

Probability Theory
Fellerian pants

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Abstract

Given a manifold M we define a Banach space E associated with the loop space $L(M)$ of M in such a way that the random pants realize a continuous map from the injective tensor product $E \hat{\otimes}_\varepsilon E$ into E . Our research is motivated by one of the axioms of conformal field theory of G. Segal. Full details will be presented in a forthcoming article. *To cite this article: Z. Brzeźniak, R. Léandre, C. R. Acad. Sci. Paris, Ser. I 343 (2006).*

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Résumé

Pantalon de Feller. Nous définissons un espace de Banach E associé à l'espace des lacets tel qu'un pantalon aléatoire réalise une application continue de $E \hat{\otimes}_\varepsilon E$ dans E . Nous sommes motivés par l'un des axiomes de G. Segal de la théorie des champs conformes. Les détails seront écrits dans un prochain article. *Pour citer cet article : Z. Brzeźniak, R. Léandre, C. R. Acad. Sci. Paris, Ser. I 343 (2006).*

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Considérons l'espace de Hilbert $H = H^{1,2}(S^1, \mathbb{R}^d)$ des applications $s \mapsto \gamma(s)$ telles que :

$$\|\gamma\|^2 = \int_{S^1} (|\gamma(s)|^2 + |\gamma'(s)|^2) ds < \infty.$$

Introduisons le mouvement Brownien W_t à valeurs dans H . Ce mouvement Brownien est en fait à valeurs dans l'espace de Besov–Slobodetski $W^{\theta,p}(S^1, \mathbb{R}^d)$ pour $p > 2$ et $0 < \theta < 1/2$. Introduisons une application π de \mathbb{R}^d dans $GL(\mathbb{R}^d)$, de dérivées à tout ordre bornées. On suppose que π est uniformément inversible. L'équation de Airault–Malliavin [2]

$$dx_t(s) = \pi(x_t(s)) dW_t(s) ; \quad x_0(s) = \gamma(s)$$

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peut être considérée comme une équation où l'on peut appliquer la théorie générale des équations différentielles de Brzeźniak–Elworthy [5] sur les variétés Banachiques associées à des espaces de Banach du type $M - 2$:

$$d_t X_t = \Pi(X_t) dW_t ; \quad X_0 = \gamma.$$

Dans notre cas particulier, Π est l'application de Nemytski associée à π , qui est globalement Lipschitzienne sur l'espace de Besov–Slobodetski.

E est l'espace de Banach des fonctions continues bornées φ sur $W^{\theta,p}(S^1, \mathbb{R}^d)$. L'équation de Brzeźniak–Elworthy définit un semi-groupe P_t sur E .

Théorème 0.1. *Le semi-groupe P_t est de Feller.*

Introduisons deux points (s_1, s_2) sur le cercle. La diffusion $t \mapsto \hat{x}_t = (x_t(s_1), x_t(s_2))$ est non dégénérée. On peut la conditionner par arriver au temps 1 en \hat{b} , en utilisant les techniques standards d'étude du pont d'une diffusion elliptique (voire [22] et [4] par exemple). En décomposant l'espace Gaussien $(t, s) \mapsto W_t(s)$ en la somme de l'espace Gaussien $t \mapsto (w_t(s_1), w_t(s_2))$ et de son orthogonal, on peut conditionner la diffusion $t \mapsto X_t(\gamma)$ par $\hat{x}_t = \hat{b}$. Nous obtenons un processus $\hat{X}_t(\gamma)$ qui réalise une sorte de cylindre aléatoire dont la fin se termine par un '8'. Nous obtenons :

Théorème 0.2. $\varphi \mapsto \{\gamma \mapsto \mathbb{E}_\gamma[\varphi(\hat{X}_t(\gamma))]\}$ est une application linéaire continue de E dans E .

