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Complex Analysis

Analytic sets extending the graphs of holomorphic mappings between domains of different dimensions ☆

Ensembles analytiques prolongeant les graphes d'applications holomorphes entre domaines de dimensions différentes

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ABSTRACT

Let D, D' be arbitrary domains in $\mathbb{C}^n$ and $\mathbb{C}^N$ respectively, $1 < n \leqslant N$, both possibly unbounded and let $M \subset \partial D$, $M' \subset \partial D'$ be open pieces of the boundaries. Suppose that ∂D is smooth real-analytic and minimal in an open neighborhood of $\bar{M}$ and $\partial D'$ is smooth real-algebraic and minimal in an open neighborhood of $\bar{M}'$. Let $f : D \to D'$ be a holomorphic mapping. Assume that the cluster set $cl_f(M)$ does not intersect D'. It is proved that if the cluster set $cl_f(p)$ of a point $p \in M$ contains some point $q \in M'$ and the graph of f extends as an analytic set to a neighborhood of $(p, q) \in \mathbb{C}^n \times \mathbb{C}^N$, then f extends as a holomorphic map near p.

RÉSUMÉ

Soient D, D' deux domaines respectivement de $\mathbb{C}^n$ et $\mathbb{C}^N$, $1 < n \leqslant N$ et soient $M \subset \partial D$, $M' \subset \partial D'$ deux parties ouvertes des frontières. Supposons que ∂D (resp. $\partial D'$) est lisse, minimale et analytique réelle dans un voisinage de $\bar{M}$ (resp. lisse, minimale et algébrique réelle dans un voisinage de $\bar{M}'$). Soit $f : D \to D'$ une application holomorphe telle que l'ensemble des points limites $cl_f(M)$ n'intersecte pas D'. Nous montrons que si l'ensemble des points limites $cl_f(p)$ d'un point $p \in M$ contient un point $q \in M'$ et le graphe de f se prolonge comme un ensemble analytique dans un voisinage de $(p, q) \in \mathbb{C}^n \times \mathbb{C}^N$, alors f se prolonge holomorphiquement dans un voisinage de p.

Version française abrégée

Le but principal de cette Note est de généraliser un résultat de Diederich–Pinchuk [4] quand le domaine cible est algébrique réel, mais de dimension supérieure. On montre le théorème suivant :

Théorème 0.1. *Soient D, D' deux domaines respectivement de $\mathbb{C}^n$ et $\mathbb{C}^N$, $1 < n \leqslant N$ et soient $M \subset \partial D$, $M' \subset \partial D'$ deux parties ouvertes des frontières. Supposons que ∂D (resp. $\partial D'$) est lisse, minimale et analytique réelle dans un voisinage de $\bar{M}$ (resp. lisse, minimale et*

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algébrique réelle dans un voisinage de $\bar{M}'$). Soit $f : D \to D'$ une application holomorphe telle que l'ensemble des points limites $cl_f(M)$ n'intersecte pas D'. Si $cl_f(p)$ d'un point $p \in M$ contient un point $q \in M'$ et le graphe de f se prolonge comme un ensemble analytique dans un voisinage de $(p,q) \in \mathbb{C}^n \times \mathbb{C}^N$, alors f se prolonge holomorphiquement dans un voisinage de p.

Démonstration abrégée. La preuve est basée sur la propagation de l'analycité des applications holomorphes à travers les variétés de Segre et sur un résultat de Tumanov [12] qu'on utilise pour montrer que le prolongement de f comme correspondance est en fait un prolongement comme application. Le prolongement du graphe comme un ensemble analytique dans un voisinage de (p,q) assure l'existence d'un ensemble ouvert $\Gamma \subset M$ à travers lequel f se prolonge holomorphiquement, en plus $p \in \bar{\Gamma}$. Nous montrons d'abord le résultat quand p est un point générique. L'autre cas se déduit par induction sur la dimension. □

Comme application du théorème précédent, nous montrons le résultat suivant :

Théorème 0.2. *Soient D, D' deux domaines bornés, respectivement de $\mathbb{C}^n$ et $\mathbb{C}^N$, $1 < n \leqslant N$. Supposons que ∂D (resp. $\partial D'$) est lisse, analytique réelle (resp. lisse, algébrique réelle). Soit $f : D \to D'$ une application holomorphe propre. Si le graphe de f se prolonge comme un ensemble analytique dans un voisinage de $(p,q) \in \mathbb{C}^n \times \mathbb{C}^N$ pour un certain $p \in \partial D$ et $q \in cl_f(p)$, alors f se prolonge holomorphiquement dans un voisinage de $\bar{D}$.*

