Available online at www.sciencedirect.com

ScienceDirect

C. R. Acad. Sci. Paris, Ser. I 347 (2009) 959–964

Comptes Rendus Mathematique

Probability Theory

Nash equilibrium point for one kind of stochastic nonzero-sum game problem and BSDEs ☆

Jean-Pierre Lepeltier [a], Zhen Wu [b], Zhiyong Yu [c,d]

[a] *Département de Mathématiques, Université du Maine, avenue O. Messiaen, 72085 Le Mans cedex 9, France*
[b] *School of Mathematics and System Science, Shandong University, Jinan 250100, China*
[c] *School of Economics, Shandong University, Jinan 250100, China*
[d] *Département de Mathématiques, Université d'Évry Val d'Essonne, 91025 Évry cedex, France*

Received 24 June 2008; accepted after revision 2 March 2009

Available online 11 June 2009

Presented by Marc Yor

Abstract

In this Note, we deal with one kind of stochastic nonzero-sum differential game problem for N players. Using the theory of backward stochastic differential equations and Malliavin calculus, we give the explicit form of a Nash equilibrium point. ***To cite this article: J.-P. Lepeltier et al., C. R. Acad. Sci. Paris, Ser. I 347 (2009).***

Résumé

Equilibre de Nash pour un jeu de somme non nulle et équations rétrogrades. Dans cette Note nous nous intéressons à un problème particulier de jeu différentiel stochastique de somme non nulle à N joueurs. En utilisant la théorie des équations différentielles stochastiques rétrogrades et le calcul de Malliavin nous donnons la forme explicite d'un équilibre de Nash. ***Pour citer cet article : J.-P. Lepeltier et al., C. R. Acad. Sci. Paris, Ser. I 347 (2009).***

Version française abrégée

Soit (Ω, F, P) un espace de probabilité et $(W_t)_{0\leqslant t\leqslant T}$ un Brownien standard n-dimensionnel défini sur cet espace. Sa filtration naturelle est notée $(F_t)_{t\leqslant T}$.

Une trajectoire du système partant de x est simplement

$$x(t,\omega) = x + W_t.$$

☆ This work is supported by the Natural Science Foundation of China (10671112), the National Basic Research Program of China (973 Program, No. 2007CB814901 and No. 2007CB814904), the Natural Science Foundation of Shandong Province (JQ200801 and 2008BS01024) and the Doctoral Fund of Education Ministry of China.

E-mail addresses: lepeltier@univ-lemans.fr (J.-P. Lepeltier), wuzhen@sdu.edu.cn (Z. Wu), yuzhiyong@sdu.edu.cn (Z. Yu).

1631-073X/$ – see front matter
doi:10.1016/j.crma.2009.04.033

On considère N joueurs, chacun d'entre eux agissant sur une fonction de coût par un contrôle $u_i(t,\omega)$, $1 \leqslant i \leqslant N$. Un système de contrôles admissibles est alors tel que $u_i(t)$, $1 \leqslant i \leqslant N$ soient bornés.

Les fonctions de coût associées à chaque joueur sont données par (3). On cherche alors un équilibre de Nash pour le jeu de somme non nulle, c'est-à-dire un système de contrôles (u_i^*) $1 \leqslant i \leqslant N$ admissible tel que (4) soit réalisé.

Ce problème est étudié sous l'hypothèse (H_3) appelée condition minimax généralisée. Il est facile de noter que (7) est un équilibre de Nash pour la famille d'hamiltoniens donnée dans (5).

Le Lemme 2.1 consiste à établir le lien entre les fonctions de coût et les solutions de N équations différentielles stochastiques rétrogrades. On est amené à considérer la famille d'équations rétrogrades (10). On sait qu'il en existe une solution $(Y_i^*, Z_i^*)_{1\leqslant i\leqslant N}$ avec $(Z_i^*)_{1\leqslant i\leqslant N}$ à priori seulement de carré intégrable. Toutefois sous les hypothèses (H_4), en utilisant le calcul de Malliavin nous pouvons établir que les Z_i^* sont bornés.

Alors le système de contrôles donné par (11) est admissible. Il suffit d'utiliser the théorème de comparaison des solutions des équations rétrogrades (comme dans Hamadène–Lepeltier (1995) [3]) pour obtenir que le sytème (11) est un équilibre de Nash.