Introduisons deux autres mouvements Browniens sur H W_t^1 et W_t^2 mutuellement indépendants et indépendants du premier. Le lacet aléatoire $\hat{X}_t(\gamma)$ est divisé en deux lacets aléatoires $\hat{X}_t^1(\gamma)$ et $\hat{X}_t^2(\gamma)$ appartenant par la théorie précédente à $W^{\theta,p}(S^1, \mathbb{R}^d)$. Nous étudions l'équation de Airault–Malliavin issue de chaque lacet $\hat{X}_t^i(\gamma)$ associé au Brownien W_t^i . Nous obtenons une sorte de pantalon aléatoire issu de γ et se terminant par deux lacets $\hat{X}_t^i(\gamma)$.

Appelons $E \hat{\otimes}_\varepsilon E$ l'espace des fonctions continues bornées sur $W^{\theta,p}(S^1, \mathbb{R}^d) \times W^{\theta,p}(S^1, \mathbb{R}^d)$. Nous obtenons :

Théorème 0.3. $\varphi_1 \otimes \varphi_2 \mapsto \{\gamma \mapsto \mathbb{E}_\gamma[\varphi_1(\hat{X}_t^1(\gamma))\varphi_2(\hat{X}_t^2(\gamma))]\}$ définit une application linéaire continue de $E \hat{\otimes}_\varepsilon E$ dans E .

Ce théorème exprime que le pantalon est Fellerien.

1. Introduction

Throughout the whole Note by $H = H^{1,2}(S^1, \mathbb{R}^d)$ we denote the Hilbert space of maps $S^1 \ni s \mapsto \gamma(s) \in \mathbb{R}^d$ such that

$$\|\gamma\|^2 = \int_{S^1} (|\gamma(s)|^2 + |\gamma'(s)|^2) ds < \infty. \quad (1)$$

For fixed $p > 2$ and $1/p < \theta < 1/2$, we denote by $M = W^{\theta,p}(S^1, \mathbb{R}^d)$ the Besov–Slobodetski space of all maps $S^1 \ni s \mapsto \gamma(s) \in \mathbb{R}^d$ such that

$$|\gamma|^p := \int_{S^1} |\gamma(s)|^p ds + \int_{S^1} \int_{S^1} \frac{|\gamma(s_2) - \gamma(s_1)|^p}{|s_2 - s_1|^{1+\theta p}} ds_1 ds_2 < \infty. \quad (2)$$

It is well known that $W^{\theta,p}(S^1, \mathbb{R}^d)$ is a Banach space with norm $|\cdot|$ and that $H^{1,2}(S^1, \mathbb{R}^d) \subset W^{\theta,p}(S^1, \mathbb{R}^d) \subset C^0(S^1, \mathbb{R}^d)$.

Let $(W_t)_{t \geq 0}$ be a $W^{\theta,p}(S^1, \mathbb{R}^d)$ -valued Wiener process, defined on some complete filtered probability space, such that the Cameron–Martin space of the law of W_1 is equal to H . This process induces a Gaussian random field $(W_t(s))_{(t,s) \in [0,1] \times S^1}$. Let $\pi : \mathbb{R}^d \rightarrow \mathcal{L}(\mathbb{R}^d, \mathbb{R}^d)$ be a smooth map with bounded derivatives of all orders. For a given initial loop $\gamma \in M$ we consider the Airault–Malliavin [2] equation in the Stratonovich form:

$$dx_t(s) = \pi(x_t(s)) \circ dW_t(s), \quad t \geq 0, s \in S^1; \quad x_0(s) = \gamma(s), \quad s \in S^1. \quad (3)$$

