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C. R. Acad. Sci. Paris, Ser. I 342 (2006) 421–426

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Differential Geometry

Totally geodesic Riemannian foliations with locally symmetric leaves

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Received 2 December 2005; accepted 10 January 2006

Available online 10 February 2006

Presented by Étienne Ghys

Abstract

We prove the arithmeticity of totally geodesic Riemannian foliations, with a dense leaf, on complete finite volume Riemannian manifolds when the leaves are isometrically covered by an irreducible symmetric space of noncompact type and rank at least 2. ***To cite this article: R. Quiroga-Barranco, C. R. Acad. Sci. Paris, Ser. I 342 (2006).***

Résumé

Feuilletages riemanniens totalement géodésiques avec des feuilles localement symétriques. Nous prouvons le caractère arithmétique des feuilletages riemanniens totalement géodésiques, possédant une feuille dense, sur une variété riemannienne complète de volume fini, quand les feuilles sont revêtues de façon isométrique par un espace symétrique irréductible de type noncompact et de rang au moins 2. ***Pour citer cet article : R. Quiroga-Barranco, C. R. Acad. Sci. Paris, Ser. I 342 (2006).***

Version française abrégée

Pour étudier la géométrie des feuilletages il s'est avéré utile de considérer des structures riemanniennes adaptées à ces objets géométriques. Des exemples remarquables sont donnés par la théorie de structure de Molino pour les feuilletages riemanniens et par la théorie duale de Cairns pour les feuilletages totalement géodésiques (voir [8] et [3]). Celles-ci permettent de décrire les feuilletages riemanniens ou totalement géodésiques de petite codimension (1, 2 et 3). Des descriptions précises des variétés feuilletées sont obtenues dans [5] pour les feuilletages totalement géodésiques et dans [4] pour les feuilletages riemanniens totalement ombilicaux, dans les deux cas sur les 4-variétés compactes.

D'autre part, les groupes de Lie semi-simples ont un comportement rigide bien connu, ce qui a permis de décrire les propriétés des feuilletages ayant des feuilles localement symétriques (voir [11]) et de classifier certaines actions de groupes de Lie simples non compacts (voir [9] et [10]).

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[1] Research supported by SNI-México and Conacyt Grant 44620.

1631-073X/$ – see front matter
doi:10.1016/j.crma.2006.01.015

Le but de ce travail est de considérer ces deux théories et d'appliquer certaines de leurs techniques pour décrire complètement les feuilletages riemanniens totalement géodésiques sur les variétés compactes quand les feuilles sont isométriquement revêtues par un espace symétrique irréductible de type non compact.

Soient G un groupe de Lie simple adjoint non compact, de rang réel au moins 2, K un sous-groupe compact maximal de G et $X_G = G/K$ l'espace symétrique associé à G.

Théorème 1. *Soit $(M, \mathcal{F})$ une variété feuilletée avec une métrique riemannienne h quasi-fibrée complète de volume fini, pour laquelle $\mathcal{F}$ est totalement géodésique et les feuilles sont isométriquement revêtues par X_G. En particulier, $\mathcal{F}$ est un feuilletage riemannien totalement géodésique. Si $\mathcal{F}$ a une feuille dense, alors $\mathcal{F}$ possède, a un revêtement fini près, une structure arithmétique. Autrement dit, il y a* :

(1) *un revêtement $\widetilde{M} \to M$,*
(2) *une variété riemannienne homogène $Y = H/L$, où H est un groupe de Lie adjoint semisimple et $L \subset H$ est un sous-groupe compact, et*
(3) *un réseau arithmétiqaue irréductible $\Gamma \subset G \times H$, tels que $(\widetilde{M}, \tilde{h}) = X_G \times Y$ isométriquement, où $\tilde{h}$ est le relèvement de la métrique h à $\widetilde{M}$ et $X_G \times Y$ a le métrique produit, et tels que $\Gamma \backslash \widetilde{M}$ est un revêtement fini de M. En outre, le feuilletage relevé sur $\widetilde{M}$ est donné par la projection naturelle $X_G \times Y \to Y$.*

Ceci nous permet de prouver que les feuilletages considérés sont étroitement liés aux espaces symétriques.

