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On the behaviour of power series in the absence of Hadamard–Ostrowski gaps

Sur le comportement des séries entières en l'absence de lacunes de Hadamard–Ostrowski

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ABSTRACT

We show that the partial sums $(S_n f)_{n\in\mathbb{N}}$ of a power series f with radius of convergence one tend to ∞ in capacity on (arbitrarily large) compact subsets of the complement of the closed unit disk, if f does not have so-called Hadamard–Ostrowski gaps. Regarding a recent result of Gardiner, this covers a large class of functions f holomorphic in the unit disk.

RÉSUMÉ

Nous montrons que les sommes partielles $(S_n f)_{n\in\mathbb{N}}$ d'une série entière f de rayon de convergence 1 tendent vers ∞ en capacité sur les ensembles compacts (arbitrairement grands) du complémentaire du disque unité fermé si f ne contient pas de lacunes de Hadamard–Ostrowski. Tenant compte d'un résultat récent de Gardiner, ceci couvre une grande classe de fonctions f holomorphes sur le disque unité.

1. Introduction and main result

For a power series $f(z)=\sum_{\nu=0}^{\infty} a_\nu z^\nu$ with radius of convergence one, we denote the partial sums by:

$$S_n f(z)=\sum_{\nu=0}^{n} a_\nu z^\nu.$$

Then $(S_n f)(z_0)$ is unbounded at each point z_0 with $|z_0|>1$. More precisely, we have:

$$\limsup_{n\to\infty}\left|S_n f(z_0)\right|^{1/n}=|z_0| \tag{1}$$

and, for $R\geqslant 1$,

$$\limsup_{n\to\infty}\max_{|z|=R}\left|S_n f(z)\right|^{1/n}=R. \tag{2}$$

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This does not prevent subsequences of $(S_n f)$ from being convergent, even on large subsets of the plane. Indeed, it is well known (see, e.g., the expository articles [6] and [8]) that there exist functions f holomorphic in the unit disk $\mathbb{D}$ having the property that the sequence $(S_n f)_{n\in\mathbb{N}}$ is universal outside $\mathbb{D}$ in the sense that for each compact set $K\subset\mathbb{C}\setminus\mathbb{D}$ with connected complement and each continuous function $h: K\to\mathbb{C}$ which is holomorphic in the interior of K, there is a subsequence of $(S_n f)_{n\in\mathbb{N}}$ tending to h uniformly on K.

It turns out that for such power series the sequence of coefficients $(a_\nu)_{\nu\in\mathbb{N}_0}$ necessarily exhibits a strong kind of irregularity in the sense of having so-called Ostrowski gaps (see [5,10,12]). In contrast, if the sequence $(a_\nu)_{\nu\in\mathbb{N}_0}$ behaves regularly, as for example in the case of the geometric series $f(z)=\sum_{\nu=0}^{\infty} z^\nu$, the partial sums $(S_n f(z))_{n\in\mathbb{N}}$ tend to be attracted by ∞ for z outside the closed unit disk. Our aim is to show that, for a reasonable class of power series, this turns out to be true in a certain sense.

In the sequel, we use the term capacity for logarithmic capacity. For unexplained notions from potential theory, see [13].

Definition 1.1. A sequence $(h_n)_{n\in\mathbb{N}}$ of Borel-measurable functions is said to *converge in capacity* to ∞ on a set $D\subset\mathbb{C}$, if for every $M>0$ we have:

$$\lim_{n\to\infty}\operatorname{cap}\bigl(\bigl\{z\in D\colon \bigl|h_n(z)\bigr|\leqslant M\bigr\}\bigr)=0.$$

Furthermore, if D is open, (h_n) is said to *converge locally in capacity* on D to ∞, if the sequence converges in capacity to ∞ on every (non-polar) compact subset of D.

This definition is in accordance with the definition of convergence in capacity to finite limit functions, as considered, e.g., in [9]. The notion of convergence in capacity is well known in Padé approximation (see, e.g., [1, Section 6.6]). Since $\operatorname{cap}(K)\geqslant\sqrt{|K|/\pi}$, where $|K|$ denotes the area of a compact plane set K, convergence in capacity implies convergence in (plane Lebesgue) measure.

Definition 1.2. Let $f(z)=\sum_{\nu=0}^{\infty}a_\nu z^\nu$ have radius of convergence one. We say that f possesses *Hadamard–Ostrowski gaps* if there are sequences $(p_k)_{k\in\mathbb{N}}$ and $(q_k)_{k\in\mathbb{N}}$ in $\mathbb{N}$ with $p_1<q_1\leqslant p_2<q_2\leqslant\cdots$, such that:

1. $\liminf_{k\to\infty}\frac{q_k}{p_k}>1$,
2. $\limsup_{I\ni\nu\to\infty}|a_\nu|^{\frac{1}{\nu}}<1$ for $I:=\bigcup_{k\in\mathbb{N}}\{p_k+1,p_k+2,\ldots,q_k-1\}$.