Pour terminer nous pouvons remarquer

1) Dans le cas Markovien, le problème avec un terme supplémentaire quadratique dans les fonctions de coût a été étudié par Bensoussan–Frehse (2000). Dans le cas non Markovien nous aurions besoin d'un résultat d'existence pour la solution d'équations rétrogrades à coefficient quadratique en z ; ce problème est encore ouvert.
2) Il nous semble que lorsque x satisfait

$$\mathrm{d}x_t = \sigma(t,x_t)\,\mathrm{d}W_t + b(x_t)\,\mathrm{d}t$$

le calcul de Malliavin soit encore opérationnel sous des hypothèses raisonnables (à définir) sur b et σ.

1. Introduction and formulation of the problem

The objective of this Note consists in applying the existence results on backward stochastic differential equations (BSDEs in short) to get the explicit form of a Nash equilibrium point for one kind of stochastic nonzero-sum differential game problem for N players.

Let $(\Omega,\mathcal{F},P)$ be a probability space and $(W(t))_{0\leqslant t\leqslant T}$ a standard n-dimensional Brownian motion, defined on this space, whose natural filtration is $\mathcal{F}_t = \sigma\{W(s), 0\leqslant s\leqslant t\}_{t\leqslant T}$; $\mathcal{P}$ is the σ-algebra generated by the progressively measurable processes on $[0,T]\times\Omega$.

A state trajectory starting at x, is simply

$$x(t,\omega) = x + W(t). \tag{1}$$

We now consider N players, who have no influence on the system (1), each of them acting through a control $u_i(t,\omega)$, $i = 1,2,\ldots,N$, to protect his advantages. We assume that $u(t) = (u_1(t),\ldots,u_N(t))$ is a progressively measurable process with bounded values in $\mathbb{R}^{nN}$ which we call an admissible control.

Let g be a function from $\Omega\times[0,T]\times\mathbb{R}^n$ into $\mathbb{R}^n$ which satisfies

Assumption (H1). *$g(t,x)$ is $\mathcal{P}\otimes\mathcal{B}(\mathbb{R}^n)$ measurable and bounded.*

For each x and admissible control $u(t)$, we associate the process $\beta_{x,u}(t) = g(t,x(t)) + \sum_{i=1}^N u_i(t)$, and the probability $P_{x,u}$ such that $\frac{\mathrm{d}P_{x,u}}{\mathrm{d}P}|_{\mathcal{F}_t} = \exp\{\int_0^t \beta_{x,u}^\tau(s)\,\mathrm{d}W(s) - \frac{1}{2}\int_0^t \|\beta_{x,u}(s)\|^2\,\mathrm{d}s\}$, where τ denotes transpose in the above and all the following. From Girsanov's Theorem, we know under $P_{x,u}$ the process $W_{x,u}(t) = W(t) - \int_0^t \beta_{x,u}(s)\,\mathrm{d}s$, $0\leqslant t\leqslant T$ is an $\mathcal{F}_t$-Brownian motion and $x(t)$ is a weak solution of:

$$\mathrm{d}x(t) = \left[g\big(t,x(t)\big) + \sum_{i=1}^N u_i(t)\right]\mathrm{d}t + \mathrm{d}W_{x,u}(t), \quad x(0) = x, \quad 0\leqslant t\leqslant T. \tag{2}$$

Let also f_i be functions from $\Omega\times[0,T]\times\mathbb{R}^n$ into $\mathbb{R}$ and Φ_i be functions from $\Omega\times\mathbb{R}^n$ into $\mathbb{R}$, $i = 1,2,\ldots,N$, such that

Assumption (H2). *The functions f_i are $\mathcal{P} \otimes \mathcal{B}(\mathbb{R}^n)$ measurable and bounded and the functions Φ_i are $\mathcal{F}_T \otimes \mathcal{B}(\mathbb{R}^n)$ measurable and bounded, $i = 1, 2, \ldots, N$.*

We define the cost functions corresponding to each player by:

$$J_i(x, u) = \mathbb{E}_{x,u}\left[\Phi_i\big(x(T)\big) + \int_0^T \left(f_i\big(t, x(t)\big) + \theta u_i(t) \cdot \sum_{j \neq i} u_j(t)\right) \mathrm{d}t\right], \quad i = 1, 2, \ldots, N, \tag{3}$$

where $\theta \neq 0$ is a constant. We shall also use the notation $u = (u_i, \bar{u}^i)$, where $\bar{u}^i$ represents all the components of the admissible control u other than u_i. Our problem is then to find an admissible control $u^* = (u_i^*, \bar{u}^{i*})$, called a Nash equilibrium point for the nonzero-sum game, such that

$$J_i\big(x, u_i^*, \bar{u}^{i*}\big) \leqslant J_i\big(x, u_i, \bar{u}^{i*}\big), \quad i = 1, 2, \ldots, N, \tag{4}$$

for any admissible control $u = (u_i, \bar{u}^i)$.

