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Group Theory

On representation zeta functions of groups and a conjecture of Larsen–Lubotzky

Sur les fonctions zêta de représentations de groupes et sur une conjecture de Larsen–Lubotzky

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ABSTRACT

We study zeta functions enumerating finite-dimensional irreducible complex linear representations of compact p-adic analytic and of arithmetic groups. Using methods from $\mathfrak{p}$-adic integration, we show that the zeta functions associated to certain p-adic analytic pro-p groups satisfy functional equations. We prove a conjecture of Larsen and Lubotzky regarding the abscissa of convergence of arithmetic groups of type A_2 defined over number fields, assuming a conjecture of Serre on lattices in semisimple groups of rank greater than 1.

RÉSUMÉ

On étudie les fonctions zêta dénombrant les représentations linéaires complexes irréductibles de dimension finie de groupes compacts p-adiques analytiques et de groupes arithmétiques. En utilisant une méthode d'intégration $\mathfrak{p}$-adique, on démontre que celles de ces fonctions qui sont associées à certains pro-p-groupes p-adiques analytiques satisfont à des équations fonctionnelles. En admettant une conjecture de Serre sur les réseaux dans les groupes semi-simples de rang supérieur 1, on démontre une conjecture de Larsen et Lubotzky pour les groupes algébriques de type A_2 définis sur des corps de nombres.

Version française abrégée

Soit G un groupe et, pour tout $n \in \mathbb{N}$, soit $r_n(G)$ le nombre de classes d'équivalence de représentations linéaires complexes irréductibles de dimension n de G ; si G est un groupe topologique (resp. un groupe algébrique), on suppose que les représentations sont continues (resp. rationnelles). Si G est rigide, c'est-à-dire si $r_n(G)$ est fini pour tout $n \in \mathbb{N}$, il est naturel d'étudier la suite $r_n(G)$ et son comportement asymptotique quand n tend vers l'infini. Si, de plus, G est de croissance de représentation polynomiale, c'est-à-dire si $r_n(G)$ est borné par un polynôme en n, un outil puissant pour l'étude de la croissance de représentations est la fonction zêta de représentations associée

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$$\zeta_G(s) := \sum_{n=1}^{\infty} r_n(G) n^{-s},$$

où s est une variable complexe. Son abscisse de convergence $\alpha(G)$ donne le degré de la croissance polynomiale de $\sum_{n \leqslant N} r_n(G)$ quand N tend vers l'infini. Dans les articles [2,3] on étudie les fonctions zêta de représentations de groupes compacts p-adiques analytiques et de groupes arithmètiques.

Nos premiers principaux résultats (théorèmes A et B) concernent les membres génériques de familles de pro-p-groupes p-adiques analytiques provenant, via la théorie de Lie p-adique, des localisations des réseaux de Lie « parfaits » définis sur des anneaux d'entiers de corps de nombres. Une famille importante d'exemples est formée par les sous-groupes de congruence principaux de groupes p-adiques analytiques tel que $\mathrm{SL}_n(\mathfrak{o})$, où $\mathfrak{o}$ est un anneau de valuation discrète compact de caractéristique zéro.

Dans le théorème A, on donne une formule « uniforme » pour les fonctions zêta de représentations de ces groupes en termes de nombres de points rationnels de certaines variétés algébriques associées au réseau de Lie sous-jacent et l'on montre qu'elles satisfont certaines équations fonctionnelles locales. Dans le théorème B, on établit l'existence de bornes uniformes pour les parties réelles des pôles des fonctions zêta de représentations en question et montre que les abscisses de convergence sont monotones sous les extensions d'anneaux.

Les groupes arithmétiques auquels nous nous intéressons sont des réseaux arithmétiques dans des groupes algébriques semi-simples définis sur des corps de nombres, comme par exemple les groupes $\mathrm{SL}_n(\mathcal{O}_S)$, où $\mathcal{O}_S$ est l'anneau des S-entiers d'un corps de nombres. On sait que, si un groupe arithmétique a la propriété de sous-groupe de congruence (CSP), alors il est de croissance de représentations polynomiale et sa fonction zêta de représentations a une décomposition en produit eulérien, voir (4) ci-dessous. Les facteurs non-archimédiens de ce produit sont des fonctions zêta de représentations de groupes p-adiques analytiques compacts. Les théorèmes A et B s'appliquent aux sous-groupes de congruence principaux de ces groupes.

