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Partial Differential Equations

A stability estimate for ill-posed elliptic Cauchy problems in a domain with corners

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Abstract

We prove in this Note a stability estimate for ill-posed elliptic Cauchy problems in a domain with corners. This result completes an earlier result obtained for a smooth domain. ***To cite this article: L. Bourgeois, C. R. Acad. Sci. Paris, Ser. I 345 (2007).***

Résumé

Une inégalité de stabilité pour les problèmes de Cauchy elliptiques mal posés dans un domaine comportant des coins. Nous montrons dans cette Note une inégalité de stabilité pour les problèmes de Cauchy elliptiques mal posés dans un domaine comportant des coins. Ce résultat complète un résultat antérieur obtenu pour un domaine régulier. ***Pour citer cet article : L. Bourgeois, C. R. Acad. Sci. Paris, Ser. I 345 (2007).***

Version française abrégée

Dans cette Note, nous montrons un résultat de stabilité pour l'opérateur $P=-\Delta.-k.$ dans un domaine $\Omega\subset\mathbb{R}^2$ borné et connexe, dont la frontière $\partial\Omega$ est régulière (au moins de classe C^3), excepté en un nombre fini de coins à bords droits, la mesure β du plus petit satisfaisant $0<\beta<\pi$. Si Γ_0 est un ouvert de $\partial\Omega$ tel qu'il existe $x_0\in\Gamma_0$ et $\delta>0$ avec $\partial\Omega\cap B(x_0,\delta)\subset\Gamma_0$, alors
$\exists C,\varepsilon_0>0,\forall\varepsilon\in\,]0,\varepsilon_0[,\forall u\in H^2(\Omega)$,

$$\|u\|_{H^1(\Omega)}\leqslant \mathrm{e}^{\mathrm{e}^{C/\varepsilon}}\left(\|Pu\|_{L^2(\Omega)}+\|u\|_{H^1(\Gamma_0)}+\|\partial_n u\|_{L^2(\Gamma_0)}\right)+\varepsilon^{\beta/\pi}\|u\|_{H^2(\Omega)}. \tag{1}$$

Pour tout ouvert $\omega\Subset\Omega$, on a une inégalité similaire en remplaçant $\|u\|_{H^1(\Gamma_0)}+\|\partial_n u\|_{L^2(\Gamma_0)}$ par $\|u\|_{H^1(\omega)}$ dans (1). Ce résultat complète celui de [8], c'est à dire l'inégalité (2) valable lorsque le domaine Ω est régulier (au moins de classe C^3). Nous vérifions également que dans le cas où Ω ne présente que des coins rentrants, soit $\beta\geqslant\pi$, l'inégalité (2) demeure valide.

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Ce type d'inégalité joue un rôle important dans les domaines de la contrôlabilité [8] et celui des problèmes inverses [1].

Nous commençons par l'étude de l'hypothèse d'hypoellipticité d'Hörmander pour l'opérateur P au voisinage d'un coin de mesure $\beta \leqslant \pi$, qui implique l'opérateur h-pseudodifférentiel $P_\phi = h^2 e^{\phi/h} \circ P \circ e^{-\phi/h}$ pour une fonction régulière ϕ. Précisément, le domaine considéré est $K_\varepsilon^0 = \{x = (r, \theta) \in [0, R_0] \times [0, \beta], \psi_0(x) \geqslant \varepsilon\}$ avec $R_0 > 0$ et $\varepsilon \geqslant 0$, et nous posons $\phi = e^{\alpha\psi_0}$, $\alpha > 0$, ψ_0 étant une fonction harmonique qui s'annule sur les arêtes du coin. Lorsque $\beta < \pi$, il en ressort une nécessaire dépendance de α en fonction de ε pour satisfaire l'hypothèse d'hypoellipticité, qui s'écrit $\alpha > \alpha_0(\varepsilon) = 2(1 - \beta/\pi)(1/\varepsilon)$, et par conséquent une dégradation prévisible de l'inégalité de stabilité dans un domaine présentant des coins par rapport à celle dans un domaine régulier.

Nous établissons ensuite une inégalité de Carleman globale au coin (Proposition 3.1), dans le domaine $K_\varepsilon = \{x = (r, \theta) \in [0, R_0] \times [0, \beta], \psi(x) \geqslant \varepsilon\}$, le poids utilisé étant $\phi = e^{\alpha\psi}$ avec $\alpha > \alpha_0(\varepsilon)$ et la fonction ψ étant judicieusement choisie. Plus précisément, ψ est telle que $\psi = \psi_0$ au voisinage du sommet du coin et telle que la ligne de niveau $\psi = 0$ s'écarte des bords du coin loin du sommet.

