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Existence result for a one-dimensional eikonal equation

Résultat d'existence pour l'équation eikonale unidimensionnelle

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ABSTRACT

In this Note, we consider the eikonal equation in one-dimensional space describing the evolution of interfaces moving with non-signed velocity. We prove a global existence result of discontinuous viscosity solutions in a weak sense by considering *BV* initial data.

RÉSUMÉ

Dans cette Note, nous considérons l'équation eikonale en une dimension d'espace décrivant le mouvement d'interfaces avec une vitesse non signée. Nous prouvons un résultat d'existence globale de solutions de viscosité discontinues dans un sens faible en considérant des données initiales *BV*.

Version française abrégée

Dans cette Note, nous considérons l'équation eikonale unidimensionnelle, donnée par :

$$\begin{cases} \partial_t u(x,t) = c(x,t)\left|\partial_x u(x,t)\right| & \text{dans } \mathbb{R}\times(0,T) \\ u(x,0) = u_0(x) & \text{dans } \mathbb{R}, \end{cases} \tag{1}$$

avec les hypothèses :

$$(H_1) \quad u_0 \in L^\infty(\mathbb{R}) \cap BV(\mathbb{R})$$

$$(H_2) \quad c \in L^\infty\big(\mathbb{R}\times(0,T)\big) \cap L^\infty\big((0,T); BV(\mathbb{R})\big),$$

où $BV(\mathbb{R})$ est l'espace des fonctions $L^1_{loc}(\mathbb{R})$ à variations bornées.

L'objet de cette Note est d'établir un résultat d'existence globale en temps d'une solution de viscosité discontinue de (1) dans un sens assez faible, en supposant les hypothèses (H_1) et (H_2).

Notre idée principale était, tout d'abord, de régulariser la vitesse et la donnée initiale (par convolutions classiques, comme dans (5)), en considérant, pour tout $\epsilon > 0$, le problème suivant :

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$$\begin{cases} \partial_t u_\epsilon(x,t) = c_\epsilon(x,t)\left|\partial_x u_\epsilon(x,t)\right| & \text{dans } \mathbb{R}\times(0,T) \\ u_\epsilon(x,0) = u_{0,\epsilon}(x) & \text{dans } \mathbb{R}. \end{cases} \tag{2}$$

Ensuite, par le principe du maximum, nous avons obtenu une estimation L^∞ permettant de déduire que les semi-limites relaxées au sens de Barles et Perthame [5], notées par :

$$\bar{u}(x,t) = \limsup_{\substack{\epsilon\to 0\\ (y,s)\to(x,t)}} u_\epsilon(y,s), \qquad \underline{u}(x,t) = \liminf_{\substack{\epsilon\to 0\\ (y,s)\to(x,t)}} u_\epsilon(y,s), \tag{3}$$

sont, respectivement, sous- et sur-solutions de viscosité discontinues de (1).

De plus, grâce à une estimation dans BV uniforme en ϵ sur u_ϵ et la propriété de propagation à vitesse finie de l'équation, nous avons montré que $\bar{u}$ et $\underline{u}$ sont égales presque partout en espace, uniformément en temps. Ce qui conduit au résultat suivant.

Théorème 0.1 *(Existence globale). Soit $T > 0$, sous les hypothèses (H_1) et (H_2), nous avons :*

i) il existe une unique solution de viscosité Lipschitz u_ϵ du problème (2). De plus, les semi-limites relaxées $\bar{u}$ et $\underline{u}$ sont, respectivement, sous- et sur-solutions de viscosité discontinues du problème (1) ;

ii) quitte à extraire une sous-suite, la fonction u_ϵ converge, quand ϵ tend vers 0, vers une fonction

$$u \in L^\infty\big(\mathbb{R}\times(0,T)\big) \cap L^\infty\big((0,T); BV(\mathbb{R})\big) \cap C\big([0,T); L^1_{loc}(\mathbb{R})\big)$$