Dans le théorème C on établit que l'abscisse de convergence $\alpha(\Gamma)$ de la fonction zêta de représentations d'un réseau arithmétique Γ dans un groupe algébrique de type A_2 est égal à 1. En admettant la conjecture de Serre concernant (CSP) pour les réseaux irréductibles dans les groupes semi-simples de rang supérieur, ceci confirme et précise dans ce cas la conjecture de Larsen et Lubotzky qui affirme que, pour des réseaux irréductibles Γ_1 et Γ_2 donnés dans un tel groupe, on a toujours $\alpha(\Gamma_1) = \alpha(\Gamma_2)$.

La preuve du résultat global du théorème C repose sur le résultat local du théorème E, qui donne une formule explicite pour les fonctions zêta de représentations des sous-groupes de congruence principaux des groupes $\mathrm{SL}_3(\mathfrak{o})$ et $\mathrm{SU}_3(\mathfrak{O}, \mathfrak{o})$, où $\mathfrak{O}$ est une extension quadratique non-ramifiée de $\mathfrak{o}$. Le théorème E illustre les théorèmes A et B. Dans le théorème F, on décrit une formule explicite pour la fonction zêta de représentations des groupes $\mathrm{SL}_3(\mathfrak{o})$, qui est donnée dans [3]. (Les deux théorèmes Theorems F et E supposent que la caractéristique résiduelle de $\mathfrak{o}$ est suffisamment grande.) Dans le théorème G, on montre qu'il existe $c \in \mathbb{R}_{>0}$ tel que

$$\sum_{n=1}^{N} r_n\big(\mathrm{SL}_3(\mathcal{O}_S)\big) \sim c \cdot N \log N.$$

Notre méthodologie utilise la méthode des orbites de Kirillov pour les groupes compacts p-adiques analytiques, ainsi que des techniques provenant de l'intégration $\mathfrak{p}$-adique et de la théorie de Clifford. Le théorème F est obtenu par une analyse explicite de classes de similitude dans des quotients finis de $\mathfrak{gl}_3(\mathfrak{o})$. La démonstration du théorème G repose sur des prolongements analytiques de fonctions zêta de représentations en question et sur l' application des théorèmes Taubériens.

1. Statement of results

Let G be a group and, for $n \in \mathbb{N}$, denote by $r_n(G)$ the number of equivalence classes of n-dimensional irreducible complex representations of G; if G is a topological or an algebraic group, it is tacitly understood that representations are continuous or rational, respectively. We assume henceforth that G is (representation) *rigid*, i.e. that $r_n(G)$ is finite for all $n \in \mathbb{N}$. In the subject of representation growth one investigates the arithmetic properties of the sequence $r_n(G)$ and its asymptotic behaviour as n tends to infinity. Recent key advances in this area were made by Larsen and Lubotzky in [6].

The group G is said to have *polynomial representation growth* (PRG) if the sequence $R_N(G) := \sum_{n=1}^{N} r_n(G)$ is bounded by a polynomial. An important tool to study the representation growth of a PRG group G is its *representation zeta function*, viz. the Dirichlet series

$$\zeta_G(s) := \sum_{n=1}^{\infty} r_n(G) n^{-s},$$

where s is a complex variable. It is well known that the *abscissa of convergence* $\alpha(G)$ of the series $\zeta_G(s)$, i.e. the infimum of all $\alpha \in \mathbb{R}$ such that $\zeta_G(s)$ converges on the complex right half-plane $\{s \in \mathbb{C} \mid \mathrm{Re}(s) > \alpha\}$, gives the precise degree of polynomial growth: $R_N(G) = O(1 + N^{\alpha(G)+\epsilon})$ for every $\epsilon \in \mathbb{R}_{>0}$.