Nous en déduisons ensuite une inégalité de stabilité dans un coin (Proposition 4.3), qui exprime le contrôle approché de la norme H^1 d'une fonction dans le coin par la somme de la norme L^2 de son image par P dans le coin et de la norme H^1 de la fonction dans un ouvert relativement compact du coin. Cette inégalité provient du recollement d'une estimation loin du sommet issue de l'inégalité de Carleman, et d'une estimation dans le domaine complémentaire issue d'une inégalité de Hardy, comme dans [8].

L'inégalité (1) est alors obtenue en combinant la Proposition 4.3 avec trois autres propositions démontrées dans [8] et rappelées ici (Propositions 5.2, 5.3 et 5.4).

1. Introduction

In [8], the following stability estimate was obtained for the operator $P = -\Delta . - k.$ in a bounded and connected domain $\Omega \subset \mathbb{R}^n$ of class C^∞. If Γ_0 is an open domain of $\partial\Omega$ such that there exist $x_0 \in \Gamma_0$ and $\delta > 0$ with $\partial\Omega \cap B(x_0, \delta) \subset \Gamma_0$, then it is proved in [8] that:

$\forall k \in \mathbb{R}$, $\forall r \in]0, 1[$, $\exists C, \varepsilon_0 > 0$ such that $\forall \varepsilon \in]0, \varepsilon_0[$, $\forall u \in H^2(\Omega)$,

$$\|u\|_{H^1(\Omega)} \leqslant e^{C/\varepsilon}\big(\|Pu\|_{L^2(\Omega)} + \|u\|_{H^1(\Gamma_0)} + \|\partial_n u\|_{L^2(\Gamma_0)}\big) + \varepsilon^r \|u\|_{H^2(\Omega)}. \tag{2}$$

A similar estimate holds with $\|u\|_{H^1(\omega)}$ replacing $\|u\|_{H^1(\Gamma_0)} + \|\partial_n u\|_{L^2(\Gamma_0)}$ in (2) for any open domain $\omega \Subset \Omega$. This kind of stability estimates, which are strongly related to the ill-posed Cauchy problem for the operator P, plays an important role in the fields of controllability [8] and inverse problems [1]. The proof of (2) is mainly based on Carleman estimates [5,6]. An additional analysis of the work exposed in [6] would show that (2) still holds for domains of class C^3.

However, to the author's knowledge, no estimate such as (2) seems to exist for less smooth domains. In the following Note, we obtain a Carleman estimate near a corner in $\mathbb{R}^2$, which enables us to derive an estimate such as (2) for a domain of $\mathbb{R}^2$, the boundary of which has a finite number of corners with straight edges and is smooth elsewhere, like a polygon.

Detailed proofs of the results announced in this Note are given in [1].

2. A preliminary remark on hypoellipticity

In this section, for $\beta \in]0, \pi]$ and $R_0 > 0$, we consider the corner domain $K = \{x = (r, \theta) \in [0, R_0] \times [0, \beta]\}$, and the harmonic function ψ_0 defined in K by

$$\psi_0(x) = r^{\pi/\beta} \sin\left(\frac{\pi}{\beta}\theta\right). \tag{3}$$

The level curve $\psi_0 = 0$ is the boundary $\{\theta = 0\} \cap \{\theta = \beta\}$ of K. Next, we define for a given $\varepsilon \geqslant 0$, $K_\varepsilon^0 = \{x \in K, \psi_0(x) \geqslant \varepsilon\}$ and $\partial K_\varepsilon^0 = \{x \in K, \psi_0(x) = \varepsilon\}$. We now look for the $\alpha > 0$ for which $\phi = e^{\alpha\psi_0}$ fulfills the hypoellipticity assumption of Hörmander in domain K_ε^0 for the operator P, when $\varepsilon > 0$. To this end, we introduce

h-pseudodifferential operators as in [2], in particular the operator $P_\phi = h^2 e^{\phi/h} \circ P \circ e^{-\phi/h}$, and its principal symbol $p_\phi(x,\xi)$. The hypoellipticity assumption amounts to the existence of $c_1(\varepsilon) > 0$ such that