Démonstration abrégée. Soit M_h l'ensemble des points du bord, où f se prolonge holomorphiquement. D'après le Théorème 0.1, l'ensemble M_h est non vide. Pour montrer que $M_h = \partial D$, il suffit de montrer que M_h est fermé dans ∂D (puisque par définition M_h est ouvert). La preuve est identique à celle dans [1]. Elle est par l'absurde et elle est basée sur la construction d'une famille d'ellipsoïdes utilisée par Merker et Porten dans [7]. Cette construction nous ramène à étudier le prolongement de f au voisinage des points génériques. Cette étude se déduit de la preuve du Théorème 0.1. □

1. Introduction and main results

It was proved in [4] that a proper holomorphic mapping $f : D \to D'$ between bounded domains in $\mathbb{C}^n$ with smooth real-analytic boundaries extends holomorphically to a neighborhood of any point $p \in \partial D$, if the graph of f extends as an analytic set near (p,q) for some $q \in cl_f(p)$. The purpose of this Note is to study this result when the boundary of the target domain is smooth real-algebraic but of higher dimension.

Theorem 1.1. *Let D, D' be arbitrary domains in $\mathbb{C}^n$ and $\mathbb{C}^N$ respectively, $1 < n \leqslant N$, both possibly unbounded and let $M \subset \partial D$, $M' \subset \partial D'$ be open pieces of the boundaries. Suppose that ∂D is smooth real-analytic and minimal in an open neighborhood of $\bar{M}$ and $\partial D'$ is smooth real-algebraic and minimal in an open neighborhood of $\bar{M}'$. Let $f : D \to D'$ be a holomorphic mapping. Assume that the cluster set $cl_f(M)$ does not intersect D'. If the cluster set $cl_f(p)$ of a point $p \in M$ contains some point $q \in M'$ and the graph of f extends as an analytic set to a neighborhood of $(p,q) \in \mathbb{C}^n \times \mathbb{C}^N$, then f extends as a holomorphic map near p.*

The proof of Theorem 1.1 is based on the method of analytic continuation along Segre varieties and a result of Tumanov [12]. Here, f is not assumed to be proper and we do not require compactness of M'. Also, we do not assume that $cl_f(M) \subset M'$. Therefore, a priori $cl_f(p)$ may contain the point at infinity or boundary points which do not lie in M'. In particular, this is the main reason why our result cannot be directly derived from [11], even in the case where M' is strictly pseudoconvex. Note that the assumption that f sends D to D' may be replaced by $f : D \to \mathbb{C}^N$ with $cl_f(M) \subset M'$.

As an application of Theorem 1.1, one has the following:

Theorem 1.2. *Let D, D' be smoothly bounded domains in $\mathbb{C}^n$ and $\mathbb{C}^N$ respectively, $1 < n \leqslant N$, ∂D is real-analytic and $\partial D'$ is real-algebraic. Let $f : D \to D'$ be a proper holomorphic mapping. If the graph of f extends as an analytic set to a neighborhood of $(p,q) \in \mathbb{C}^n \times \mathbb{C}^N$ for some $p \in \partial D$ and $q \in cl_f(p)$, then f extends as a holomorphic map in a neighborhood of $\bar{D}$.*

Theorem 1.2 generalizes [4] when the boundary of the target domain is real-algebraic but of higher dimension. The algebraicity of D' allows to show that the extension is in a neighborhood of $\bar{D}$ and not only near p. If f is a proper holomorphic map as in Theorem 1.2 that extends smoothly in a neighborhood of some boundary point p, then according to [2] and [8] f extends holomorphically across p. Hence, Theorem 1.2 implies that f extends holomorphically to a neighborhood of $\bar{D}$. This result was proved in [11], when D' is strictly pseudoconvex.