Let us fix two distinct points s_1 and s_2 on the unit circle S^1 and a point b on the manifold M . In what follows we assume that the map π is uniformly invertible, i.e. for all $x \in \mathbb{R}^d$, $\pi(x)$ is a linear isomorphism and $\sup_{x \in \mathbb{R}^d} \|\pi(x)^{-1}\| < \infty$. Then, since the $\widehat{\mathbb{R}}^d = \mathbb{R}^d \times \mathbb{R}^d$ -valued Wiener process $\widehat{w}_t = (W_t(s_1), W_t(s_2))$, $t \geq 0$ is non-degenerate so is the $\widehat{\mathbb{R}}^d$ -valued diffusion process $(x_t(s_1), x_t(s_2))$, $t \geq 0$. Hence, by employing the methods of Quasi-sure Analysis of Sugita and Airault–Malliavin, see e.g. [1,20,24] and [12], it can be shown that the random field $(x_t(s))_{t \in [0,1], s \in S^1}$ can be conditioned by the condition $(x_1(s_1), x_1(s_2)) = (b, b)$. The realizations of that conditioned process, denoted by \widehat{x}_t , $t \in [0, 1]$, are topological surfaces (i.e. 2-dimensional topological manifolds) in a shape of a pinched cylinder. Their boundary consists of two parts, the ingoing being the loop γ and the outgoing being of a shape of a figure eight. However, the dependence of the outgoing manifold on the ingoing one is lost in this approach. This dependence is important from the point of view of Segal’s axioms. Another approach is needed to handle this difficulty.

The basic observation leading to this alternative approach is that the following pair of stochastic processes: $\widehat{\mathbb{R}}^d$ -valued $(x_t(s_1), x_t(s_2))$ and \mathbb{R}^d -valued $x_t(s)$, where as before two distinct points s_1, s_2 and another one s are fixed on the unit circle S^1 , satisfy the following system of differential equations:

$$\begin{aligned} d(x_t(s_1), x_t(s_2)) &= g_{s_1, s_2}(x_t(s_1), x_t(s_2)) \circ dW_t, \quad t \geq 0, \\ dx_t(s) &= g_s(x_t(s)) \circ dW_t, \quad t \geq 0. \end{aligned} \tag{3a}$$

Here, with $R(H, X)$ denoting the space of all γ -radonifying operators from the Hilbert space H to a separable Banach space X , the vector fields $g_{s_1, s_2} : \mathbb{R}^d \times \mathbb{R}^d \rightarrow R(H, \mathbb{R}^d \times \mathbb{R}^d)$ and $g_s : \mathbb{R}^d \rightarrow R(H, \mathbb{R}^d)$, are defined respectively by $g(x_1, x_2)(h) := (\pi(x_1)(h(s_1)), \pi(x_2)h(s_2))$, $g(x)(h) := (\pi(x)(h(s)))$. Let $p_t(\widehat{x}, \widehat{y})$, $t > 0$, $\widehat{x}, \widehat{y} \in \widehat{\mathbb{R}}^d$ be the heat kernel corresponding to the process $(x_t(s_1), x_t(s_2))$, $t \geq 0$. It follows then by Theorem 2.1 from [8] that the law of the process $(x_t(s_1), x_t(s_2), x_t(s))$, $t \in [0, 1]$, conditioned by $(x_1(s_1), x_1(s_2)) = (b, b) =: \widehat{b}$ is the same as the law of solution to the following system of SDE’s:

$$\begin{aligned} d(\widehat{x}_t^{\widehat{b}}) &= g_{s_1, s_2}(\widehat{x}_t^{\widehat{b}}) \circ dW_t + \nabla_{\widehat{x}} p_{1-t}(\widehat{x}_t, \widehat{b}) dt, \quad t \in [0, 1), \\ dx_t^{\widehat{b}}(s) &= g_s(x_t^{\widehat{b}}(s)) \circ dW_t + g_s(x_t^{\widehat{b}}(s)) \widehat{A}(\widehat{x}_t^{\widehat{b}}, x_t^{\widehat{b}}(s)) dt, \quad t \in [0, 1), \end{aligned} \tag{3b}$$