vérifiant les estimations

$$\begin{cases} \|u\|_{L^\infty(\mathbb{R}\times(0,T))} \le \|u_0\|_{L^\infty(\mathbb{R})} \\ \big|u(\cdot,t)\big|_{BV(\mathbb{R})} \le |u_0|_{BV(\mathbb{R})}, & \text{pour presque tout } t\in[0,T) \\ \big\|u(\cdot,t)-u(\cdot,s)\big\|_{L^1(\mathbb{R})} \le \big(\|c\|_{L^\infty(\mathbb{R}\times(0,T))}|u_0|_{BV(\mathbb{R})}\big)|t-s|, & \text{pour tout } s,t\in[0,T) \end{cases}$$

et l'égalité suivante

$$\bar{u}(\cdot,t) = \underline{u}(\cdot,t) = u(\cdot,t), \quad \text{sauf sur un ensemble dénombrable dans } \mathbb{R}, \text{ pour tout } t\in[0,T).$$

Ici, $|\cdot|_{BV(\mathbb{R})}$ désigne la semi-norme associée à la variation totale. Nous renvoyons à [2, Def. 3.1], pour la définition des solutions de viscosité discontinues classiques. Par ailleurs, nous signalons que la solution construite dans le Théorème 0.1ii) est une solution de viscosité discontinue, mais dans un sens assez faible, vu qu'elle vérifie qu'une égalité presque partout en espace entre $\bar{u}$ et $\underline{u}$.

1. Introduction and main result

In this Note, we are interested in the one-dimensional eikonal equation given, for all $T > 0$, by

$$\begin{cases} \partial_t u(x,t) = c(x,t)\left|\partial_x u(x,t)\right| & \text{in } \mathbb{R}\times(0,T) \\ u(x,0) = u_0(x) & \text{in } \mathbb{R}, \end{cases} \tag{1}$$

with the following assumptions on the initial data and the velocity

$$(H_1) \quad u_0 \in L^\infty(\mathbb{R}) \cap BV(\mathbb{R})$$

$$(H_2) \quad c \in L^\infty\big(\mathbb{R}\times(0,T)\big) \cap L^\infty\big((0,T); BV(\mathbb{R})\big),$$

where, $BV(\mathbb{R})$ is the space of $L^1_{loc}(\mathbb{R})$ functions with bounded variations.

The goal is here to establish a global existence result of discontinuous viscosity solutions to (1) in a weak sense, assuming (H_1) and (H_2).

Let us remark that Eq. (1) can be seen as the "level-set approach" equation associated with the propagation of the front $\Gamma_t := \{x : u(x,t) = 0\}$ with a normal velocity $c(x,t)$ (see Barles et al. [6]).

Our work was motivated by the dynamics of linear defects in crystals called dislocations, where the velocity is non-local and can change sign (see for instance [2] and [7]). In this framework, we can write

$$c(x,t) = c_0(\cdot,t) \star u(\cdot,t)(x), \tag{4}$$

where $\star$ denotes the convolution in space and $c_0(x,t)$ is a kernel that depends only on the physical properties of the crystal. For this problem, there are only few known existence and uniqueness results. In the case when the velocity is non-negative, global existence and uniqueness results were first obtained by Alvarez et al. [1] and then by Barles et al. [4] using different arguments. The general case, with unsigned velocity, has been studied first by Alvarez et al. in [2], yielding a short-time existence and uniqueness result. Then, the global existence of some weak solutions was obtained by Barles et al. in [7], in the framework of the L^1-viscosity solutions theory. Let us mention that the problem describing the dynamics of dislocation is initially considered in 2D; nevertheless, in a particular geometry, it can be reduced to a 1D problem.