We say that Γ_f, the graph of f, extends as an analytic set to a neighborhood of $(p,q) \in \partial D \times \partial D'$, if there exist neighborhoods $U \ni p$, $U' \ni q$, an irreducible analytic subset $\mathcal{A} \subset U \times U'$ of pure dimension n and a sequence $\{a_\nu\} \subset U \cap D$ with $a_\nu \to p$ and $f(a_\nu) \to q$ such that $\mathcal{A}$ contains an open piece of Γ_f near $(a_\nu, f(a_\nu))$ for each ν. A hypersurface is called minimal if it does not contain germs of complex hypersurfaces. We refer the reader to [3] for definitions and details on Segre varieties.

2. Proof of Theorem 1.1

Assume that $p = 0$, $q = 0'$ and 0 is not in the envelope of holomorphy of D. Let U, U' be small neighborhoods of 0 and $0'$ respectively. We denote by $\mathcal{A}$ the irreducible analytic subset in $U \times U'$ extending the graph of f. According to [4], one has the following:

Lemma 2.1. *There exists an open set $\Gamma \subset M \cap U$ such that f extends holomorphically to a neighborhood of $(U \cap D) \cup \Gamma$, and the graph of f near any point $(z, f(z))$, $z \in \Gamma$, is contained in $\mathcal{A}$. Moreover, $0 \in \overline{\Gamma}$ and $\lim_{z\to 0,\, z\in\Gamma} f(z) = 0'$.*

Since M is real-analytic, the set Γ given by Lemma 2.1 can be constructed in a way that $\partial\Gamma \cap M$ is a real-analytic set defined by a finite system of equations. If $0 \in \Gamma$, then the proof follows from Lemma 2.1. Therefore, we may assume that $0 \in \partial\Gamma$. First, we consider the case where 0 is a generic point (i.e., $\partial\Gamma \cap M$ is a smooth generic submanifold near 0).

2.1. Extension across generic submanifolds

Recall that a real submanifold $M \subset \mathbb{C}^n$ of real dimension $d \geqslant n$ is called generic if for any $z \in M$, the complex tangent space $T_z^c M$ to M at z has complex dimension equal to $d - n$. In this subsection, we consider the restriction of f on Γ (still denoted by f). This restriction $f : \Gamma \to M'$ is holomorphic in a neighborhood of Γ and its graph extends as an analytic set to a neighborhood of $(0, 0')$. In all this paragraph, we will assume that $\partial\Gamma \cap M$ *is a smooth generic submanifold near* 0. Our aim here is to prove that f extends holomorphically near 0. First, we will prove the extension of f as a holomorphic correspondence near 0. The proof is similar to the proof of Theorem 1.3 in [11] (here, M' is not assumed to be compact). For the sake of completeness, we add an abbreviated proof. In view of Proposition 4.1 in [10], there exists an open subset ω of Q_0 such that for all $b \in \omega$, $Q_b \cap \Gamma$ is non-empty. Furthermore, there exists a non-constant curve $\gamma \subset \Gamma \cap Q_b$ with the end point at 0. Thus, we may choose t and b such that $b \in Q_0$ and $t \in \gamma \subset \Gamma \cap Q_b$. For simplicity, we will also denote by $f : U_t \to \mathbb{C}^N$ a germ of a holomorphic mapping defined from the extension of f in some neighborhood U_t of t. Let V be a neighborhood of Q_t and define $X = \{(w, w') \in V \times \mathbb{C}^N\colon\ f(Q_w \cap U_t) \subset Q'_{w'}\}$. Since $w \in Q_t$ implies that $t \in Q_w$, then we may choose V such that $Q_w \cap U_t$ is non-empty for all $w \in V$. The analytic set X allows us to extend the graph of f as an analytic set along Q_t, $t \in \Gamma$. In contrast with the equidimensional case, the dimension of X may be bigger than the dimension of the graph of f and this leads us to construct another analytic set X^* from X extending the graph of f and with dimension equal to n (the same dimension as the graph of f). For this construction, we will follow the ideas in [11]. The analytic set X^* allows us to prove that f extends as a holomorphic correspondence to a neighborhood of 0. This extension is guaranteed to be single-valued near Levi non-degenerate points in M'.