where $\widehat{A} : \widehat{\mathbb{R}}^d \times \mathbb{R}^d \rightarrow H$, $\widehat{A}(\widehat{x}, x) = [g_{s_1, s_2}(\widehat{x})]^{-1}(\nabla_{\widehat{x}} p_{1-t}(\widehat{x}, \widehat{b}))$. Note that in our case the map \widehat{A} does not depend on the second variable x , hence it is reasonable to write $\widehat{A}(\widehat{x})$ instead of $\widehat{A}(\widehat{x}, x)$. Here $[g_{s_1, s_2}(\widehat{x})]^{-1}$ is the right inverse of the surjective linear map $g_{s_1, s_2}(\widehat{x}) : H \rightarrow \widehat{\mathbb{R}}^d$, chosen with range equal to the orthogonal complement of $\ker g_{s_1, s_2}(\widehat{x})$. As explained in [8], the map $\widehat{x} \mapsto [g_{s_1, s_2}(\widehat{x})]^{-1}$ is smooth. It follows from [8] that the problem (3b) has a unique continuous solution $(\widehat{x}_t^{\widehat{b}}, x_t^{\widehat{b}}(s))$, $t \in [0, 1]$ (including time 1).

Since, the first equation in (3b) can be solved independently of the second one and $A_t = A_t(y) = \widehat{A}(\widehat{x}_t^{\widehat{b}}, y) \in H$ is not depending on $y \in \mathbb{R}^d$, the second equation can be written in (3b) the following way, with coefficients depending on the solution to the former one.

$$dx_t^{\widehat{b}}(s) = g_s(x_t^{\widehat{b}}(s)) \circ dW_t + g_s(x_t^{\widehat{b}}(s)) A_t dt, \quad t \in [0, 1). \tag{3c}$$

It can be shown that moreover $(x_1^{\widehat{b}}(s_1), x_1^{\widehat{b}}(s_2)) = \widehat{x}_1^{\widehat{b}}$. Hence, if the initial data for the problem (3b) lies on a curve γ , i.e. $x_1^{\widehat{b}}(s) = \gamma(s)$, allowing $s \in S^1$ to be a parameter, the final values of the solution lie on a curve (a priori not even continuous) that self-intersects at parameters s_1 and s_2 . In other words, a typical realization of the process $x_t(s)$ is a pinched cylinder whose base, called also an incoming manifold, is the loop γ and whose end, called also the outgoing manifold, is a curve essentially like a figure eight. The above procedure can be continued with both parts of the outgoing eight curve being reparameterized and then treated as an initial, independent loops for two process x_t^i , $t \in [1, 2]$, $i = 1, 2$, governed by Eq. (3). Gluing these three surfaces together we obtain a random field whose realizations are surfaces with an incoming manifold being a loop and the outgoing manifold being a disjoint union of two loops. We will call such surfaces ‘pants’. This is described in more details below.

2. ‘Pant’ surfaces

Let $E = C_b(M)$ be the Banach space of all real bounded continuous functions on $M = W^{\theta, p}(S^1; \mathbb{R}^d)$. The Airault–Malliavin equation (3) can be viewed as a equation of Brzeźniak–Elworthy type, see [5], i.e. with Π being a gen-

eralized Nemytski map associated to π , i.e. $\Pi : M \rightarrow \mathcal{L}(M, M) \subset R(H, M)$, $\Pi(\gamma)(h)(s) := \pi(\gamma(s))(h(s))$, $\gamma \in M$, $h \in M$ or H , $s \in S^1$,

$$dX_t = \Pi(X_t) \circ dW_t; \quad X_0 = \gamma. \quad (4)$$

By [5] Eq. (4) has a unique global solution which will be denoted by $X_t(\gamma)$, $t \geq 0$. The process $X_t(\gamma)$, $t \geq 0$ is Markov and hence defines a semi-group $(P_t)_{t \geq 0}$ on $B_b(M)$,

$$P_t \varphi(\gamma) = \mathbb{E}[\varphi(X_t(\gamma))], \quad \varphi \in B_b(M), \quad \gamma \in M, \quad t \geq 0. \quad (5)$$

Using methods from [5], see also [6,9] and/or [3], one can show that $P_t \varphi \in E$ whenever $\varphi \in E$ and $t \geq 0$, i.e. the semi-group P_t is Feller.