1. Introduction

To understand the geometry of foliations it has been useful to consider Riemannian structures adapted to such geometric objects. Remarkable examples are given by Molino's structure theory for Riemannian foliations and Cairns' dual theory for totally geodesic foliations (see [8] and [3]). These have allowed to describe low codimension (1, 2 and 3) foliations which are either Riemannian or totally geodesic. Also, precise descriptions of foliated manifolds have been obtained in [5] for totally geodesic foliations and in [4] for totally umbilic Riemannian foliations, in both cases on compact 4-manifolds.

On the other hand, there is a well known rigid behavior of higher real rank semisimple Lie groups that has allowed one to describe some properties of foliations with locally symmetric leaves (see [11]) and to classify suitable locally free actions of noncompact simple Lie groups (see [9] and [10]).

The goal of this Note is to consider these two setups and apply some of their techniques to fully describe the totally geodesic Riemannian foliations, with a dense leaf, on finite volume manifolds when the leaves are isometrically covered by a higher rank irreducible symmetric space of noncompact type.

From now on, we denote by G a noncompact adjoint simple Lie group with real rank at least 2, with K a (fixed) maximal compact subgroup of G and with $X_G = G/K$ the symmetric space associated to G. Our main result is the following:

Theorem 1. *Let $(M, \mathcal{F})$ be a foliated manifold with a finite volume complete bundle-like Riemannian metric h for which $\mathcal{F}$ is totally geodesic and the leaves are isometrically covered by X_G. In particular, $\mathcal{F}$ is a totally geodesic Riemannian foliation. If M has a dense leaf, then, up to a finite covering, $\mathcal{F}$ has an arithmetic nature. More precisely, there exist*:

(1) *a covering map $\widetilde{M} \to M$,*
(2) *a homogeneous Riemannian manifold $Y = H/L$, where H is an adjoint semisimple Lie group and $L \subset H$ is a compact subgroup, and*
(3) *an irreducible arithmetic lattice $\Gamma \subset G \times H$, such that $(\widetilde{M}, \tilde{h}) = X_G \times Y$ isometrically, where $\tilde{h}$ is the lift of the metric h to $\widetilde{M}$ and $X_G \times Y$ carries the product metric, and such that $\Gamma \backslash \widetilde{M}$ is a finite covering of M. Furthermore, the lifted foliation on $\widetilde{M}$ induced by $\mathcal{F}$ is given by the natural projection $X_G \times Y \to Y$.*

By replacing in Theorem 1 the subgroup L with a maximal compact subgroup of H containing L, we obtain the following result:

Theorem 2. *Let $(M,\mathcal{F})$, h and $M_1=\Gamma\backslash\widetilde{M}$ be as in Theorem 1. Then up to fibrations with compact fibers, M is an irreducible locally symmetric space of noncompact type. More precisely, there exist an irreducible symmetric space M_2 of noncompact type and a fibration with compact fiber $\pi: M_1\to M_2$. Furthermore, π can be chosen as a foliation preserving map for a foliation on M_2 induced by X_G appearing as a de Rham factor of its universal covering space.*

2. Tangential structures associated to $\mathcal{F}$

From now on, we will consider M, $\mathcal{F}$ and h as in Theorem 1. We will denote with $T\mathcal{F}$ the tangent bundle to the leaves and with $T\mathcal{F}^\perp$ its orthogonal complement. Also, in what follows, let $x_0=eK\in G/K=X_G$.