According to [11], X is a complex analytic subset of $V \times \mathbb{C}^N$. By the invariance property of Segre varieties, X contains the germ at t of the graph of f. From the algebraicity of M', the set X extends to an analytic subset of $V \times \mathbb{P}^N$. Since $\mathbb{P}^N$ is compact and X is closed in $V \times \mathbb{P}^N$, the projection $\pi : X \to V$ is proper. It follows that $\pi(X)$ is a complex analytic subset of V. Since V is connected, $\pi(X) = V$. Otherwise; $\pi(X)$ is nowhere dense in V and therefore $\dim \pi(X) \leqslant n - 1$, which proves that π is surjective. Since X contains the germ at t of the graph of f, we may consider only the irreducible component of the least dimension which contains the graph of f. So, we may assume that $\dim(X) \equiv m \geqslant n$. For $\xi \in X$, let $I_\xi\pi \subset X$ be the germ of the fiber $\pi^{-1}(\pi(\xi))$ at ξ. For a generic point $\xi \in X$, $\dim(I_\xi\pi) = m - n$ which is the smallest possible dimension of the fiber. By Cartan–Remmert's theorem (see [5]), the set $\Sigma := \{\xi \in X\colon\ \dim(I_\xi\pi) > m - n\}$ is complex-analytic and by Remmert's proper mapping theorem, $\pi(\Sigma)$ is a complex-analytic set in V. Furthermore, $\dim \pi(\Sigma) < n - 1$. By the above considerations, we deduce that $\pi(\Sigma)$ does not contain $Q_0 \cap V$. Without loss of generality we may assume that $b \notin \pi(\Sigma)$. Since the projection π is proper, then X defines a holomorphic correspondence. Denote the corresponding multiple-valued map by $\widehat{F}$. That is, $\widehat{F} := \pi' \circ \pi^{-1} : V \to \mathbb{P}^N$, where $\pi' : X \to \mathbb{P}^N$ denotes the other coordinate projection. We choose suitable neighborhoods, U_γ of γ (including its endpoints) and U_b of b such that $U_b \cap \pi(\Sigma) = \emptyset$ and $Q_w \cap U_b$ is non-empty and connected for any $w \in U_\gamma$. Consider the set $X^* = \{(w, w') \in U_\gamma \times \mathbb{P}^N\colon\ \widehat{F}(Q_w \cap U_b) \subset Q'_{w'}\}$. The same arguments used for π show that the projection $\pi^* : X^* \to U_\gamma$ is surjective and proper. Now, define $\pi'^* : X^* \to \mathbb{P}^N$ and consider the multiple-valued mapping $\widehat{F^*} := \pi'^* \circ \pi^{*-1} : U_\gamma \to \mathbb{P}^N$. We will denote by w^s the symmetric point of $w \in U$, which is the unique point in the intersection $Q_w \cap \{z \in U\colon\ {}'z = {}'w\}$. Let now Ω be a small connected neighborhood of the path γ which connects t and 0, such that for any $w \in \Omega$, the symmetric point w^s belongs to U_γ, and let Q_w^s denote the connected component of $Q_w \cap U_\gamma$ which contains w^s. Define further $\Sigma^* = \{z \in U_\gamma\colon\ \pi^{*-1}(z)$ does not have the generic dimension$\}$. Since Σ^* is a complex analytic set of dimension at most $n - 2$, then $\Omega \setminus \Sigma^*$ is connected. According to [11], one has the following:

Lemma 2.2.

(a) *For any point $w \in \Omega \setminus \Sigma^*$ and $w' \in \widehat{F^*}(w)$, we have*:

$$\widehat{F^*}\big(Q_w^s\big) \subset Q'_{w'}. \tag{2.1}$$

(b) *X^* contains the germ of the graph of f at $(t, f(t))$.*

(c) *X^* is a complex-analytic subset of $U_\gamma \times \mathbb{P}^N$ of complex dimension n.*

From the algebraicity of M' the analytic subset $\mathcal{A} \subset U \times U'$ extending the graph of f, extends to an analytic subset in $U \times \mathbb{P}^N$. Denote this extension by $\overline{\mathcal{A}}$.

Lemma 2.3. $X^* \cap [(U \cap U_\gamma) \times \mathbb{P}^N] = \overline{\mathcal{A}} \cap [(U \cap U_\gamma) \times \mathbb{P}^N]$.