We will construct an M -valued process \widehat{X}_t , $t \in [0, 1]$, such that for each $\gamma \in M$, $\widehat{X}_1(\gamma)(s_i) = b$, $i = 1, 2$. This can be done either by a formal ‘lifting’ procedure from Eq. (3c), the same way as Eq. (4) is obtained from Eq. (3) or by generalizing Theorem 2.1 from [8] to a Banach space setting. Whichever method we choose we get the following SDE on the Banach space M :

$$d\widehat{X}_t(\gamma) = \Pi(\widehat{X}_t(\gamma)) dW_t + \Pi(\widehat{X}_t(\gamma)) A_t(\gamma) dt, \quad \widehat{X}_0(\gamma) = \gamma, \quad (6)$$

where $A_t(\gamma) = \widehat{A}(\widehat{x}_t^{\hat{b}}, 0) = [g_{s_1, s_2}(\widehat{x})]^{-1} (\nabla_{\widehat{x}} p_{1-t}(\widehat{x}_t^{\hat{b}}, \widehat{b}))$. Note that $A_t(\gamma)$, $t \geq 0$, is an H -valued process.¹ The choice of the process $A_t(\gamma)$, $t \in [0, 1]$, will guarantee that there exists a unique solution $\widehat{X}_t(\gamma)$, $t \in [0, 1]$, i.e. that is defined up to and inclusive time 1, and that $\widehat{X}_1(\gamma)(s_i) = b$, $i = 1, 2$.

In the so called flat case, i.e. when $\pi(x) = I_d$, for all $x \in \mathbb{R}^d$, X_t is a Gaussian process and an explicit formula for the heat kernel associated to a flat Brownian bridge is known. Therefore an explicit formula for the process $A_t(\gamma)$ can be found. Using this formula it can be shown that Eq. (6) has a unique solution \widehat{X}_t , $t \in [0, 1]$, and that this solution is a (time in-homogeneous) Markov process. In the general case, i.e. when π is assumed to be uniformly invertible, such an explicit formula for the heat kernel $\widehat{p}_t(\widehat{a}, \widehat{b})$ associated to the process \widehat{x}_t does not exist. Instead we can use estimates on the Hessian of the logarithm of the heat kernel $\widehat{p}_t(\widehat{a}, \widehat{b})$, uniform with respect to $t \in (0, 1]$ and $\widehat{a}, \widehat{b} \in \widehat{\mathbb{R}}^d$, see e.g. [22] or [4] to obtain certain continuity property of the process $A_t(\gamma)$ with respect to γ . We have the following key lemma:

Lemma 2.1. *The map $M \ni \gamma \mapsto A_t(\gamma) \in \mathcal{H}$ is continuous, where by \mathcal{H} we denote the space of all H -valued progressively measurable processes V_t , $t \in [0, 1]$, such that*

$$\left[\mathbb{E} \left(\int_0^1 \|V_t\|_H dt \right)^2 \right]^{1/2} < \infty. \quad (7)$$

Idea of the proof. We will not prove this result in this short note but will only indicate the reasons for the process A_t to satisfy the condition (7). Because of the well established continuity a solution of a SDE with respect to parameters, see [21], we only have to prove that for some fixed $\eta \in (0, 1)$, $[\mathbb{E}(\int_{1-\eta}^1 \|A_t\|_H dt)^2]^{1/2} < \infty$. Since for some constant $C > 0$, $\mathbb{E}|\nabla_{\widehat{x}} \log p_{1-t}(\widehat{x}_t(\gamma(s_1), \gamma(s_2)), \widehat{b})|^2 \leq C(1-t)^{-1/2}$, for $t \in [1-\eta, 1]$, the result follows. \square

Once we are able to prove existence and uniqueness of solution to problem (6) we can define the following family $(\widehat{P}_t)_{t \geq 0}$ of linear operators on $B_b(M)$ by

$$\widehat{P}_t \varphi(\gamma) = \mathbb{E}[\varphi(\widehat{X}_t(\gamma))], \quad \varphi \in B_b(M), \quad \gamma \in M, \quad t \in [0, 1]. \quad (8)$$

Summing up and recalling that $E = C_b(M)$ is the Banach space of all real bounded continuous functions on $M = W^{\theta, p}(S^1; \mathbb{R}^d)$, we have the following result.