The idea behind our result is first to regularize the velocity c and the initial data (by classical convolutions). This leads us to consider, for every $0 < \epsilon < 1$, the following equation:

$$\begin{cases} \partial_t u_\epsilon(x,t) = c_\epsilon(x,t)\left|\partial_x u_\epsilon(x,t)\right| & \text{in } \mathbb{R}\times(0,T) \\ u_\epsilon(x,0) = u_{0,\epsilon}(x) & \text{in } \mathbb{R}, \end{cases} \tag{2}$$

where

$$u_{0,\epsilon}(x) = u_0 \star \rho^1_\epsilon(x) \quad \text{and} \quad c_\epsilon(x,t) = \hat{c} \star \rho^2_\epsilon(x,t) \quad \forall x \in \mathbb{R},\ t \in \mathbb{R}, \tag{5}$$

with the function $\hat{c}$ is an extension in $\mathbb{R}^2$ of the function c by 0 and ρ^1_ϵ, ρ^2_ϵ are the standard mollifiers. Afterwards, by the maximum principle, we obtain an L^∞ uniform estimate, which implies that the relaxed semi-limits $\bar{u}$ and $\underline{u}$ defined by (3) are, respectively, sub- and super-solutions of (1). Moreover, using a BV uniform bound and the finite speed propagation property of the equation, it is possible to show that $\bar{u}$ and $\underline{u}$ are equal almost everywhere in space, uniformly in time. Now, we give our main result.

Theorem 1.1 *(Global existence result). Suppose that assumptions (H_1) and (H_2) are satisfied. Then, we have:*

i) there exists a unique Lipschitz continuous viscosity solution u_ϵ to (2). Moreover, the half relaxed semi-limits $\bar{u}$ and $\underline{u}$ defined by (3) are, respectively, discontinuous viscosity sub- and super-solutions of (1);

ii) up to extract a subsequence, the function u_ϵ converges, as ϵ goes to zero, to a function

$$u \in L^\infty\big(\mathbb{R}\times(0,T)\big) \cap L^\infty\big((0,T); BV(\mathbb{R})\big) \cap C\big([0,T); L^1_{\mathrm{loc}}(\mathbb{R})\big)$$

satisfying, for all $T > 0$, the following estimates

$$\begin{cases} \|u\|_{L^\infty(\mathbb{R}\times(0,T))} \le \|u_0\|_{L^\infty(\mathbb{R})} \\ \left|u(\cdot,t)\right|_{BV(\mathbb{R})} \le |u_0|_{BV(\mathbb{R})}, & \text{for almost all } t \in [0,T) \\ \left\|u(\cdot,t) - u(\cdot,s)\right\|_{L^1(\mathbb{R})} \le \big(\|c\|_{L^\infty(\mathbb{R}\times(0,T))}|u_0|_{BV(\mathbb{R})}\big)|t-s|, & \text{for all } s,t \in [0,T) \end{cases} \tag{6}$$

and the following equality

$$\bar{u}(\cdot,t) = \underline{u}(\cdot,t) = u(\cdot,t), \quad \text{except at most on a countable set in } \mathbb{R},\ \text{for all } t \in [0,T). \tag{7}$$

Here, $|\cdot|_{BV(\mathbb{R})}$ denotes the semi-norm associated with the total variation. We refer the reader to [2, Def. 3.1], for the definition of the classical discontinuous viscosity solutions. Furthermore, we note that, the solution constructed in Theorem 1.1ii) is a discontinuous viscosity solution, but in some weak sense, since it verifies only an almost everywhere equality in space between $\bar{u}$ and $\underline{u}$.

2. Proof of Theorem 1.1

We divide this section into three subsections. In the first one, we study the regularized problem (2). In the second one, we show the existence of the sub- and super-solutions of (1) and finally, in the last, we prove the point ii) of Theorem 1.1.

2.1. Existence and uniqueness for the regularized problem

Firstly, after regularizing the initial data and the velocity by classical convolutions and using the result of Ley [8], we prove that Eq. (2) has a unique Lipschitz continuous viscosity solution. Moreover, by the maximum principle, we can show the following L^∞ uniform estimate:

$$\|u_\epsilon\|_{L^\infty(\mathbb{R}\times(0,T))} \le \|u_0\|_{L^\infty(\mathbb{R})} \tag{8}$$

and the finite speed propagation property, given by:

$$\inf_{|y-x|\le t\|c_\epsilon\|_{L^\infty(\mathbb{R}\times(0,T))}} u_\epsilon(y,h) \le u_\epsilon(x,t+h) \le \sup_{|y-x|\le t\|c_\epsilon\|_{L^\infty(\mathbb{R}\times(0,T))}} u_\epsilon(y,h), \tag{9}$$

for all $(x,t) \in \mathbb{R}\times[0,T-h)$.