Proof. We may assume that t is close to 0 so that $U_t \subset U \cap U_\gamma$. By Lemma 2.1, f extends holomorphically across t, and the graph of f near $(t, f(t))$ is contained in $\overline{\mathcal{A}}$. The set X^* contains the graph of f near $(t, f(t))$ by Lemma 2.2. By considering dimensions of X^* and $\overline{\mathcal{A}}$, and by shrinking U_t if necessary we have: $X^*|_{U_t \times \mathbb{P}^N} = \overline{\mathcal{A}}|_{U_t \times \mathbb{P}^N}$. Now the proof follows from the uniqueness theorem for analytic sets. □

Our aim now is to prove that f extends as a holomorphic correspondence to a neighborhood of 0. First, suppose that $0 \notin \Sigma^*$. In view of Lemma 2.3, $(0, 0') \in X^*$. By Lemma 2.2, $(z, z') \in X^* \setminus {\pi^*}^{-1}(\Sigma^*)$ implies that $\widehat{F^*}(Q_z^s) \subset Q'_{z'}$. In particular, $\widehat{F^*}(z) \subset Q'_{z'}$. Hence, $z' \in Q'_{z'}$ and so $z' \in M'$. Then, for any $z \in M$ close to 0 and any z' close to $0'$, the inclusion $(z, z') \in X^*$ implies $z' \in M'$. Since $\widehat{F^*}(z)$ is contained in a countable union of complex analytic sets and M' is minimal, it follows that ${\pi^*}^{-1}(z)$ is discrete near $(0, 0')$. Therefore, we may choose U and U' so small such that $\pi'^*|_{X^* \cap (U \times U')} \circ {\pi^*}^{-1}|_U$ is the desired extension of f as a holomorphic correspondence. Now, suppose that $0 \in \Sigma^*$. Consider a sequence of points $w_j \in (\Gamma \cap \Omega) \setminus \Sigma^*$ such that $w_j \to 0$ and $f(w_j) \to 0'$. Then $\widehat{F^*}(Q_{w_j}^s) \subset Q'_{f(w_j)}$. Since $\dim \Sigma^* < \dim Q_0$, to prove that

$$\widehat{F^*}\big(Q_0^s\big) \subset Q'_{0'}, \tag{2.2}$$

it suffices to prove this inclusion in a neighborhood of any point in $Q_0^s \setminus \Sigma^*$. But this follows by analyticity of the fibers of $\pi^* : X^* \to U_\gamma$. Then as above ${\pi^*}^{-1}(0)$ is discrete near $(0, 0')$ and f extends to a neighborhood of 0 as a holomorphic correspondence. We denote this correspondence by G. To prove that the extension of f is in fact an extension as a map, we need the following result:

Theorem *(A. Tumanov). (See [12].) Let $N \subset \mathbb{C}^N$ be a real-analytic (resp. a real-algebraic) minimal submanifold. Then N can be stratified as $N = \bigcup_{j=1}^k N_j$ so that each stratum N_j is a real-analytic (resp. a real-algebraic) CR manifold and locally is contained in a Levi non-degenerate real-analytic (resp. real-algebraic) hypersurface.*

We denote by $M_s'^+$ (resp. $M_s'^-$) the set of strictly pseudoconvex points (resp. strictly pseudoconcave points) of M'. The set of points where the Levi-form of M' has eigenvalues of both signs on the complex tangent space $T^c(M')$ to M' and no zero will be denoted by $M'^{\pm}$ and by M'_0 we mean the set of points of M' where this Levi-form has at least one eigenvalue 0 on $T^c(M')$. We will discuss two cases. First assume that $0' \in M_s'^+ \cup M_s'^- \cup M'^{\pm}$. We may shrink U' so that the Segre map $\lambda' : U' \to \{Q_{w'},\ w' \in U'\}$ is one to one. Let $w' \in G(w)$ for $w \in M \cap U$. In view of (2.1) and (2.2), $G(Q_w) \subset Q'_{w'}$. In particular, $w' \in Q'_{w'}$ and hence $G(M \cap U) \subset M' \cap U'$. By using Corollary 4.2 of [3] and the fact that λ' is one to one, we may show that the correspondence G splits into several holomorphic maps, one of which extends the map f. Secondly, assume that $0' \in M'_0$. By Tumanov's theorem, $M' = \bigcup_{j=1}^k N_j$ and each N_j is locally contained in a Levi non-degenerate real-algebraic hypersurface $\tilde{M}_j$. The extension of f as a correspondence near 0 implies that f extends continuously to $U_0 \cap M$, for some neighborhood $U_0 \subset U$ of 0. Let j_0 be the largest index such that $0' \in N_{j_0}$. Using the continuity of f and by shrinking U_0 if necessary, we may assume that $f(U_0 \cap M) \subset \tilde{M}_{j_0}$. By [6], the hypersurface $\tilde{M}_{j_0}$ is minimal (since, it is Levi non-degenerate). Hence as above, we may show that f extends as a holomorphic correspondence $\tilde{G}$ near 0 and we may choose U_0 and U' so that $\tilde{G}(U_0 \cap M) \subset U' \cap \tilde{M}_{j_0}$. Now as in the first case, we may show that f extends as a holomorphic map near 0. □