$$p_\phi(x,\xi) = 0 \quad \text{and} \quad (x,\xi) \in K^0_\varepsilon \times \mathbb{R}^2 \quad \Longrightarrow \quad \{\operatorname{Re} p_\phi, \operatorname{Im} p_\phi\}(x,\xi) \geqslant c_1,$$

with $\{\operatorname{Re} p_\phi, \operatorname{Im} p_\phi\} = \nabla_\xi \operatorname{Re} p_\phi \cdot \nabla_x \operatorname{Im} p_\phi - \nabla_x \operatorname{Re} p_\phi \cdot \nabla_\xi \operatorname{Im} p_\phi$. We find after a straightforward calculation that $\operatorname{Re} p_\phi = |\xi|^2 - |\nabla\phi|^2$ and $\operatorname{Im} p_\phi = 2\xi.\nabla\phi$, and thus that for $p_\phi(x,\xi) = 0$ and $(x,\xi) \in K^0_\varepsilon \times \mathbb{R}^2$,

$$\{\operatorname{Re} p_\phi, \operatorname{Im} p_\phi\}(x,\xi) \geqslant 4\alpha^3\phi^3|\nabla\psi_0|^2 \frac{\pi}{\beta} r^{2\pi/\beta-2}\left(-2\left(\frac{\pi}{\beta}-1\right)\frac{1}{\varepsilon} + \alpha\frac{\pi}{\beta}\right).$$

As a result, if $\beta = \pi$, which corresponds to $\psi_0(x) = x_2$, the hypoellipticity assumption holds as soon as $\alpha > 0$, with α independent of ε. But if $\beta < \pi$, the hypoellipticity assumption holds as soon as

$$\alpha > \alpha_0(\varepsilon) := 2\left(1 - \frac{\beta}{\pi}\right)\frac{1}{\varepsilon}. \tag{4}$$

Hence, we expect that an estimate of $\|u\|_{H^1(\Omega)}$ will require larger ε-dependent coefficients than in (2).

3. A Carleman estimate near a corner

In this section, we assume $0 < \beta < \pi$. Let γ denote a C^2 decreasing function on $[0, R_0]$, $\gamma(r) = 1$ on $[0, r_0]$ with $0 < r_0 < R_0$, and $\gamma(R_0) > 0$. We introduce the function ψ defined for $(r,\theta) \in K$ by

$$\psi(r,\theta) = \gamma(r)\psi_0(r,\theta) + \big(1-\gamma(r)\big)\psi_1(r,\theta), \quad \psi_1(r,\theta) = -r^{\pi/\beta}\cos\left(\frac{2\pi}{\beta}\theta\right),$$

and the associated domains $K_\varepsilon = \{x \in K, \psi(x) \geqslant \varepsilon\}$ and $\partial K_\varepsilon = \{x \in K, \psi(x) = \varepsilon\}$. Note that the level curve $\psi = 0$ deviates from $\{\theta = 0\}$ and $\{\theta = \beta\}$ inside K for $r > r_0$. If we denote for $\varepsilon \geqslant 0$, $I_\varepsilon = K_\varepsilon \cap \overline{B(0,r_0)}$ and $E_\varepsilon = K_\varepsilon \setminus B(0,r_0)$, the useful properties of ψ are that for $\varepsilon > 0$, ψ is C^2 in $I_\varepsilon \cup E_0$, with $\nabla\psi \neq 0$ in $I_\varepsilon \cup E_0$, and $\psi = \psi_0$ in I_ε.

If $H^2_0(K)$ denotes the set of the restrictions to K of functions in $H^2_0(B(0,R_0))$, we obtain the following Carleman estimate:

Proposition 3.1. *There exist $L, \varepsilon_0, \lambda_0 > 0$ such that $\forall\varepsilon \in]0,\varepsilon_0[$, $\forall\lambda > \lambda_0$, $\forall\alpha > \alpha_0(\varepsilon)$ with α_0 given by* (4)*, $\forall u \in H^2_0(K)$,*

$$\alpha^4\lambda^3\int_{K_\varepsilon}\phi^3|\nabla\psi|^4u^2e^{2\lambda\phi}\,dx + \alpha^2\lambda\int_{K_\varepsilon}\phi|\nabla\psi|^2|\nabla u|^2e^{2\lambda\phi}\,dx$$
$$\leqslant L\int_{K_\varepsilon}|Pu|^2e^{2\lambda\phi}\,dx + L\alpha\lambda\int_{\partial K_\varepsilon}\phi|\nabla\psi||\nabla u|^2e^{2\lambda\phi}\,d\Gamma + L\alpha^3\lambda^3\int_{\partial K_\varepsilon}\phi^3|\nabla\psi|^3u^2\,e^{2\lambda\phi}\,d\Gamma.$$

