Séminaire Dubreil.
Algèbre et théorie
des nombres

Blanka Kolibiarová

# On the construction of some semigroups

*Séminaire Dubreil. Algèbre et théorie des nombres*, tome 23, nº 2 (1969-1970), exp. nº DG 12, p. DG1-DG2

<http://www.numdam.org/item?id=SD_1969-1970__23_2_A11_0>

Séminaire DUBREIL-PISOT
(Algèbre et Théorie des nombres)
23e année, 1969/70
n° 12, 2 p.



# ON THE CONSTRUCTION OF SOME SEMIGROUPS

by Blanka KOLIBIAROVÁ

We study the properties and the construction of semigroups $S$, each left ideal of which contains a unique right identity.

Denote by $I(S)$ the set of all idempotents of $S$ (elements of $I(S)$ by $e$, with indices if needed), by $L(x)$ $(R(x))$ the set of all elements of $S$ generating left (right) principal ideal $(x)_L$ $((x)_R)$. Then, $I(S)$ is a commutative subsemigroup of $S$, $I(S)$ is a well ordered set according to the relation

$$e_k \leqslant e_i \iff e_i\, e_k = e_k\, e_i = e_k .$$

In $S$, each element $x$ belongs to some $L(e_i)$ and to some $R(e_k)$ class. Denote

$$L^*(e) = \{L(e) \cap R(e_i) \; ; \quad e_i \leqslant e\} .$$

Then $L^*(e)$ is a subsemigroup of $S$ with the two-sided identity $e$. Further, $\mathfrak{L}(e) = \{L(e_i) \; ; \quad e_i \leqslant e\}$ is a subsemigroup of $S$ with the two-sided identity $e$. The same for $R^*(e)$, $\mathcal{R}(e)$.

Let $S$ be a finite semigroup. Then $S$ is a finite well ordered set of finite groups.

Every semigroup $S$, with $I(S)$ of the type $\omega$, and with $L(e_i) \cap R(e_k) = D_{ik}$ containing just one element for each $e_i$, $e_k$, is isomorphic to the bicyclic semigroup. If $S$ has the classes $D_{ik}$ containing at least one element, then $S$ can be homomorph mapped into bicyclic semigroup (preserving $D_{ik}$).

The semigroups of required properties, if $D_{ik}$ contain at most one element, can be constructed in a simple manner if $I(S)$ is of the type $\omega$. Firstly, we define in $I(S)$

$$e_i\, e_k = e_k\, e_i = e_k \quad \text{if} \quad e_k \leqslant e_i .$$

Then we assign to each $e_i \in I(S)$ two well ordered sets $L^*(e_i)$ and $R^*(e_i)$ orderisomorphic if both are non empty, having the property : Let $e$ be the first element of $I(S)$ with a non empty $L^*(e)$. We begin to define the multiplication between $L^*(e)$ and $I(S)$ ; for $x \in L^*(e)$, and $e_i$ where $e \leqslant e_i$, let

$$xe_i = e_i\, x = x .$$

To each $x$, we choose a fixed $e_k < e$, this choice is to be orderpreserving (the mapping of $L^*(e)$ into $I(S)$ denote by $\alpha$ ), and $e_m x = x$ for $e_k \leqslant e_m$ . Now we define the multiplication in $L^*(e)$ :

$$x_1 , x_2 \in L(e) , \alpha x_1 = e_1 < \alpha x_2 = e_2 ,$$

then

$$x_1 x_2 = x_2 x_1 = x_3 \in L^*(e) \text{ with } \alpha x_3 = e_3 \text{ where } \langle e_1 , e_3 \rangle \simeq \langle e , e_2 \rangle .$$

Further to each $e_i < e$ , we assign $L(e_i)$ orderisomorphic to $L^*(e)$ (the mapping $\varphi$ ). We continue in the definition of the multiplication between $L^*(e)$ and $L^*(e_i)$ , using $\alpha$ and $\varphi$ . Similarly, for $R^*(e)$ , $R^*(e_i)$ (mappings $\beta$ and $\psi$ ), at last between $R^*(e)$ and $L^*(e_i)$ , using $\alpha$ , $\beta$ , $\varphi$ , $\psi$ .

Remark. - If we have choosen $L^*(e)$ for some $e$ , and in $L^*(e_i)$ would exist some element $x$ which is not an image of an element of $L^*(e)$ , this $x$ requires, for each $y \in L^*(e)$ , $\alpha y = e_1$ , the existence of $y' \in L^*(e)$ where, if $\alpha x = e_k$ , then $\alpha y' = e_2$ with $\langle e_1 , e_2 \rangle \simeq \langle e_i , e_k \rangle$ . Respecting this, we can construct each semigroup of required property.

This construction can be adapted to the other cases.

Blanka KOLIBIAROVÁ
Ass. Prof. Techn. Univ.
Gottwaldovo nám., 2
BRATISLAVA (Tchécoslovaquie)

(Texte reçu le 15 septembre 1970)