Remark 1.3. A recent result of Gardiner [3, Cor. 3] states that a power series $f(z)=\sum_{\nu=0}^{\infty}a_\nu z^\nu$ which converges on $\mathbb{D}$ and is analytically continuable to a domain $\Omega\supset\mathbb{D}$, but not to a neighbourhood of a given point $\xi\in\partial\mathbb{D}$, has no Hadamard–Ostrowski gaps, if $\mathbb{C}\setminus\Omega$ is thin at ξ. In particular, it follows that all functions holomorphic in $\mathbb{D}$ that have an isolated singularity at some point ξ on the boundary of $\mathbb{D}$ do not have Hadamard–Ostrowski gaps. It is easily seen that the same is true for all (repeated) antiderivatives of such functions. Thus, for a large class of functions holomorphic in $\mathbb{D}$, this turns out to be the case.

Our main result is the following. The proof is given in the next section.

Theorem 1.4. *Let $f(z)=\sum_{\nu=0}^{\infty}a_\nu z^\nu$ be a power series with radius of convergence one and without Hadamard–Ostrowski gaps. Then $S_n f\to\infty$ locally in capacity on $\mathbb{C}\setminus\overline{\mathbb{D}}$.*

Under the conditions of the theorem, $S_n f$ does not need to tend to ∞ (globally) in capacity on any open annulus $\{z\colon 1<|z|<R\}$, where $R>1$. As an example, on the semi-circular arcs:

$$B_n:=\bigl\{z\in\mathbb{C}\colon \operatorname{Re}z\leqslant 0,\ |z|=1+1/n\bigr\}$$

the partial sums $S_n f(z)=(z^{n+1}-1)(z-1)^{-1}$ of the geometric series f are uniformly bounded; more precisely,

$$\max_{z\in B_n}\bigl|S_n f(z)\bigr|\leqslant 5/\sqrt{2},$$

while $\operatorname{cap}(B_n)=(1+1/n)\operatorname{cap}(B)\geqslant\operatorname{cap}(B)$, where B is the corresponding semi-circular arc on the unit circle. This also shows that pointwise convergence to ∞ on the annulus does not imply convergence in capacity there.

Remark 1.5. Let h_n be a sequence of meromorphic functions in the plane. According to [11, Thm. 2], convergence in capacity of $h_n\to h$ on a bounded set $D\subset\mathbb{C}$ implies that there exists a subsequence $(n_k)_{k\in\mathbb{N}}$ in $\mathbb{N}$ such that $h_{n_k}\to h$ *quasi-everywhere* on D, i.e.:

$$\operatorname{cap}\bigl(\bigl\{z\in D\colon h_{n_k}(z)\nrightarrow h(z)\bigr\}\bigr)=0.$$

Corollary 1.6. *Let $f(z)=\sum_{\nu=0}^{\infty}a_\nu z^\nu$ be a power series with radius of convergence one and without Hadamard–Ostrowski gaps. Then every subsequence $(S_{n_k}f)_{k\in\mathbb{N}}$ of $(S_nf)_{n\in\mathbb{N}}$ contains a subsequence $(S_{n_{k_j}}f)_{j\in\mathbb{N}}$ with $1/S_{n_{k_j}}f\to 0$ quasi-everywhere on $\mathbb{C}\setminus\overline{\mathbb{D}}$.*

The proof follows with a standard diagonal sequence argument from Theorem 1.4:

The annuli $K_l:=\{z\colon\ 1+1/l\leqslant|z|\leqslant 1+l\}$ $(l\in\mathbb{N})$ form a compact exhaustion of $\mathbb{C}\backslash\overline{\mathbb{D}}$. Let $(S_{n_k}f)_{k\in\mathbb{N}}$ be any subsequence of $(S_nf)_{n\in\mathbb{N}}$. By Theorem 1.4 and Remark 1.5, there exists a subsequence $(n_k^{(1)})_{k\in\mathbb{N}}$ of $(n_k)_{k\in\mathbb{N}}$ such that:

$$1/S_{n_k^{(1)}}f\to 0\quad\text{quasi-everywhere on } K_1.$$

Proceeding in this way, there exists a subsequence $(n_k^{(2)})_{k\in\mathbb{N}}$ of $(n_k^{(1)})_{k\in\mathbb{N}}$ such that:

$$1/S_{n_k^{(2)}}f\to 0\quad\text{quasi-everywhere on } K_2,$$

and so on. The diagonal sequence $(n_k^{(k)})_{k\in\mathbb{N}}$ is a subsequence of $(n_k)_{k\in\mathbb{N}}$ for which $\operatorname{cap}(\{z\in K_l\colon\ 1/S_{n_k^{(k)}}f\nrightarrow 0\})=0$ for all $l\in\mathbb{N}$. Since countable unions of polar sets are polar, this implies $\operatorname{cap}(\{|z|>1\colon\ 1/S_{n_k^{(k)}}f\nrightarrow 0\})=0$.

2. Proof of the main theorem

For $r>1$ we set $\mathbb{D}_r:=\{\zeta\in\mathbb{C}\colon\ |\zeta|<r\}$. The following purely potential theoretic result will play the key role for our argumentation.

Lemma 2.1. *Suppose that (u_j) is a sequence of upper bounded and subharmonic functions on $\mathbb{D}$. If*

$$\limsup_{j\to\infty}\sup_{\mathbb{D}}u_j\leqslant 0$$

and if there is a sequence (E_j) of compact subsets of $\mathbb{D}_\delta$ for some $\delta<1$ with $\operatorname{cap}(E_j)\geqslant\alpha>0$ and

$$\beta:=\limsup_{j\to\infty}\max_{E_j}u_j<0$$

then, for all $r\in(\delta,1)$, a positive constant $c(\delta,r)$ exists so that:

$$\limsup_{j\to\infty}\max_{\overline{\mathbb{D}_r}}u_j\leqslant\beta\frac{c(\delta,r)}{\log(1/\alpha)}.$$

Proof. In view of the maximum principle (for subharmonic functions), we may suppose that the sets $\mathbb{C}\setminus E_j$ are connected. Let $\varepsilon>0$ be fixed. Then there exists an integer $j_0=j_0(\varepsilon)$ with:

$$\sup_{\mathbb{D}}u_j<\varepsilon\quad\text{and}\quad\max_{E_j}u_j<\beta+\varepsilon\quad(j\geqslant j_0).$$

Let $\omega_{\mathbb{D}\setminus E_j}$ denote the harmonic measure of $\mathbb{D}\setminus E_j$. According to the two-constant-theorem (see, e.g. [13, Thm. 4.3.7]) we get, for $\zeta\in\mathbb{D}\setminus E_j$ and $j\geqslant j_0$,

$$u_j(\zeta)\leqslant(\beta+\varepsilon)\omega_{\mathbb{D}\setminus E_j}(\zeta,E_j)+\varepsilon\bigl(1-\omega_{\mathbb{D}\setminus E_j}(\zeta,E_j)\bigr)=\beta\omega_{\mathbb{D}\setminus E_j}(\zeta,E_j)+\varepsilon.$$

Moreover, from a result in [4, p. 123], we obtain the existence of a positive constant $c(\delta,r)$ with:

$$\max_{|\zeta|=r}\omega_{\mathbb{D}\setminus E_j}(\zeta,E_j)\geqslant\frac{c(\delta,r)}{\log(1/\operatorname{cap}(E_j))}\geqslant\frac{c(\delta,r)}{\log(1/\alpha)}$$

and thus the maximum principle yields:

$$\sup_{\overline{\mathbb{D}_r}}u_j\leqslant\beta\frac{c(\delta,r)}{\log(1/\alpha)}+\varepsilon\quad(j>j_0).$$

Since $\varepsilon>0$ was arbitrary, the conclusion follows. □

Now, we are prepared for the *Proof of Theorem* 1.4:

1. We show in a first step that $S_nf(1/\zeta)\to\infty$ locally in capacity on $\mathbb{D}$.

For this purpose, let $f(z)=\sum_{\nu=0}^{\infty}a_\nu z^\nu$ be a power series with radius of convergence one. Assume that $(\zeta\mapsto S_nf(1/\zeta))_{n\in\mathbb{N}}$ does not tend to ∞ in capacity on a non-polar compact set $E\subset\mathbb{D}$. We have to show that f has Hadamard–Ostrowski gaps.