The proof of Proposition 3.1 is based on a classical technique that allows to derive global Carleman estimates simply by integrating by parts. Such a technique is for example detailed in [3]. The novelty here is that the above Carleman estimate is uniform in ε. Besides, it follows from the property $\Delta\psi = \Delta\psi_0 = 0$ in I_ε that any value of parameter α given by (4), which results from the hypoellipticity assumption, is sufficient to satisfy the Carleman estimate.

Remark 1. Proposition 3.1 remains true for $\beta = \pi$ if assumption (4) is replaced by the uniform inequality $\alpha > \alpha_0$ for some $\alpha_0 > 0$.

4. A stability estimate at a corner

We begin with an important remark concerning corners of measure $\beta > \pi$, which enables us to stick to the analysis of stability at corners of measure $\beta < \pi$ in the following:

Remark 2. In the vicinity of a corner of measure $> \pi$, it is clear that the stability is the same as in the vicinity of a smooth point. Actually, by simply extending the two edges of $\partial\Omega$ adjacent to the corner inside Ω, the norm of any function in the corner is smaller than the sum of the norms of the restriction of the same function in two different half-discs centered at the corner. Hence we come back to the case $\beta = \pi$.

We continue with the two following lemmas. The first one is a trace inequality on ∂K_ε which is uniform in ε. It is obtained by using the two bisectors of the corner as a new system of coordinates, and by applying the standard technique of [7], p. 15. The second one is deduced from a Hardy's inequality and is proved in [4], p. 29:

Lemma 4.1. *There exist $c > 0$ (independent of ε) and $\varepsilon_0 > 0$ such that $\forall \varepsilon \in]0, \varepsilon_0[$, $\forall u \in H^2(K)$,*

$$\|u\|_{H^1(\partial K_\varepsilon)} + \|\partial_n u\|_{L^2(\partial K_\varepsilon)} \leqslant c\|u\|_{H^2(K_\varepsilon)}.$$

Lemma 4.2. *Let Ω be an open, connected and bounded domain with Lipschitz continuous boundary $\partial\Omega$, $\rho(x)$ the distance from x to $\partial\Omega$. Then $\forall r \in]0, \frac{1}{2}[$, $\exists c > 0$ such that*

$$\forall u \in H^r(\Omega), \quad \left\| \frac{u}{\rho^r} \right\|_{L^2(\Omega)} \leqslant c\|u\|_{H^r(\Omega)}.$$

For $\beta \in]0, \pi[$, the following estimate holds in the corner domain K:

Proposition 4.3. *There exists an open domain $\omega \Subset \overset{\circ}{K}$, there exist $C, \varepsilon_0 > 0$ such that,*

$$\forall \varepsilon \in]0, \varepsilon_0[, \forall u \in H^2(K), \quad \|u\|_{H^1(K \cap B(0,r_0))} \leqslant e^{e^{C/\varepsilon}} \big(\|Pu\|_{L^2(K)} + \|u\|_{H^1(\omega)}\big) + \varepsilon^{\beta/\pi} \|u\|_{H^2(K)}.$$

Sketch of proof. the different steps are approximately the same as in [8], Lemma 3.2.

The first step consists in finding a stability estimate in I_ε by using Proposition 3.1 with α such that (4) is fulfilled and by using Lemma 4.1. We conclude that there exist $L, C, C', \varepsilon_0, \lambda_0 > 0$ such that $\forall \varepsilon \in]0, \varepsilon_0[$, $\forall \lambda > \lambda_0$, $\forall \alpha > \alpha_0(\varepsilon)$, $\forall u \in H_0^2(K)$,

$$\|u\|_{H^1(K_{2\varepsilon})} \leqslant L e^{\lambda e^{C/\varepsilon}} \|Pu\|_{L^2(K_\varepsilon)} + L \frac{\lambda}{\varepsilon^{3/2-\beta/\pi}} e^{-C'\lambda} \|u\|_{H^2(K_\varepsilon)}.$$