Denote by M^* the space of isometric coverings from X_G onto the leaves of $\mathcal{F}$, and define $\rho: M^*\to M$ by $\rho(\varphi)=\varphi(x_0)$. Then, M^* is a principal right K-bundle over M so that the K-action extends to a locally free right G-action whose orbits are precisely the inverse images under ρ of the leaves of $\mathcal{F}$. For such G-action, the stabilizer of every $x\in M^*$ is isomorphic to the fundamental group of the leaf through $\rho(x)$.

In this section we will follow [1] and [3] where one can find detailed accounts of the facts stated here. Denote with $B_{tg}(M,\mathcal{F})$ the principal fiber bundle of the vector bundle $T\mathcal{F}$. Since $\mathcal{F}$ is totally geodesic, the vector bundle $T\mathcal{F}^\perp$ induces a vector subbundle $\mathcal{H}$ of the tangent bundle of $B_{tg}(M,\mathcal{F})$ which is called the lifted horizontal bundle. Such bundle can be used to define tangential structures; more precisely, for a subgroup F of $\mathrm{GL}(p)$ (where $p=\dim(X_G)$), a tangential F-structure for $(M,\mathcal{F})$ is an F-reduction Q of $B_{tg}(M,\mathcal{F})$ such that $\mathcal{H}_\alpha\subset T_\alpha Q$ for every $\alpha\in Q$. For $F=\{e\}$, the tangential structure is called a tangential parallelism. Another example is given by $O_{tg}(M,\mathcal{F})$, the principal fiber bundle of orthonormal frames for $T\mathcal{F}$, which defines a tangential $O(p)$-structure. The pull-back in $O_{tg}(M,\mathcal{F})$ of the leaves of $\mathcal{F}$ defines a foliation $\mathcal{F}_{tg}$. The foliation $\mathcal{F}_{tg}$ can be endowed with a natural parallelism given by the standard horizontal vector fields with respect to the Levi-Civita connection along the leaves and the vector fields defined by the $O(p)$-action on $O_{tg}(M,\mathcal{F})$. After choosing some inner product on $\mathfrak{o}(p)$, we can define a lifted Riemannian metric $\hat{h}$ on $O_{tg}(M,\mathcal{F})$ for which the foliation $\mathcal{F}_{tg}$ is totally geodesic. With respect to such structure, the natural parallelism of $\mathcal{F}_{tg}$ is in fact a tangential parallelism.

Let $\gamma:[0,1]\to M$ be a curve starting at x and perpendicular to $\mathcal{F}$. We recall from [7] the existence of a unique one-parameter family of isometries $\psi_t: V_x\to V_{\gamma(t)}$, where $V_{\gamma(t)}$ is a neighborhood of $\gamma(t)$ in the leaf containing $\gamma(t)$, such that $\psi_t(x)=\gamma(t)$, the orbits of $(\psi_t)_t$ are perpendicular to $\mathcal{F}$ and ψ_0 is the identity. As in [1], we will refer to such maps as the elements of horizontal holonomy associated to γ.

Proposition 2.1. *Consider $O_{tg}(M,\mathcal{F})$ as the bundle of linear isometries from $T_{x_0}X_G$ onto the fibers of $T\mathcal{F}$. Then, the map $M^*\to O_{tg}(M,\mathcal{F})$ given by $\varphi\mapsto d\varphi_{x_0}$ realizes M^* as a tangential K-structure of $\mathcal{F}$.*

Proof. Clearly, the given map is an embedding. Also, note that for the homomorphism from K into $O(T_{x_0}X_G)\simeq O(p)$ given by $\varphi\mapsto d\varphi_{x_0}$, such embedding is equivariant and so M^* is a K-reduction of $O_{tg}(M,\mathcal{F})$. It remains to show that $\mathcal{H}_\alpha\subset T_\alpha M^*$ for every $\alpha\in M^*$.