By assumption, there exist $M > 0$, $\alpha > 0$, and a sequence $(n_j)_{j\in\mathbb{N}}$ in $\mathbb{N}$ tending to infinity such that the compact sets

$$E_j := \{\zeta \in E\colon \left|S_{n_j} f(1/\zeta)\right| \leqslant M\} \quad (j \in \mathbb{N})$$

all satisfy $\operatorname{cap}(E_j) \geqslant \alpha$. We define:

$$u_j(\zeta) := \log\left|S_{n_j} f(1/\zeta)\zeta^{n_j}\right|^{1/n_j} = \log|\zeta| + \frac{1}{n_j}\log\left|S_{n_j} f(1/\zeta)\right| \quad (\zeta \in \mathbb{D}).$$

Then the u_j are subharmonic in $\mathbb{C}$ (note that $S_n f(1/\zeta)\zeta^n$ is a polynomial in ζ). Moreover, (2) implies:

$$\limsup_{n\to\infty} \max_{|\zeta|=1} \left|S_n f(1/\zeta)\right|^{1/n} = 1$$

and therefore (by the maximum principle):

$$\limsup_{j\to\infty} \sup_{\mathbb{D}} u_j \leqslant 0.$$

Since $E_j \subset E \subset \mathbb{D}_\delta$ for some $\delta < 1$, we further obtain:

$$\max_{E_j} u_j \leqslant \log(\delta) + \frac{1}{n_j}\log M$$

and thus:

$$\limsup_{j\to\infty} \max_{E_j} u_j \leqslant \log(\delta) < 0. \tag{3}$$

Since $\operatorname{cap}(E_j) \geqslant \alpha$, Lemma 2.1 yields:

$$\limsup_{j\to\infty} \max_{\overline{\mathbb{D}_r}} u_j \leqslant \log(\delta)\frac{c(\delta, r)}{\log(1/\alpha)} < 0$$

for $\delta < r < 1$ (note that $\alpha \leqslant \operatorname{cap}(E_j) \leqslant \operatorname{cap}(\mathbb{D}_\delta) = \delta < 1$). If we fix such an $r \in (\delta, 1)$, for $R := 1/r \in (1, 1/\delta)$ this estimate implies:

$$\limsup_{j\to\infty} \max_{|z|=R} \left|S_{n_j} f(z)\right|^{1/n_j} < R.$$

Hence, there is $\gamma < 1$ so that:

$$\max_{|z|=R} \left|S_{n_j} f(z)\right| < (\gamma R)^{n_j}$$

for j sufficiently large. For every $\nu \leqslant n_j$, Cauchy's formula implies:

$$|a_\nu|^{\frac{1}{\nu}} = \left|\frac{1}{2\pi i}\int_{|z|=R} \frac{S_{n_j} f(z)}{z^{\nu+1}}\,\mathrm{d}z\right|^{\frac{1}{\nu}} \leqslant \gamma^{\frac{n_j}{\nu}} \cdot R^{\frac{n_j}{\nu}-1} \leqslant \gamma \cdot R^{\frac{n_j}{\nu}-1}.$$

Now, if $q \in (0, 1)$ is arbitrary and $qn_j \leqslant \nu \leqslant n_j$, we obtain:

$$|a_\nu|^{\frac{1}{\nu}} \leqslant \gamma \cdot R^{\frac{1}{q}-1},$$

and hence

$$\limsup_{j\to\infty} \max_{qn_j\leqslant\nu\leqslant n_j} |a_\nu|^{\frac{1}{\nu}} \leqslant \gamma \cdot R^{\frac{1}{q}-1}.$$

Finally, we can choose $q < 1$ close enough to 1 so that $\gamma R^{\frac{1}{q}-1} < 1$, which gives:

$$\limsup_{j\to\infty} \max_{qn_j\leqslant\nu\leqslant n_j} |a_\nu|^{\frac{1}{\nu}} < 1.$$

This implies that f has Hadamard–Ostrowski gaps.

2. From [13, Thm. 5.3.1], it is easily seen that convergence in capacity is preserved under bi-Lipschitz mappings. Let $K \subset \mathbb{C}$ be a compact set and $\varphi : K \to \mathbb{C}$ bi-Lipschitz. For any sequence (h_n) of Borel-measurable functions on $\varphi(K)$, we have $h_n \to \infty$ in capacity on $\varphi(K)$ if and only if $h_n \circ \varphi \to \infty$ in capacity on K. If we apply this for arbitrary compact subsets K of $\mathbb{C} \setminus \overline{\mathbb{D}}$ and $\varphi(z) = 1/z$ on K, the conclusion follows from the first part of the proof, with $h_n(\zeta) = S_n f(1/\zeta)$. □