In [2] we introduce new methods from the theory of $\mathfrak{p}$-adic integration to study representation zeta functions associated to compact p-adic analytic groups and arithmetic groups. In [3] we compute explicit formulae for the representation zeta functions of the groups $\mathrm{SL}_3(\mathfrak{o})$, where $\mathfrak{o}$ is a compact discrete valuation ring of characteristic 0, in the case that the residue field characteristic is large compared to the ramification index of $\mathfrak{o}$.

A finitely generated profinite group G is rigid if and only if it is FAb, i.e. if every open subgroup of G has finite abelianisation. In [5], Jaikin-Zapirain proved rationality results for the representation zeta functions of FAb compact p-adic analytic groups using tools from model theory. In particular, the representation zeta function of a FAb p-adic analytic pro-p group is a rational function in p^{-s}, for $p > 2$. Key examples of FAb compact p-adic analytic groups are the special linear groups $\mathrm{SL}_n(\mathfrak{o})$ and their principal congruence subgroups $\mathrm{SL}_n^m(\mathfrak{o})$, where $\mathfrak{o}$ is a compact discrete valuation ring of characteristic 0 and residue field characteristic p. For fixed n, and varying m and $\mathfrak{o}$, the latter also yield important examples of families of pro-p groups which arise from a global Lie lattice, in this case $\mathfrak{sl}_n(\mathbb{Z})$.

To be more precise, let $\mathcal{O}$ be the ring of integers of a number field k, and let Λ be an $\mathcal{O}$-Lie lattice such that $k \otimes_{\mathcal{O}} \Lambda$ is a perfect k-Lie algebra of dimension d. Let $\mathfrak{o} = \mathcal{O}_v$ be the ring of integers of the completion k_v of k at a non-archimedean place v, lying above a rational prime p. Given a finite extension $\mathfrak{O}$ of $\mathfrak{o}$, we write $\mathfrak{P}$ for the maximal ideal of $\mathfrak{O}$, $e(\mathfrak{O}|\mathfrak{o})$ for the ramification index and $f(\mathfrak{O}|\mathfrak{o})$ for the residue class field extension degree. Let $\mathfrak{g}(\mathfrak{O}) := \mathfrak{O} \otimes_{\mathcal{O}} \Lambda$. For all sufficiently large m, the Lie lattice $\mathfrak{g}^m(\mathfrak{O}) := \mathfrak{P}^m\mathfrak{g}(\mathfrak{O})$ corresponds, by p-adic Lie theory, to a FAb, potent, saturable pro-p group $\mathsf{G}^m(\mathfrak{O}) := \exp(\mathfrak{g}^m(\mathfrak{O}))$. We call such m *permissible* for the Lie lattice $\mathfrak{g}(\mathfrak{O})$. For example, for unramified extensions $\mathfrak{O}$ of $\mathbb{Z}_p$ and p odd, every $m \in \mathbb{N}$ is permissible. In [2], we prove

Theorem A. *In the above setup, there exists a finite set S of places of k, a natural number r and a rational function $W(X_1, \ldots, X_r, Y) \in \mathbb{Q}(X_1, \ldots, X_r, Y)$ such that, for every non-archimedean place v of k with $v \notin S$, the following is true*:

There exist algebraic integers $\lambda_1 = \lambda_1(v), \ldots, \lambda_r = \lambda_r(v)$ such that for all finite extensions $\mathfrak{O}$ of $\mathfrak{o} = \mathcal{O}_v$ and for all $m \in \mathbb{N}$ which are permissible for $\mathfrak{g}(\mathfrak{O})$ one has

$$\zeta_{\mathsf{G}^m(\mathfrak{O})}(s) = q_v^{fdm} W\bigl(\lambda_1^f, \ldots, \lambda_r^f, q_v^{-fs}\bigr), \tag{1}$$

where q_v denotes the residue field cardinality of $\mathfrak{o}$, $f = f(\mathfrak{O}|\mathfrak{o})$ and $d = \mathrm{rank}_{\mathfrak{O}}(\mathfrak{g}(\mathfrak{O})) = \dim_k(k \otimes_{\mathcal{O}} \Lambda)$. Furthermore, the functional equation

$$\zeta_{\mathsf{G}^m(\mathfrak{O})}(s)\big|_{\substack{q_v \to q_v^{-1} \\ \lambda_i \to \lambda_i^{-1}}} = q_v^{fd(1-2m)} \zeta_{\mathsf{G}^m(\mathfrak{O})}(s) \tag{2}$$

holds.