Let $\alpha\in M^*$ be given and suppose that α projects to x in M, so that $\alpha: T_{x_0}X_G\to T_x\mathcal{F}$. For a given curve $\gamma:[0,1]\to M$ starting at x and perpendicular to $\mathcal{F}$ let $(\psi_t)_t$ be the elements of horizontal holonomy associated to γ. Then for the curve $\bar\gamma(t)=d(\psi_t)_x\circ\alpha$ we have $\bar\gamma'(0)\in\mathcal{H}_\alpha$ and every element in $\mathcal{H}_\alpha$ is of this form (see [1]). It suffices to show that $\bar\gamma$ lies in M^* for any such α and γ. Let $\varphi\in M^*$ be such that $\varphi(x_0)=x$ and $d\varphi_{x_0}=\alpha$. In particular, we have $\bar\gamma(t)=d(\psi_t\circ\varphi)_{x_0}$ for every t. By the properties of symmetric spaces, for every t there exists an isometric covering $\varphi_t: X_G\to L_{\gamma(t)}$ onto the leaf $L_{\gamma(t)}$ containing $\gamma(t)$ such that $\varphi_t=\psi_t\circ\varphi$ in a neighborhood of x_0 in X_G. In particular, $\bar\gamma(t)=d(\varphi_t)_{x_0}$ and since $\varphi_t\in M^*$ for every t, we conclude that $\bar\gamma$ lies in M^* as required. □

It follows that the restriction of $\mathcal{H}$ to M^* is tangent to M^*. We will denote such restriction with $\mathcal{H}^*$.

Lemma 2.2. *Let $O(X_G)$ be the orthonormal frame bundle of X_G viewed as the space of linear isometries of $T_{x_0}X_G$ onto the fibers of TX_G. Then, for the embedding $G\hookrightarrow O(X_G)$ given by $g\mapsto dg_{x_0}$, the standard horizontal vector fields on $O(X_G)$ are tangent to G. Also, if $X=\Gamma\backslash X_G$ is a Riemannian quotient of X_G, where $\Gamma\subset G$ is discrete, then there is an induced embedding $\Gamma\backslash G\hookrightarrow O(X)$ so that the standard horizontal vector fields are tangent to $\Gamma\backslash G$.*

Proof. First we observe that the embedding $G \hookrightarrow O(X_G)$ is G-equivariant for the natural left G-action on G and the G-action on $O(X_G)$ that lifts from the isometric G-action on X_G.

Choose $v \in T_{x_0} X_G$ and denote with $(T_t)_t$ the one-parameter subgroup of transvections of G whose orbit is the geodesic γ_v with initial velocity vector v. Then $\hat{\gamma}_v(t) = d(T_t)_{x_0}$ defines the parallel transport along the geodesic γ_v and so it is the horizontal lift at α_0 in $O(X_G)$ with respect to the Levi-Civita connection, where $\alpha_0 \in O(X_G)$ is the identity map of $T_{x_0} X_G$. Hence, $\hat{v} = \hat{\gamma}_v'(0)$ is horizontal at α_0; moreover, all horizontal vectors at α_0 are obtained through this construction. Since $T_t \in G$ for every t, then $\hat{\gamma}_v$ lies in G with respect to the given embedding. Hence, $T_e G$ contains the horizontal subspace of $O(X_G)$ at α_0 defined by the Levi-Civita connection. Since the embedding $G \hookrightarrow O(X_G)$ is G-equivariant and the G-action preserves the Levi-Civita connection on $O(X_G)$ the first part follows. The last claim follows by modding out by the left Γ-action and using the properties obtained so far. □

Proposition 2.3. *The bundle $\mathcal{H}^*$ is G-invariant.*

Proof. Let $\mathcal{P} = (X_1, \ldots, X_p, A_1, \ldots, A_l)$ be a set of vector fields over $O_{tg}(M, \mathcal{F})$ such that $(X_i)_{i=1}^p$ is a basis for the standard horizontal vector fields along the leaves of $\mathcal{F}$ and $(A_i)_{i=1}^l$ comes from the right action of the one-parameter subgroups defined by a basis of $\mathfrak{k}$ (the Lie algebra of K); in particular, $\mathcal{P}$ is a subset of the natural tangential parallelism of $O_{tg}(M, \mathcal{F})$ (see [3]). Also, by Lemma 2.2 and the properties of M^*, the set $\mathcal{P}|_{M^*}$ of restrictions to M^* of the elements in $\mathcal{P}$ defines a parallelism for $T\mathcal{F}$.