Remark. In [11], Shafikov and Verma proved that if M and M' are hypersurfaces as in Theorem 1.1, M' is compact, $\Gamma \subset M$ is a connected open set and f is a holomorphic map in a neighborhood of Γ with $f(\Gamma) \subset M'$, then f extends as a holomorphic correspondence near any generic point in $\partial\Gamma \cap M$. So, as above we may use the result of Tumanov to prove that this extension is in fact an extension as a map.

3. Conclusion of the proof of Theorem 1.1

First, suppose that $0 \in \mathrm{Reg}(\partial\Gamma)$. Then near 0, $\partial\Gamma \cap M$ is a generic submanifold of dimension $2n-2$ and the proof follows from Section 2.1. Suppose now that $0 \in \mathrm{Sing}(\partial\Gamma)$. Since $\partial\Gamma$ is a real-analytic set defined by a finite system of equations, it follows from [9] that there exists a real-analytic set Γ_1 of real dimension at most $2n-3$, which is also defined by a finite system of equations such that $\mathrm{Sing}(\partial\Gamma) \subset \Gamma_1$. If $0 \in \mathrm{Reg}(\Gamma_1)$, then we may shrink U if necessary so that $U \cap \Gamma_1$ is contained in some generic submanifold $\widetilde{\Gamma_1}$ of M, of dimension $2n-2$, and we may show that f extends holomorphically near 0 by repeating the argument above. The singular part of Γ_1 is now contained in a real-analytic set of dimension $2n-4$, then if $0 \in \mathrm{Sing}(\Gamma_1)$, by induction on dimension we may complete the proof. □

4. Proof of Theorem 1.2

Let $M_h := \{z \in \partial D\colon\ f$ extends holomorphically to a neighborhood of $z\}$. The set M_h is open by construction and non-empty by Theorem 1.1. To prove the theorem, it suffices to show that M_h is closed in ∂D. By contradiction, assume that $\overline{M_h} \neq M_h$, and let $q \in \partial M_h$. Following the ideas developed in [1] and [11] there exists a CR-curve γ passing through q and entering M_h. After shortening γ, we may assume that γ is a smoothly embedded segment. Then γ can be described as a part of an integral curve of some non-vanishing smooth CR-vector field L near q. By integrating L we obtain a smooth coordinate system $(t,s) \in \mathbb{R} \times \mathbb{R}^{2n-2}$ on ∂D such that for any fixed s_0 the segments (t, s_0) are contained in the trajectories of L. We may assume that $(0,0) \in \gamma \cap M_h$ sufficiently close to q. For $\epsilon > 0$ and $\tau > 0$, define the family of ellipsoids on ∂D centered at 0 by $E_\tau = \{(t,s)\colon\ |t|^2/\tau + |s|^2 < \epsilon\}$, where $\epsilon > 0$ is so small that for some $\tau_0 > 0$ the ellipsoid E_{τ_0} is compactly contained in M_h. Observe that every ∂E_τ is transverse to the trajectories of L out off the set $\Lambda := \{(0,s)\colon\ |s|^2 = \epsilon\}$. So, ∂E_τ is generic at every point except the points of Λ. Note that Λ is contained in M_h. Let τ_1 be the smallest positive number such that f does not extend holomorphically to some point $b \in \partial E_{\tau_1}$. Note that $\tau_1 > \tau_0$ and b may be different from q. Near b, ∂E_{τ_1} is a smooth generic manifold of ∂D; since the non-generic points of ∂E_{τ_1} are contained in Λ, which is contained in M_h. Then, we are in the situation of the Section 2.1. Consequently, f extends as a holomorphic map to a neighborhood of b. This contradiction finishes the proof of Theorem 1.2. □

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