A similar nonzero-sum game problem was first studied by Bensoussan and Frehse (2000) [2]. However, their model is in the Markovian case and there is one extra quadratic term for u_i in the cost functions and the terminal time is a special stopping time. Using the solution of a Bellman equation which is one kind of second order partial differential equation, they got a Nash point for the problem. It is obvious that since our problem is under the non-Markovian frame, their method cannot be used in our case.

Using the results on BSDEs, Hamadène and Lepeltier (1995) [3] obtained the existence of a saddle-point strategy under the Isaac's condition for a zero-sum differential game problem and the existence of an optimal strategy for an optimal stochastic control problem. Then, Hamadène, Lepeltier and Peng (1997) [4] applied some results on BSDEs to prove the existence of a Nash equilibrium point for some nonzero-sum Markovian stochastic differential games.

In this Note, we intend to use the solutions of BSDEs to get an explicit Nash equilibrium point for our stochastic nonzero-sum game problem for N players in the non-Markovian case. The main tool is the existence result on BSDEs when the coefficient has a quadratic growth in Z, which can be found in Kobylanski (2000) [6] and Lepeltier and San Martin (1997) [7]. We are able to construct optimal controls by using $Z_i(\cdot)$ ($i = 1, 2, \ldots, N$), solutions of the corresponding BSDEs, for each player. Then it is necessary to use Malliavin calculus technique to obtain the representation and boundness of $Z_i(\cdot)$. The optimality has to be taken in the sense of Nash (1950) [8], see also J.P. Aubin (1976) [1].

As far as we know, this kind of non-Markovian stochastic game problem has not been considered before and it can be seen as one good application of BSDEs theory combined with Malliavin calculus techniques.

2. One explicit Nash equilibrium point

For $(\omega, t, x, p_i, u) \in \Omega \times [0, T] \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^{nN}$ we define the Hamiltonian functions associated with the game by

$$H_i(t, x, p_i, u) = p_i \cdot g(t, x) + p_i \cdot \sum_{j=1}^N u_j + f_i(t, x) + \theta u_i \cdot \sum_{j \neq i} u_j, \quad i = 1, 2, \ldots, N. \tag{5}$$

For the resolution of our nonzero-sum game problem, we assume the following assumption called the generalized mini–max condition which is the analogue of Isaac's condition in the zero-sum game frame.

Assumption (H3). *There exist N measurable functions $u_i^*(t, x, p_1, p_2, \ldots, p_N)$, which satisfy:*

$$H_i\big(t, x, p_i, u^*(t, x, p_1, \ldots, p_N)\big) \leqslant H_i\big(t, x, p_i, u_i, \bar{u}^{i*}(t, x, p_1, \ldots, p_N)\big), \quad i = 1, 2, \ldots, N, \tag{6}$$

here $u^*(t, x, p_1, \ldots, p_N) = (u_i^*(t, x, p_1, \ldots, p_N), \bar{u}^{i*}(t, x, p_1, \ldots, p_N))$, u_i is the ith component of any admissible control u.

Using Assumption (H3), we look for the possible form of a Nash equilibrium point $u^* = (u_1^*, u_2^*, \ldots, u_N^*)$. From (6), $u^*(\cdot)$ for H should satisfy: $p_i + \theta \sum_{j \neq i} u_j^* = 0$, $i = 1, 2, \ldots, N$. Then it is easy to get:

$$u_i^* = \frac{(N-2)p_i - \sum_{j\neq i} p_j}{(N-1)\theta}, \tag{7}$$

$$H_i\big(t, x, p_i, u^*(t, x, p_1, \ldots, p_N)\big) = H_i\big(t, x, p_i, u_i, \bar{u}^{i*}(t, x, p_1, \ldots, p_N)\big) = p_i \cdot g(t,x) + f_i(t,x) - \frac{p_i^2}{\theta}, \tag{8}$$

for $i = 1, 2, \ldots, N$.