On the other hand, from the proof of Lemma 2.2, the properties of M^* and the fact that G preserves the horizontal standard vector fields on X_G, it follows that the elements in $\mathcal{P}|_{M^*}$ are locally given by left invariant vector fields on G. We recall the (elementary) fact that on a Lie group the left invariant vector fields have flows corresponding to right actions of one-parameter subgroups. From this fact, again the properties of M^* and since $\mathcal{P}|_{M^*}$ is a parallelism, it is easy to see that $\mathcal{P}|_{M^*}$ consists of vector fields whose local flows generate the right G-action on M^*.

Let $v^* \in \mathcal{H}^*$ be given. Choose curves γ in M (everywhere perpendicular to $\mathcal{F}$) and $\bar{\gamma}$ in M^* as in the proof of Proposition 2.1, so that $\bar{\gamma}'(0) = v^*$ and $\bar{\gamma}(t) = d(\psi_t)_x \circ \alpha$, where $(\psi_t)_t$ are the elements of horizontal holonomy associated to γ, α is the basepoint of v^* and x is the projection of α. Clearly, the curves $\delta_y(t) = \psi_t(y)$, where $y \in V_x$ with our previous notation, are the integral curves of a local vector field Z^* such that $Z^*_\alpha = v^*$; furthermore, Z^* is a section of $\mathcal{H}^*$ (see [1]). Since the local flow of Z^* is given by maps of the form $y \mapsto \delta_y(t)$ and the latter restrict to the maps $(\psi_t)_t$ along the leaves of $\mathcal{F}$, it follows that the flow of Z^* fixes the elements in $\mathcal{P}|_{M^*}$. Hence, every element of $\mathcal{P}|_{M^*}$ commutes with Z^*, and so the flows given by the elements in $\mathcal{P}|_{M^*}$ fix Z^*. Then, the G-action maps v^* into $\mathcal{H}^*$ from which the result follows. □

3. Completion of the proof of Theorem 1

Lemma 3.1. *There exists a G-invariant pseudoRiemannian metric h^* on M^* for which the projection $\rho : M^* \to M$ is a pseudoRiemannian submersion.*

Proof. First we observe that $\rho : (M^*, \hat{h}) \to (M, h)$ is a Riemannian submersion where $\hat{h}$ is the restriction to M^* of the lifted horizontal metric of $O_{tg}(M, \mathcal{F})$. Such metric is given by declaring $\mathcal{H}^*$ and $\mathcal{F}_{tg}$ to be perpendicular, $d\rho|_{\mathcal{H}^*} : \mathcal{H}^* \to T\mathcal{F}^\perp$ to be fiberwise isometric and the elements in $\mathcal{P}|_{M^*}$ (as in the proof of Proposition 2.3) to be orthonormal (see [1] and [3]). As remarked in the proof of Proposition 2.3, the elements in $\mathcal{P}|_{M^*}$ are locally given by left invariant vector fields. Hence, by starting with suitable choices of such vector fields, we can consider a pseudoRiemannian metric h_1 along the leaves of $\mathcal{F}_{tg}$ in M^* such that the linear span of the elements in $\mathcal{P}|_{M^*}$ are isometric to $\mathfrak{g}$ (the Lie algebra of G) with its Killing form. Since the latter is invariant under the adjoint action of G, it is easy to prove that h_1 is G-invariant. Also, by Proposition 2.3, and since the metric h on M is bundle-like, there exists a Riemannian metric h_2 on $\mathcal{H}^*$ which is G-invariant.