Our proof of Theorem A implies in particular that the real parts of the poles of the zeta functions $\zeta_{\mathsf{G}^m(\mathfrak{O})}(s)$ are rational numbers. More precisely, we prove the following:

Theorem B. *In the above setup, there exists a finite set $P \subset \mathbb{Q}$ such that for all non-archimedean places v of k, all finite extensions $\mathfrak{O}$ of $\mathfrak{o} = \mathcal{O}_v$, and all permissible m for $\mathfrak{g}(\mathfrak{O})$ one has*

$$\bigl\{\mathrm{Re}(s) \bigm| s \text{ a pole of } \zeta_{\mathsf{G}^m(\mathfrak{O})}(s)\bigr\} \subseteq P.$$

Furthermore, if $v \notin S$, where S is a finite set of places arising from Theorem A, *and if $\mathcal{O}_v \subseteq \mathfrak{O}_1 \subseteq \mathfrak{O}_2$, then for every $m \in \mathbb{N}$ which is permissible for $\mathfrak{g}(\mathfrak{O}_1)$ and $\mathfrak{g}(\mathfrak{O}_2)$,*

$$\alpha\bigl(\mathsf{G}^m(\mathfrak{O}_1)\bigr) \leqslant \alpha\bigl(\mathsf{G}^m(\mathfrak{O}_2)\bigr). \tag{3}$$

Notice that, if the groups $\mathsf{G}^m(\mathfrak{O})$ are principal congruence subgroups of a FAb compact p-adic analytic group $\mathsf{G}(\mathfrak{O})$ consisting of the $\mathfrak{O}$-points of an algebraic group $\mathbf{G}$, such as $\mathbf{G} = \mathrm{SL}_n$, then (3) implies the monotonicity of the abscissae of convergence $\alpha(\mathsf{G}(\mathfrak{O}))$ under ring extensions. This follows from the fact that these abscissae are commensurability invariants. The set P of candidate poles is obtained by means of a resolution of singularities which leads to the generic formula (1). Theorems A and B are illustrated by the explicit formulae given in Theorem E below. The proofs of Theorems A and B are based on the Kirillov orbit method for the groups in question [4,5], and methods from $\mathfrak{p}$-adic integration [8,9].

The arithmetic groups we are interested in are arithmetic lattices in semisimple algebraic groups defined over number fields. More precisely, let $\mathbf{G}$ be a connected, simply connected semisimple algebraic group, defined over a number field k, together with a fixed k-embedding into some GL_n. Let $\mathcal{O}_S$ denote the ring of S-integers in k, for a finite set S of places of k including all the archimedean ones. We consider groups Γ which are commensurable to $\mathbf{G}(\mathcal{O}_S) = \mathbf{G}(k) \cap \mathrm{GL}_n(\mathcal{O}_S)$. In [7], Lubotzky and Martin showed that such a group Γ has PRG if and only if Γ has the Congruence Subgroup Property (CSP). Suppose that Γ has these properties. Then, according to a result of Larsen and Lubotzky [6, Proposition 1.3], the representation zeta function of Γ admits an Euler product decomposition. Indeed, if $\Gamma = \mathbf{G}(\mathcal{O}_S)$ and if the congruence kernel of Γ is trivial, this decomposition is particularly easy to state: it takes the form

$$\zeta_\Gamma(s) = \zeta_{\mathbf{G}(\mathbb{C})}(s)^{[k:\mathbb{Q}]} \cdot \prod_{v \notin S} \zeta_{\mathbf{G}(\mathcal{O}_v)}(s). \tag{4}$$

Here each archimedean factor $\zeta_{\mathbf{G}(\mathbb{C})}(s)$ enumerates rational representations of the group $\mathbf{G}(\mathbb{C})$; their contribution to the Euler product reflects Margulis super-rigidity. The groups $\mathbf{G}(\mathcal{O}_v)$ are FAb compact p-adic analytic groups whose principal congruence subgroups fit into the framework of Theorems A and B; the product of the zeta functions of these local groups captures the finite image representations of Γ.