Next, for any $s > 0$ we now define λ such that $\lambda = -s \log \varepsilon$. Hence, for some new constant L,

$$\|u\|_{H^1(K_{2\varepsilon})} \leqslant L e^{e^{(C+s)/\varepsilon}} \|Pu\|_{L^2(K_\varepsilon)} + L \varepsilon^{S(s)} \|u\|_{H^2(K_\varepsilon)},$$

with $S(s) = C's/2 - 3/2 + \beta/\pi$. Since $I_{2\varepsilon} \subset K_{2\varepsilon}$ and $K_\varepsilon \subset K_0 \subset K$, we obtain the following estimate in $I_{2\varepsilon}$,

$$\|u\|_{H^1(I_{2\varepsilon})} \leqslant L e^{e^{(C+s)/\varepsilon}} \|Pu\|_{L^2(K_0)} + L \varepsilon^{S(s)} \|u\|_{H^2(K)}. \tag{5}$$

The second step consists in finding an estimate in the complementary part of I_ε in I_0, which is denoted J_ε, by using Lemma 4.2.

In Lipschitz continuous domain I_0, we have that for $r \in]0, 1/2[$, for $u \in H_0^2(K)$, $\|v/\rho^r\|_{L^2(I_0)} \leqslant c\|v\|_{H^r(I_0)}$, with $v = u$ or $v = \partial_i u$, $i = 1, 2$. Since we derive that $\rho \leqslant \varepsilon^{\beta/\pi}$ in J_ε,

$$\|v\|_{L^2(J_\varepsilon)} \leqslant c\varepsilon^{r\beta/\pi} \|v\|_{H^r(I_0)} \leqslant c\varepsilon^{r\beta/\pi} \|v\|_{H^{1/2}(I_0)}.$$

By using a classical interpolation inequality, we find that $\forall \eta > 0$,

$$\|u\|_{H^1(J_\varepsilon)} \leqslant c' \left(\frac{\varepsilon^{2r\beta/\pi}}{\eta} \|u\|_{H^2(I_0)} + \eta \|u\|_{H^1(I_0)} \right). \tag{6}$$

Since $\|u\|_{H^1(I_0)} \leqslant \|u\|_{H^1(I_{2\varepsilon})} + \|u\|_{H^1(J_{2\varepsilon})}$, if we combine (5) and (6) by choosing $s > 0$ such that $S(s) = 2r\beta/\pi$ and $\eta > 0$ such that $c'\eta = 1/2$, it follows that $\forall S < \beta/\pi$, $\exists C, \varepsilon_0 > 0$ such that $\forall \varepsilon \in]0, \varepsilon_0[$, $\forall u \in H_0^2(K)$,

$$\|u\|_{H^1(I_0)} \leqslant e^{e^{C/\varepsilon}} \|Pu\|_{L^2(K_0)} + \varepsilon^S \|u\|_{H^2(K)}.$$

A deeper analysis of the different steps of the above proof shows that C and ε_0 can be chosen independently of $S < \beta/\pi$, which allows us to take the limit $S \to \beta/\pi$. Thus, the above estimate remains true for $S = \beta/\pi$.

The third step consists in using a cut-off function in order to come back to a function in $H^2(K)$.

For $u \in H^2(K)$, we set $v = \chi u$, where χ is a C^2 function such that $\chi = 1$ in $\overline{B(0, r_0')}$ and $\chi = 0$ in $K/B(0, r_0'')$, with $0 < r_0 < r_0' < r_0'' < R_0$.

Since $P(\chi u) = \chi Pu - 2\nabla\chi \cdot \nabla u - (\Delta\chi)u$, if we plug $v = \chi u$ in the previous estimate with $S = \beta/\pi$, we infer that there exists $L > 0$ such that

$$\|u\|_{H^1(I_0)} \leqslant L e^{e^{C/\varepsilon}} \big(\|Pu\|_{L^2(K_0)} + \|u\|_{H^1(K_0(r_0', r_0''))}\big) + L\varepsilon^{\beta/\pi} \|u\|_{H^2(K)}.$$

Here, $K_0(r_0', r_0'')$ denotes the set $\{x \in K_0, r_0' \leqslant r \leqslant r_0''\}$.