Now we need to get p_i and check that $u^* = (u_1^*, u_2^*, \ldots, u_N^*)$ defined by (7) is one Nash equilibrium point. We need first the following result:

Lemma 2.1. *For any admissible control u, if (Y, Z) is the solution of the following N-dimensional BSDEs*:

$$\begin{cases} -\mathrm{d}Y_i(t) = [Z_i(t)\cdot g(t, x(t)) + Z_i(t)\cdot \sum_{j=1}^N u_j(t) + f_i(t, x(t)) + \theta u_i(t)\cdot \sum_{j\neq i} u_j(t)]\,\mathrm{d}t - Z_i^\tau(t)\,\mathrm{d}x(t), \\ Y_i(T) = \Phi_i(x(T)), \quad 0 \leqslant t \leqslant T, \quad i = 1, 2, \ldots, N, \end{cases} \tag{9}$$

then we have $J_i(x, u) = Y_i(0)$, $i = 1, 2, \ldots, N$.

The proof is almost the same like Theorem 3.2 in Hamadène and Lepeltier (1995) [3]. We omit it. We notice that, in fact, the generator of BSDEs (9) is just $H_i(t, x(t), Z_i(t), u(t))$. So, from Eq. (8), we only need to consider the following N-dimensional BSDEs with quadratic growth in Z_i:

$$\begin{cases} -\mathrm{d}Y_i^*(t) = [Z_i^*(t)\cdot g(t, x(t)) + f_i(t, x(t)) - \frac{(Z_i^*(t))^2}{\theta}]\,\mathrm{d}t - Z_i^{*\tau}(t)\,\mathrm{d}x(t), \\ Y_i(T) = \Phi_i(x(T)), \quad 0 \leqslant t \leqslant T, \quad i = 1, 2, \ldots, N. \end{cases} \tag{10}$$

From Theorem 1 in Lepeltier and San Martin (1997) [7] or Theorem 2.3 in Kobylanski (2000) [6], there exists $(Y^*, Z^*) = (Y_1^*, \ldots, Y_N^*, Z_1^*, \ldots, Z_N^*)$ satisfying (10), with $\|Y_i^*\|$, $i = 1, 2, \ldots, N$, bounded. Normally Z_i^*, $i = 1, 2, \ldots, N$, are only square integrable from general BSDE's theory. However if, under suitable conditions we can prove that Z_i^*, $i = 1, 2, \ldots, N$, are bounded, then from (7), we can take:

$$u_i^* = \frac{(N-2)Z_i^* - \sum_{j\neq i} Z_j^*}{(N-1)\theta} \tag{11}$$

admissible, and get $J_i(x, u^*) = Y_i^*(0)$. It follows by the comparison theorem on BSDE's that $J_i(x, u^*) \leqslant J_i(x, u_i, \bar{u}^{i*})$, $i = 1, 2, \ldots, N$.

We look back BSDEs (10) again and make the change of variables $\bar{Y}_i(t) = \exp(-\frac{2}{\theta}Y_i^*(t))$, then get:

$$\bar{Y}_i(t) = \exp\bigg(-\frac{2}{\theta}\Phi_i\big(x(T)\big)\bigg) + \int_t^T \bigg[-\frac{2}{\theta}\bar{Y}_i(s) f_i\big(s, x(s)\big) + \Lambda_i(s)\cdot g\big(s, x(s)\big)\bigg]\mathrm{d}s - \int_t^T \Lambda_i^\tau(s)\,\mathrm{d}x(s); \tag{12}$$

here $\Lambda_i(t) = -\frac{2}{\theta}\bar{Y}_i(t) Z_i^*(t)$. Since BSDEs (12) are linear equations, we can get the explicit form of $\bar{Y}_i(t)$:

$$\bar{Y}_i(t) = \mathbb{E}\bigg[\exp\bigg(-\frac{2}{\theta}\Phi_i\big(x(T)\big)\bigg) Q_i(T)\,\bigg|\,\mathcal{F}_t\bigg], \quad 0 \leqslant t \leqslant T, \tag{13}$$

where $Q_i(T) = \exp\{\int_t^T [-\frac{2}{\theta} f_i(s, x(s)) - \frac{1}{2} g^2(s, x(s))]\,\mathrm{d}s + \int_t^T g^\tau(s, x(s))\,\mathrm{d}x(s)\}$. Using Malliavin calculus, we show in the next section that Λ_i^*, and thus Z_i^*, $i = 1, 2, \ldots, N$, are bounded.

