on i and j .

Lemma 4: Let M_1, M_2 be subsets of $\{1, \dots, n\}$ with $\text{card } M_1 = \text{card } M_2$ and let i, j satisfy $1 \leq i, j \leq n$ then

$$(a) \quad \left| \int_{B_i} \left| \sum_{k \in M_1} h_k \right|^p - \int_{B_i} \left| \sum_{k \in M_2} h_k \right|^p \right| < c(\epsilon)$$

$$(b) \quad \left| \int_{B_i} \left| g_i + \sum_{k \in M_1} h_k \right|^p - \int_{B_j} \left| g_j + \sum_{k \in M_2} h_k \right|^p \right| < c(\epsilon)$$

for some function $c(\epsilon)$ depending on p and ϵ alone and such that
 $c(\epsilon) \rightarrow 0$ as $\epsilon \rightarrow 0$.

Proof: By the remark before the statement of the lemma, it is enough to assume that $M_1 = M_2 = \{1, \dots, \ell\}$ for some $1 \leq \ell < n$ and that $1 \leq i \leq \ell < j \leq n$. First notice that, since

$$\text{dist}(g_i, \sum_{k=1}^{\ell} h_k) = \text{dist}(g_r, \sum_{k=1}^{\ell} h_k) \text{ for all } 1 \leq r \leq \ell,$$

$$(4) \quad \left(\int_{B_i} \left| \sum_{k=1}^{\ell} h_k \right|^p \right)^{1/p} = \left(\frac{1}{\ell} \sum_{r=1}^{\ell} \int_{B_r} \left| \sum_{k=1}^{\ell} h_k \right|^p \right)^{1/p} \leq \frac{1}{\ell^{1/p}} \left\| \sum_{k=1}^{\ell} h_k \right\|$$

$$\leq \frac{1}{\ell^{1/p}} \left(\left\| \sum_{k=1}^{\ell} f_k \right\| + \left\| \sum_{k=1}^{\ell} g_k \right\| \right) \leq 2 + \epsilon$$

Now, since

$$\text{dist}(g_j, \sum_{k=1}^{\ell} h_k) = \text{dist}(g_i, \sum_{\substack{k=1 \\ k \neq i}}^{\ell+1} h_k),$$

and since g_i and h_i are disjointly supported,

$$\begin{aligned} \left(\int_{B_j} \left| \sum_{k=1}^{\ell} h_k \right|^p \right)^{1/p} &= \left(\int_{B_i} \left| \sum_{\substack{k=1 \\ k \neq i}}^{\ell+1} h_k \right|^p \right)^{1/p} = \left(\int_{B_i} \left| \sum_{k=1}^{\ell+1} h_k \right|^p \right)^{1/p} \\ &\leq \left(\int_{B_i} \left| \sum_{k=1}^{\ell} h_k \right|^p \right)^{1/p} + \epsilon \end{aligned}$$

and similarly

$$\left(\int_{B_j} \left| \sum_{k=1}^{\ell} h_k \right|^p \right)^{1/p} \geq \left(\int_{B_i} \left| \sum_{k=1}^{\ell} h_k \right|^p \right)^{1/p} - \epsilon$$

so that, by the mean value theorem and (1), we get (a) with

$c(\epsilon) = p(2+2\epsilon)^{p-1}\epsilon$. (b) is proved in a similar way, noting that

$$\left(\int_{B_i} \left| g_i + \sum_{k=1}^{\ell} h_k \right|^p \right)^{1/p} \leq \|g_i\| + 2 + \epsilon \leq 3 + \epsilon.$$

Proof of Lemma 2: By Lemma 4(b) for each i and j ,

$$\int_{B_j} \left| g_j + \sum_{k=1}^{\ell} h_k \right|^p \leq \int_{B_i} \left| g_i + \sum_{k=1}^{\ell} h_k \right|^p + c(\epsilon)$$

Summing over j we get that for every i

$$\begin{aligned} \int_0^1 \left| \sum_{j=1}^n g_j + \sum_{k=1}^{\ell} h_k \right|^p &= \sum_{j=1}^n \int_{B_j} \left| g_j + \sum_{k=1}^{\ell} h_k \right|^p \leq \\ &\leq n \int_{B_i} \left| g_i + \sum_{k=1}^{\ell} h_k \right|^p + n \cdot c(\epsilon) \end{aligned}$$

summing over $1 \leq i \leq \ell$ and dividing by ℓ we get

$$(5) \quad \int_0^1 \left| \sum_{j=1}^n g_j + \sum_{k=1}^{\ell} h_k \right|^p \leq \frac{n}{\ell} \sum_{i=1}^{\ell} \int_{B_i} \left| g_i + \sum_{k=1}^{\ell} h_k \right|^p + n \cdot c(\epsilon)$$

$$\begin{aligned}
&= \frac{n}{\ell} \int_{\bigcup_{i=1}^{\ell} B_i} \left| \sum_{i=1}^{\ell} g_i + \sum_{k=1}^{\ell} h_k \right|^p + n c(\epsilon). \\
&= \frac{n}{\ell} \int_0^1 \left| \sum_{i=1}^{\ell} f_i \right|^p - \frac{n}{\ell} \int_{\bigcup_{i=\ell+1}^n B_i} \left| \sum_{k=1}^{\ell} h_k \right|^p + n c(\epsilon).
\end{aligned}$$

By Lemma 4(a), for every i and j ,

$$\int_{B_i} \left| \sum_{k=1}^{\ell} h_k \right|^p \geq \int_{B_j} \left| \sum_{k=1}^{\ell} h_k \right|^p - c(\epsilon)$$

so, for every j ,

$$\int_{\bigcup_{i=\ell+1}^n B_i} \left| \sum_{k=1}^{\ell} h_k \right|^p \geq (n-\ell) \int_{B_j} \left| \sum_{k=1}^{\ell} h_k \right|^p - (n-\ell) c(\epsilon),$$

summing over $1 \leq j \leq n$ we get

$$(6) \quad \int_{\bigcup_{i=\ell+1}^n B_i} \left| \sum_{k=1}^{\ell} h_k \right|^p \geq \frac{n-\ell}{n} \int_0^1 \left| \sum_{k=1}^{\ell} h_k \right|^p - (n-\ell) c(\epsilon)$$

combining (5) and (6) we get

$$\int_0^1 \left| \sum_{j=1}^{\ell} g_j + \sum_{k=1}^{\ell} h_k \right|^p \leq \frac{n}{\ell} \int_0^1 \left| \sum_{i=1}^{\ell} f_i \right|^p - \frac{n-\ell}{\ell} \int_0^1 \left| \sum_{k=1}^{\ell} h_k \right|^p + \frac{n^2}{\ell} c(\epsilon).$$

The otherside inequality is proved similarly.

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