Since G-preserves the decomposition $TM^* = T\mathcal{F}_{tg} \oplus \mathcal{H}^*$ it follows that $(T\mathcal{F}_{tg}, h_1) \oplus (\mathcal{H}^*, h_2)$ defines a G-invariant pseudoRiemannian metric h^* on M^*. By using the fact that the canonical projection $G \to X_G$ is a pseudoRiemannian submersion for G endowed with the bi-invariant metric given by the Killing form of $\mathfrak{g}$, it follows that ρ is indeed a pseudoRiemannian submersion for our choice of metrics. □

We need the following variation of the results found in [9] and [10]. We note that the completeness assumption in the statement below allows to simply carry over the arguments in [10] to obtain the proof. Such proof relies heavily on Gromov's results on rigid geometric structures from [6], but it also uses arguments from the proof of Gromov's centralizer theorem as found in [2] as well as Zimmer's arithmeticity results from [11].

Theorem 3.2. ([9,10]) *Let X be a manifold with a locally free right G-action preserving a finite volume pseudoRiemannian metric that induces a transverse Riemannian structure for the foliation by G-orbits, and so that the geodesics perpendicular to the G-orbits are complete. If the G-action is faithful and has a dense orbit, then there exist*:

(1) *a finite covering $\widehat{X} \to X$,*
(2) *a connected finite center semisimple Lie group H with a compact subgroup L, and*
(3) *an irreducible arithmetic lattice Γ of $G \times H$, for which the G-action on X lifts to $\widehat{X}$ so that $\widehat{X}$ is G-equivariantly diffeomorphic to $\Gamma\backslash(G \times H/L)$. Also, the homogeneous manifold H/L carries an H-invariant Riemannian metric so that the natural projection $G \times H/L \to H/L$ defines the transverse Riemannian structure that lifts to $G \times H/L$ from X.*

Given the above results, to prove Theorem 1, note that by its construction (M^*, h^*) satisfies the hypothesis of Theorem 3.2. The transverse completeness of h^* follows from the completeness of (M, h) and the fact that ρ is a pseudoRiemannian submersion. This provides groups H, L and Γ and a finite covering $\widehat{M}^* \to M^*$ satisfying the conclusions of Theorem 3.2.

Let us now consider $\widetilde{M} = (G \times H)/(K \times L) = G/K \times H/L$. By using the freeness of the K-action on M^* it is easy to follow through the proof of Theorem 3.2 to show that Γ can be chosen so that K acts freely on $\Gamma\backslash(G \times H/L)$. Hence, the space $\Gamma\backslash\widetilde{M}$ is a smooth manifold and a finite covering of M. The rest of the claims of Theorem 1 are now straightforward to check.

4. Final remarks

The above arguments show that $\mathcal{P}|_{M^*}$ is a tangential $\mathfrak{g}$-Lie parallelism for $(M^*, \mathcal{F}_{tg})$ (see [5] for the definition of a tangential Lie parallelism). Hence, for compact M as in Theorem 1, we can apply the results from [5] and the notion of approximative center found therein to prove that $T\mathcal{F}^\perp$ is integrable. Furthermore, Professor Ghys has pointed out to us that such argument is valid for finite volume manifolds as well. Then one can use de Rham's theorem to obtain a covering of M that diffeomorphically splits as a product of coverings of a leaf of $\mathcal{F}$ and a leaf of $T\mathcal{F}^\perp$. Given these facts, the arguments from Sections 5 and 6 in [10] can be used to obtain Theorem 1.

The proof presented in the previous sections and the one we just briefly described make use of a curvature tensor that measures the obstruction for the integrability of $T\mathcal{F}^\perp$ (cf. [5] and [10]). The tensors used in these two proofs are different but quite similar in nature. Moreover, besides the way one proves the integrability of $T\mathcal{F}^\perp$, both proofs actually follow the same arguments.

Acknowledgements

I would like to thank Professor Étienne Ghys for pointing me out references and facts that allowed me to greatly improve a first draft of this work. I also thank Professor Fausto Ongay from Cimat for his helpful advice.

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