Proposition 2.2. *Suppose that the map $\pi : \mathbb{R}^d \rightarrow \mathcal{L}(\mathbb{R}^d, \mathbb{R}^d)$ is C_b^∞ and uniformly invertible. Then for each $\gamma \in M$ the problem (6) has a unique admissible M -valued solution \widehat{X}_t , $t \in [0, 1]$. Moreover, for all $t \in [0, 1]$, the map \widehat{P}_t is Feller, i.e. $\widehat{P}_t \varphi \in E$ whenever $\varphi \in E$.*

¹ It depends on γ because the process $\widehat{x}_t^{\hat{b}}$ depends on it via the initial condition $\widehat{x}_0^{\hat{b}} = (\gamma(s_1), \gamma(s_2))$.

The random loop $\widehat{X}_1(\gamma) \in M$ splits naturally into two loops, which, after re-parametrization we denote by $\widehat{X}_1^1(\gamma)$ and $\widehat{X}_1^2(\gamma)$. Both these loops are elements of M . Let us recall that the injective tensor product $E \widehat{\otimes}_\varepsilon E$ is isomorphic to the space of bounded continuous functions on $M \times M$ with the isomorphism $\vartheta : C_b(M) \widehat{\otimes}_\varepsilon C_b(M) \rightarrow C_b(M^2)$ satisfying $\vartheta(f_1 \otimes f_2)(t_1, t_2) = f_1(t_1)f_2(t_2)$, $t_1, t_2 \in M$. Because of the relationship of our work with Segal's axioms we will use the former notation. Next define a map $\widehat{Q}_1 : B_b(M^2) \rightarrow B_b(M)$ by formula

$$\widehat{Q}_1\varphi(\gamma) = \mathbb{E}[\varphi(\widehat{X}_1^1(\gamma), \widehat{X}_1^2(\gamma))], \quad \varphi \in B_b(M \times M), \quad \gamma \in M. \quad (9)$$

Theorem 2.3. *Suppose that the map $\pi : \mathbb{R}^d \rightarrow \mathcal{L}(\mathbb{R}^d, \mathbb{R}^d)$ is C_b^∞ and uniformly invertible. Then, the map \widehat{Q}_1 is Feller, i.e. \widehat{Q}_1 maps $E \widehat{\otimes}_\varepsilon E$ into E .*

In the next step we consider two independent Wiener processes W_t^i , $t \geq 1$, $i = 1, 2$, both with the same Cameron–Martin space H and independent from the original Wiener process W_t . We consider two solutions $\widehat{X}_i(t)$, $i = 1, 2$, $t \geq 1$, of the Brzeźniak–Elworthy equation (4) with initial time $t = 1$, initial random curve $\widehat{X}_i^1(\gamma)$ and with $(W_t^i)_{t \geq 1}$ as the driving Wiener process. In particular, at time $t = 2$ we get two loops: $\widehat{X}_2^1(\gamma)$ and $\widehat{X}_2^2(\gamma)$. Hence, we can define an operator \widehat{Q}_2 in the same manner as we have defined the operator \widehat{Q}_1 . From Theorem 2.3 and Proposition 2.2 we infer our main result:

Theorem 2.4. *Suppose that the map $\pi : \mathbb{R}^d \rightarrow \mathcal{L}(\mathbb{R}^d, \mathbb{R}^d)$ is C_b^∞ and uniformly invertible. Then the linear map \widehat{Q}_2 is Feller.*

Remark 2.5. Our research is concerned with 2-dimensional conformal field theory and Segal's axioms in particular, see [23] and [10,11]. However, instead of Hilbert space we use Banach spaces. Hence instead of using a unique Hilbertian tensor product, we use an injective tensor product of Banach spaces because of its nice property mentioned after Proposition 2.2. An interested reader should consult papers [15,16] and [17], where a 2-dimensional field theory satisfying one of Segal's axioms is constructed. Other related works are [13,14,18] and [19].

Remark 2.6. The results in this paper are restricted to a simple manifold $M = \mathbb{R}^d$. The proofs will be presented in [7] in a more general framework of Cartan–Hadamard manifolds.