3. Malliavin derivatives

We borrow some notions about Malliavin derivatives on Wiener space from El Karoui, Peng and Quenez (1997) [5] and Nualart (1995) [9]. We need the following:

Assumption (H4). (i) *$g(t, x)$, $f_i(t, x)$, $\Phi_i(x)$ $(i = 1, 2, \ldots, N)$ are continuously differentiable in x and the partial derivatives are bounded.* (ii) *For each x, $\Phi_i(x) \in \mathbb{D}_{1,2}$ $(i = 1, 2, \ldots, N)$ and the Malliavin derivative denoted by $D_t\Phi(x)$ are bounded for all $t \in [0, T]$.* (iii) *For a.e. $(t, x) \in [0, T] \times \mathbb{R}^n$, $g(t, x) \in \mathbb{D}_{1,2}$, $f_i(t, x) \in \mathbb{D}_{1,2}$ $(i = 1, 2, \ldots, N)$, the Malliavin derivatives denoted by $D_t g(s, x)$, $D_t f_i(s, x)$ are bounded for all $s, t \in [0, T]$, $Dg(t, x)$ and $Df_i(t, x)$ are $\mathcal{P} \otimes \mathcal{B}(\mathbb{R}^n)$ measurable.*

By Proposition 5.3 in El Karoui, Peng and Quenez (1997) [5], we know that

$$\Lambda_i(t) = D_t \bar{Y}_i(t) = D_t \mathbb{E}\big[e^{-\frac{2}{\theta}\Phi_i(x(T))} Q_i(T) \bigm| \mathcal{F}_t\big] = \mathbb{E}\big[D_t\big(e^{-\frac{2}{\theta}\Phi_i(x(T))} Q_i(T)\big) \bigm| \mathcal{F}_t\big]. \tag{14}$$

We calculate the Malliavin derivative and get:

$$\begin{aligned} &D_t\big(e^{-\frac{2}{\theta}\Phi_i(x(T))} Q_i(T)\big) \\ &\quad = e^{-\frac{2}{\theta}\Phi_i(x(T))} Q_i(T)\Bigg\{-\frac{2}{\theta}\bigg(\frac{\partial \Phi_i}{\partial x}\big(x(T)\big) + D_t\Phi_i\big(x(T)\big)\bigg) + \int_t^T \bigg[\frac{\partial g}{\partial x}\big(s, x(s)\big) + D_t g\big(s, x(s)\big)\bigg]\,\mathrm{d}x(s) \\ &\qquad + \int_t^T \bigg[-\frac{2}{\theta}\bigg(\frac{\partial f_i}{\partial x}\big(s, x(s)\big) + D_t f_i\big(s, x(s)\big)\bigg) - \bigg(\frac{\partial g}{\partial x}\big(s, x(s)\big) + D_t g\big(s, x(s)\big)\bigg) g\big(s, x(s)\big)\bigg]\,\mathrm{d}s\Bigg\}, \end{aligned}$$

then $\|D_t(e^{-\frac{2}{\theta}\Phi_i(x(T))} Q_i(T))\| \leqslant C Q_i(T) + e^{-\frac{2}{\theta}\Phi_i(x(T))} Q_i(T)\|N(T)\|$, where C is a universal constant and change from line to line, and $N(T) = \int_t^T [\frac{\partial g}{\partial x}(s, x(s)) + D_t g(s, x(s))]\,\mathrm{d}x(s)$. We get finally:

$$\begin{aligned} \|\Lambda_i(t)\| &\leqslant C\mathbb{E}\big[Q_i(T) \mid \mathcal{F}_t\big] + \mathbb{E}\big[e^{-\frac{2}{\theta}\Phi_i(x(T))} Q_i(T)\|N(T)\| \bigm| \mathcal{F}_t\big] \\ &\leqslant C + \frac{1}{2}\mathbb{E}\big[e^{-\frac{4}{\theta}\Phi_i(x(T))} Q_i^2(T) \bigm| \mathcal{F}_t\big] + \frac{1}{2}\mathbb{E}\big[N^2(T) \bigm| \mathcal{F}_t\big] \\ &\leqslant C + \frac{1}{2}\mathbb{E}\Bigg[\int_t^T \bigg(\frac{\partial g}{\partial x}\big(s, x(s)\big) + D_t g\big(s, x(s)\big)\bigg)^2 \mathrm{d}s \biggm| \mathcal{F}_t\Bigg] \leqslant C. \end{aligned}$$

Theorem 3.1. *Under Assumptions* (H1)–(H4), $u^* = (u_1^*, \ldots, u_i^*, \ldots, u_N^*)$, *where* u_i^* *defined by* (11), *is one Nash equilibrium point for the stochastic nonzero-sum differential game problem,* $J_i(x, u^*) = Y_i^*(0)$ *and* $J_i(x, u^*) \leqslant J_i(x, u_i, \bar{u}^{i*})$, u_i *is the* i*th component of any admissible control* u, $i = 1, 2, \ldots, N$, *and* $(Y_i^*(\cdot), Z_i^*(\cdot))$ *is one solution of the BSDE* (10).