Several of the key results of [6] concern the abscissae of convergence of the 'local' representation zeta functions occurring as Euler factors on the right-hand side of (4). With regards to abscissae of convergence of the 'global' representation zeta functions Avni proved that, for an arithmetic group Γ with the CSP, the abscissa of convergence of $\zeta_\Gamma(s)$ is always a rational number; see [1]. In [6, Conjecture 1.5], Larsen and Lubotzky conjectured that, for any two irreducible lattices Γ_1 and Γ_2 in a higher-rank semisimple group H, one has $\alpha(\Gamma_1) = \alpha(\Gamma_2)$, i.e. that the abscissa of convergence only depends on the ambient group. This can be regarded as a refinement of Serre's conjecture on the Congruence Subgroup Property. In [6, Theorem 10.1], Larsen and Lubotzky prove their conjecture in the case that H is a product of simple groups of type A_1, assuming Serre's conjecture. In [2], we prove

Theorem C. *Let Γ be an arithmetic lattice of a connected, simply connected simple algebraic group of type A_2 defined over a number field. If Γ has the CSP, then $\alpha(\Gamma) = 1$.*

Corollary D. *Assuming Serre's conjecture, Larsen and Lubotzky's conjecture holds for groups of the form $H = \prod_{i=1}^r \mathbf{G}_i(K_i)$, where each K_i is a local field of characteristic 0 and each $\mathbf{G}_i$ is an absolutely almost simple K_i-group of type A_2 such that $\sum_{i=1}^r \mathrm{rk}_{K_i}(\mathbf{G}_i) \geqslant 2$ and none of the $\mathbf{G}_i(K_i)$ is compact.*

Key to our proof of Theorem C is the following local result, which we formulate in accordance with the notation introduced before Theorem A:

Theorem E. *Let $\mathfrak{o}$ be a compact discrete valuation ring of characteristic 0, with residue field of cardinality q. Let $\mathfrak{g}(\mathfrak{o})$ be one of the following two $\mathfrak{o}$-Lie lattices of type A_2:*

(a) $\mathfrak{sl}_3(\mathfrak{o}) = \{\mathbf{x} \in \mathfrak{gl}_3(\mathfrak{o}) \mid \mathrm{Tr}(\mathbf{x}) = 0\}$;
(b) $\mathfrak{su}_3(\mathfrak{O}, \mathfrak{o}) = \{\mathbf{x} \in \mathfrak{sl}_3(\mathfrak{O}) \mid \mathbf{x}^\sigma = -\mathbf{x}^{\mathrm{t}}\}$, *where $\mathfrak{O}|\mathfrak{o}$ is an unramified quadratic extension with nontrivial automorphism σ.*

For $m \in \mathbb{N}$, let $\mathsf{G}^m(\mathfrak{o})$ be the m-th principal congruence subgroup of the corresponding group $\mathrm{SL}_3(\mathfrak{o})$ or $\mathrm{SU}_3(\mathfrak{O}, \mathfrak{o})$. Assume that the residue field characteristic of $\mathfrak{o}$ is not equal to 3. Then, for all $m \in \mathbb{N}$ which are permissible for $\mathfrak{g}(\mathfrak{o})$, one has

$$\zeta_{\mathsf{G}^m(\mathfrak{o})}(s) = q^{8m} \frac{1 + u(q)q^{-3-2s} + u(q^{-1})q^{-2-3s} + q^{-5-5s}}{(1 - q^{1-2s})(1 - q^{2-3s})},$$

where

$$u(X) = \begin{cases} X^3 + X^2 - X - 1 - X^{-1} & \text{if } \mathfrak{g}(\mathfrak{o}) = \mathfrak{sl}_3(\mathfrak{o}), \\ -X^3 + X^2 - X + 1 - X^{-1} & \text{if } \mathfrak{g}(\mathfrak{o}) = \mathfrak{su}_3(\mathfrak{O}, \mathfrak{o}). \end{cases}$$