3. Further remarks

1. A closer inspection of the proof of our main theorem shows that the existence of a sequence of compact subsets K_j of an annulus $\{z\colon\ R \leqslant |z| \leqslant S\}$, where $1 < R \leqslant S$, with $\inf_{j\in\mathbb{N}} \operatorname{cap}(K_j) > 0$ and

$$\limsup_{j\to\infty} \max_{z\in K_j} \frac{1}{|z|} \left|S_{n_j} f(z)\right|^{1/n_j} < 1 \tag{4}$$

implies that f has Hadamard–Ostrowski gaps (in this case, we still have:

$$\limsup_{j\to\infty} \max_{E_j} u_j < 0$$

in the estimate (3), where $E_j := 1/K_j$). In particular, (4) is satisfied if:

$$\limsup_{j\to\infty} \max_{z\in K_j} \left|S_{n_j} f(z)\right|^{1/n_j} < R.$$

With regard to (1) and (2), this means that the subsequence (S_{n_j}) has a reduced growth compared to the full sequence on the compact sets K_j.

2. Consider a power series f of radius of convergence one and being analytically continuable to a domain Ω strictly larger than $\mathbb{D}$. From a classical result of Ostrowski on overconvergence (see e.g. [7, Thm. 16.7.1]), it follows that the conclusion of Corollary 1.6 does no longer hold if f has Hadamard–Ostrowski gaps.

More precisely, let $\xi \in \Omega \cap \partial\mathbb{D}$. If $(p_k)_{k\in\mathbb{N}}$ and $(q_k)_{k\in\mathbb{N}}$ are as in Definition 1.2, the subsequence $(S_{p_k} f)_{k\in\mathbb{N}}$ converges uniformly on some open disk U with centre ξ (to the continuation of f). Thus, in particular, no subsequence of $(S_{p_k} f)_{k\in\mathbb{N}}$ can tend to infinity quasi-everywhere on $U \setminus \overline{\mathbb{D}}$.

In contrast, there exist f having $\partial\mathbb{D}$ as its natural boundary and having Hadamard–Ostrowski gaps so that $(S_n f)(z) \to \infty$ for all $|z| > 1$. A simple example is given by the gap series:

$$f(z) = \sum_{k=0}^{\infty} z^{2^k},$$

where one may choose $p_k = 2^k$ and $q_k = p_{k+1}$.

3. In [2] it was shown that, for functions f having no Hadamard–Ostrowski gaps, pointwise (finite) limit functions of $(S_n f)_{n\in\mathbb{N}}$ on sets $E \subset \mathbb{C} \setminus \overline{\mathbb{D}}$ can only exist if E is polar. This also appears now as consequence of Corollary 1.6.

4. Let H_0 denote the space of all functions holomorphic in the punctured sphere $\mathbb{C}_\infty \setminus \{1\}$ (where $\mathbb{C}_\infty := \mathbb{C} \cup \{\infty\}$) and vanishing at infinity. From the result of Gardiner mentioned in Remark 1.3, it follows that functions in $H_0\backslash\{0\}$ do not have Hadamard–Ostrowski gaps. According to 3., finite limit functions can only exist on polar sets $E \subset \mathbb{C} \setminus \overline{\mathbb{D}}$. As is shown by the geometric series f, finite limit functions may exist on sets of positive capacity on the boundary $\partial\mathbb{D}$ of the unit disk. Indeed, if $K \subset \partial\mathbb{D} \setminus \{1\}$ is a Dirichlet set (see e.g. [8]), then a subsequence of

$$S_n f(z) = \left(z^{n+1} - 1\right)(z-1)^{-1}$$

tends to 0 uniformly on K. It is known that Dirichlet sets of Hausdorff dimension 1 exist. Polar sets, however, necessarily have vanishing Hausdorff dimension.

On the other hand, from a result of Melas [9], it easily follows that for all countable sets $E \subset \mathbb{C} \setminus \mathbb{D}$, there is a residual set in H_0 (where H_0 is endowed with the topology of locally uniform convergence) consisting of functions that are universal on E in the sense that for each function $h : E \to \mathbb{C}$, a subsequence of $(S_n f)_{n\in\mathbb{N}}$ tends to h pointwise on E. Moreover, [2, Thm. 2] shows that there is a residual set of functions in H_0 so that $\{S_n f\colon\ n \in \mathbb{N}\}$ is (uniformly) dense in the space $C(K)$ of continuous functions on K for "many" perfect sets $K \subset \mathbb{C} \setminus \mathbb{D}$. This shows in particular that, for a residual set of functions in H_0, uncountable exceptional sets $E \subset \mathbb{C} \setminus \overline{\mathbb{D}}$ exist on which the sequence $(S_n f)_{n\in\mathbb{N}}$ turns out to be far away from tending pointwise to ∞.

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