In the Markovian case for our game problem, we can weaken Assumption (H4) in:

Assumption (H5). (i) $g(t, x)$ *and* $f_i(t, x)$ $(i = 1, 2, \ldots, N)$ *are continuous in* t. (ii) *There exists a constant* $L > 0$ *such that for any* $x, x' \in \mathbb{R}^n$,

$$\big|g(t, x) - g(t, x')\big| + \big|f_i(t, x) - f_i(t, x')\big| + \big|\Phi_i(x) - \Phi_i(x')\big| \leqslant L|x - x'|, \quad i = 1, 2, \ldots, N.$$

Corollary 3.2. *In the Markovian case for the stochastic nonzero-sum differential game problem, under Assumptions* (H1), (H2), (H3) *and* (H5), $u^* = (u_1^*, \ldots, u_i^*, \ldots, u_N^*)$, *where* u_i^* *is defined by* (11), *is one Nash equilibrium point,* $J_i(x, u^*) = Y_i^*(0)$ *and* $J_i(x, u^*) = J_i(x, u_i, \bar{u}^{i*})$, u_i *is* i*th component of any admissible control* u, $i = 1, 2, \ldots, N$, *where* $(Y_i^*(\cdot), Z_i^*(\cdot))$ *is one solution of the BSDEs* (10).

Remark 1. In the Markovian case, Bensoussan and Frehse (2000) [2] studied the stochastic nonzero-sum game problem with an extra quadratic control term in the cost functions. For this problem in the non-Markovian case, we need an existence result for multi-dimensional BSDEs with quadratic growth in multi-dimensional Z. Up to now, this kind of result in the general case, is still an open problem.

Remark 2. We feel that, in the more general case where x satisfies a stochastic differential equation of the form

$$\mathrm{d}x(t) = b\big(t, x(t)\big)\,\mathrm{d}t + \sigma\big(t, x(t)\big)\,\mathrm{d}W(t), \quad t \geqslant 0,$$

the Malliavin calculus is still efficient to get a similar result, the specific assumptions on the coefficients b and σ remaining open to discussion at this stage of our investigation.

Acknowledgements

Partial revised work of this Note was accomplished during a stay of Z. Yu at Université d'Évry, France. He would like to thank Federation bancaire Francaise for providing the post-doctor position. The authors are grateful to the two anonymous referees for their constructive comments. In particular, a relaxed assumption of the state trajectory, which was adopted in Remark 2 of this version, was suggested by one of the referees.

References

[1] J.P. Aubin, Mathematical Methods of Game and Economic Theory, Studies in Mathematics and Its Applications, North-Holland, Amsterdam, 1976.
[2] A. Bensoussan, J. Frehse, Stochastic games for N players, J. Optim. Theory Appl. 105 (2000) 543–565.
[3] S. Hamadène, J.P. Lepeltier, Zero-sum stochastic differential games and backward equations, Systems Control Lett. 24 (1995) 259–263.
[4] S. Hamadène, J.P. Lepeltier, S. Peng, BSDEs with continuous coefficients and stochastic differential games, in: El. Karoui, L. Mazliak (Eds.), Pitman Research Notes in Mathematics Series, vol. 364, Langman, Harlow, 1997, pp. 115–128.
[5] N. El Karoui, S. Peng, M.C. Quenez, Backward stochastic differential equations in finance, Math. Finance 7 (1) (1997) 1–71.
[6] M. Kobylanski, Backward stochastic differential equations and partial differential equations with quadratic growth, Ann. Probab. 28 (2000) 558–602.
[7] J.P. Lepeltier, J. San Martin, Existence for BSDE with superlinear-quadratic coefficient, Stochastics Stochastics Rep. 63 (1997) 227–240.
[8] J. Nash, Equilibrium points in n-person games, Proc. Natl. Acad. Sci. USA 36 (1950) 48–49.
[9] D. Nualart, The Malliavin Calculus and Related Topics, Probability and Its Applications, Springer-Verlag, New York and Berlin, 1995.