We remark that $K_0(r_0', r_0'')$, given the choice made for ψ, is contained in an open set ω that satisfies $\omega \Subset \mathring{K}$. This completes the proof of Proposition 4.3. □

5. Conclusion

In the following, a smooth point of the boundary $\partial\Omega$ denotes a point where the boundary is smooth, that is at least of class C^3. Our main result is the following theorem:

Theorem 5.1. *Let Ω be a bounded and connected domain $\Omega \subset \mathbb{R}^2$ with a smooth boundary $\partial\Omega$ (at least of class C^3) except at a finite number of corners with straight edges, the smaller measure of which is $\beta > 0$. If Γ_0 is an open domain of $\partial\Omega$ such that there exist $x_0 \in \Gamma_0$ and $\delta > 0$ with $\partial\Omega \cap B(x_0, \delta) \subset \Gamma_0$, then $\exists C, \varepsilon_0 > 0$, $\forall \varepsilon \in]0, \varepsilon_0[$, $\forall u \in H^2(\Omega)$,*

$$\|u\|_{H^1(\Omega)} \leqslant e^{e^{C/\varepsilon}} \big(\|Pu\|_{L^2(\Omega)} + \|u\|_{H^1(\Gamma_0)} + \|\partial_n u\|_{L^2(\Gamma_0)}\big) + \varepsilon^{\beta/\pi} \|u\|_{H^2(\Omega)}. \tag{7}$$

If $\beta \geqslant \pi$ or if $\partial\Omega$ is of class C^3, the improved inequality (2) *holds.*

Remark 3. An estimate similar to (7) holds with $\|u\|_{H^1(\omega)}$ instead of $\|u\|_{H^1(\Gamma_0)} + \|\partial_n u\|_{L^2(\Gamma_0)}$ in (7) for any open domain $\omega \Subset \Omega$.

The proof of our theorem follows from the combination of Proposition 4.3 and of the three propositions below, which are proved in [8]. Proposition 5.2 enables us to 'propagate' Cauchy data on Γ_0 to a vicinity of any smooth point x_0 of Γ_0, in particular to an open domain $\omega_0 \Subset \Omega$. Proposition 5.3 enables us to 'propagate' data from this open domain ω_0 to any other open domain $\omega_1 \Subset \Omega$. Lastly, Proposition 5.4 (resp. 4.3) enables us to propagate data on an open domain $\omega_1 \Subset \Omega$ up to a vicinity of any smooth point x (resp. of any corner x) with $x \in \partial\Omega$.

Proposition 5.2. *Let $x_0 \in \Gamma_0$ (x_0 is a smooth point of the boundary). There exists a vicinity ω_0 of x_0, there exist $s, c, \varepsilon_0 > 0$ such that $\forall \varepsilon \in]0, \varepsilon_0[$, $\forall u \in H^2(\Omega)$,*

$$\|u\|_{H^1(\Omega\cap\omega_0)} \leqslant \frac{c}{\varepsilon}\big(\|Pu\|_{L^2(\Omega)} + \|u\|_{H^1(\Gamma_0)} + \|\partial_n u\|_{L^2(\Gamma_0)}\big) + \varepsilon^s \|u\|_{H^1(\Omega)}.$$

Proposition 5.3. *Let ω_0, ω_1 be two open domains such that $\omega_0, \omega_1 \Subset \Omega$. There exist $s, c, \varepsilon_0 > 0$ such that $\forall \varepsilon \in]0, \varepsilon_0[$, $\forall u \in H^2(\Omega)$,*

$$\|u\|_{H^1(\omega_1)} \leqslant \frac{c}{\varepsilon}\big(\|Pu\|_{L^2(\Omega)} + \|u\|_{H^1(\omega_0)}\big) + \varepsilon^s \|u\|_{H^1(\Omega)}.$$

Proposition 5.4. *Let $x \in \partial\Omega$ (x is a smooth point of the boundary). There exist a vicinity ω of x and an open domain $\omega_1 \Subset \Omega$ such that $\forall r \in]0, 1[$, there exist $c, \varepsilon_0 > 0$, $\forall \varepsilon \in]0, \varepsilon_0[$, $\forall u \in H^2(\Omega)$,*

$$\|u\|_{H^1(\Omega\cap\omega)} \leqslant e^{c/\varepsilon}\big(\|Pu\|_{L^2(\Omega)} + \|u\|_{H^1(\omega_1)}\big) + \varepsilon^r \|u\|_{H^2(\Omega)}.$$

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