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References

- [1] H. Airault, P. Malliavin, Quasi-sure analysis, Publication of University Paris VI, 1990.
- [2] H. Airault, P. Malliavin, Analysis over loop groups, Publication of University Paris VI, 1991.
- [3] Ya.L. Belopolskaya, Yu.L. Daletski, Stochastic Equations and Differential Geometry, Mathematics and Its Applications, vol. 30, Kluwer Academic Publishers, Dordrecht, 1990.
- [4] J.-M. Bismut, Large deviations and the Malliavin Calculus, Progress in Mathematics, vol. 45, Birkhäuser, Boston, 1984.
- [5] Z. Brzeźniak, K.D. Elworthy, Stochastic differential equations on Banach manifolds; applications to diffusions on loop spaces, MFAT 6 (1) (2000) 43–84, (a special volume dedicated to the memory of Professor Yuri Daletski).
- [6] Z. Brzeźniak, R. Léandre, Horizontal lift of an infinite dimensional diffusion, Potential Anal. 12 (2000) 249–280.
- [7] Z. Brzeźniak, R. Léandre, Stochastic pants over a Riemannian manifold, in preparation.
- [8] A. Carverhill, Conditioning a lifted stochastic system in a product space, Ann. Probab. 16 (4) (1988) 1840–1853.
- [9] K.D. Elworthy, Stochastic Differential Equations on Manifolds, London Math. Soc. Lecture Notes Ser., vol. 70, Cambridge University Press, 1982.
- [10] G. Felder, K. Gawędzki, A. Kupiainen, Spectra of Wess–Zumino–Witten models with arbitrary simple groups, Comm. Math. Phys. 117 (1) (1988) 127–158.
- [11] K. Gawędzki, Conformal field theory, Séminaire Bourbaki, Vol. 1988/89; Astérisque No. 177–178, Exp. No. 704 (1989) 95–126

- [12] Z.Y. Huang, J.G. Ren, Quasi sure stochastic flows, *Stochastics Stochastics Rep.* 33 (3–4) (1990) 149–157.
- [13] H.H. Kuo, Diffusion and Brownian Motion on infinite dimensional manifolds, *Trans. Amer. Math. Soc.* 169 (1972) 439–459.
- [14] R. Léandre, Analysis over loop space and topology, *Math. Notes* 72 (2002) 212–229.
- [15] R. Léandre, Brownian surfaces with boundary and Deligne cohomology, *Rep. Math. Phys.* 52 (2003) 353–362.
- [16] R. Léandre, Markov property and operads, in: I. Nikulov, D. Sheehan (Eds.), *Quantum Limits in the Second Law of Thermodynamics*, *Entropy* 6 (2004) 180–215.
- [17] R. Léandre, Two examples of stochastic field theories, *Osaka J. Math.* 42 (2005) 353–365.
- [18] R. Léandre, Brownian pants and Deligne cohomology, *J. Math. Phys.* 46 (1–20) (2005) 033503.
- [19] R. Léandre, Galton–Watson tree and branching loops, in: I. Mladenov, A. Hirschfeld (Eds.), *Geometry, Integrability and Quantization*, *Softek*, 2005, pp. 267–283.
- [20] P. Malliavin, *Stochastic Analysis*, Springer, Berlin, 1997.
- [21] P.A. Meyer, Flot d’une équation différentielle stochastique, in: J. Azéma, M. Yor (Eds.), *Séminaire de Probabilités XV*, in: *Lecture Notes in Math.*, vol. 850, Springer, Berlin, 1981, pp. 100–117.
- [22] S. Molchanov, Diffusion processes and Riemannian geometry, *Russian Math. Surveys* 30 (1975) 1–63.
- [23] G. Segal, Two-dimensional conformal field theories and modular functors, in: *IXth International Congress on Mathematical Physics* (Swansea, 1988), Hilger, Bristol, 1989, pp. 22–37.
- [24] H. Sugita, Positive generalized Wiener functions and potential theory over abstract Wiener spaces, *Osaka J. Math.* 25 (1988) 665–696.