The close resemblance between the representation zeta functions of the special linear and the special unitary groups is noteworthy and reminiscent of the Ennola duality for the characters of the corresponding finite groups of Lie type. We also give an explicit formula for $\zeta_{\mathrm{SL}_3^m(\mathfrak{o})}(s)$ in the exceptional case where $\mathfrak{o}$ is unramified and has residue field characteristic 3. Note that Theorem E implies that the abscissae of convergence $\alpha(\mathrm{SL}_3(\mathfrak{o}))$ and $\alpha(\mathrm{SU}_3(\mathfrak{O}, \mathfrak{o}))$ are each equal to $2/3$, as the abscissa of convergence is a commensurability invariant. Theorem E is proved using the techniques from $\mathfrak{p}$-adic integration which allow us to establish Theorems A and B.

The explicit formula for $\zeta_{\mathrm{SL}_3(\mathfrak{o})}(s)$, which we present in [3], is deduced by means of the Kirillov orbit method, a description of the similarity classes in finite quotients of $\mathfrak{gl}_3(\mathfrak{o})$, and Clifford theory.

Theorem F. *There exist finitely many polynomials $f_{\tau,i}, g_{\tau,i} \in \mathbb{Q}[x]$, indexed by $(\tau, i) \in \{1, -1\} \times I$, such that for every compact discrete valuation ring $\mathfrak{o}$ of characteristic 0, with residue field characteristic $p > 3e(\mathfrak{o}|\mathbb{Z}_p)$, one has*

$$\zeta_{\mathrm{SL}_3(\mathfrak{o})}(s) = \frac{\sum_{i \in I} f_{\tau,i}(q)(g_{\tau,i}(q))^{-s}}{(1 - q^{1-2s})(1 - q^{2-3s})},$$

where q denotes the size of the residue field of $\mathfrak{o}$ and $q \equiv \tau \pmod 3$.

An explicit set of such polynomials $f_{\tau,i}, g_{\tau,i}$ is computed in [3]. This result should be seen against the background of [5, Theorem 1.1], which establishes the rationality of representation zeta functions of FAb compact p-adic analytic groups. For groups of the form $\mathrm{SL}_3(\mathfrak{o})$, Theorem F specifies that these rational functions vary 'uniformly' as a function of the residue field cardinality q. Theorem F enables us to analyse the global representation zeta functions of the arithmetic groups $\mathrm{SL}_3(\mathcal{O}_S)$.

Theorem G. *Let $\mathcal{O}_S$ be the ring of S-integers of a number field k, where S is a finite set of places of k including all the archimedean ones. Then there exists $\epsilon > 0$ such that the representation zeta function of $\mathrm{SL}_3(\mathcal{O}_S)$ admits a meromorphic continuation to the half-plane $\{s \in \mathbb{C} \mid \mathrm{Re}(s) > 1 - \epsilon\}$. The continued function is holomorphic on the line $\{s \in \mathbb{C} \mid \mathrm{Re}(s) = 1\}$ except for a double pole at $s = 1$. There is a constant $c \in \mathbb{R}_{>0}$ such that*

$$\sum_{n=1}^{N} r_n\bigl(\mathrm{SL}_3(\mathcal{O}_S)\bigr) \sim c \cdot N \log N,$$

where $f(N) \sim g(N)$ means $\lim_{N\to\infty} f(N)/g(N) = 1$.

In [2], we also give simple geometric estimates for the abscissae of convergence of representation zeta functions of compact p-adic analytic groups and we compute representation zeta functions associated to norm-1 groups in non-split quaternion algebras.

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