Then, differentiating (2), we can see formally that, if we assume that u_ϵ is a smooth enough function tending to 0, when $|x| \to +\infty$, we have:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}} |\partial_x u_\epsilon|\,\mathrm{d}x = \int_{\mathbb{R}} \operatorname{sgn}(\partial_x u_\epsilon)(\partial_x u_\epsilon)_t\,\mathrm{d}x = \int_{\mathbb{R}} \operatorname{sgn}(\partial_x u_\epsilon)\partial_x\big(c_\epsilon|\partial_x u_\epsilon|\big)\,\mathrm{d}x = \int_{\mathbb{R}} \partial_x(c_\epsilon\,\partial_x u_\epsilon)\,\mathrm{d}x = 0.$$

Integrating over $(0, t)$, the above equation, we get the following *BV* estimate:

$$\left\|\partial_x u_\epsilon(\cdot, t)\right\|_{L^1(\mathbb{R})} \le |u_0|_{BV(\mathbb{R})} \quad \text{for all } t \in [0, T]. \tag{10}$$

Note that the proof of this *BV* estimate can be justified rigorously. To do this, it is sufficient to regularize Eq. (2) by adding the viscosity term $\eta \partial^2_{xx}(\partial_x u_\epsilon)$, for $\eta > 0$, and also the absolute value function by using a smooth function $\beta_\delta(\cdot) = \sqrt{(\cdot)^2 + \delta^2}$, for $\delta > 0$; then, repeating the same procedure as before, we obtain the desired estimate, by passing to the limit, when $\delta \to 0$ and $\eta \to 0$.

Furthermore, the fact that u_ϵ satisfies the equation in (2) almost everywhere, we get the following estimate:

$$\left\|\partial_t u_\epsilon(\cdot, t)\right\|_{L^1(\mathbb{R})} \le \|c\|_{L^\infty(\mathbb{R}\times(0,T))} |u_0|_{BV(\mathbb{R})} \quad \text{for all } t \in [0, T]. \tag{11}$$

2.2. Sub- and super-solutions

First, thanks, to the L^∞ uniform bound on u_ϵ and on c_ϵ, we deduce, by the stability result of discontinuous viscosity solutions (see [3, Th. 4.1]) that $\bar{u}$ (resp. $\underline{u}$) is a sub-solution (resp. is a super-solution) to

$$\partial_t u = c^\star(x, t)|\partial_x u|, \quad \text{in } \mathbb{R} \times (0, T) \quad \big(\text{resp. of } \partial_t u = c_\star(x, t)|\partial_x u|, \text{ in } \mathbb{R} \times (0, T)\big),$$

where $c^\star$ and $c_\star$ are, respectively, the upper and the lower semi-continuous envelopes of c.

Then, since we have

$$\bar{u}(x, 0) = \lim_{n\to+\infty} u_{\epsilon_n}(x_{\epsilon_n}, t_{\epsilon_n}),$$

where $(\epsilon_n, x_{\epsilon_n}, t_{\epsilon_n}) \to (0, x, 0)$ when $n \to +\infty$, from (9) with $h = 0$ and using the classical properties of the mollifiers, we can see that, for every $\alpha > 0$, there exists $n_\alpha > 0$, such that, for all $n \ge n_\alpha$, we have:

$$u_{\epsilon_n}(x_{\epsilon_n}, t_{\epsilon_n}) \le \sup_{|z-x|\le\alpha(2+\|c\|_{L^\infty(\mathbb{R}\times(0,T))})} u_0(z).$$

Passing to the limit $n \to +\infty$ and then $\alpha \to 0$, we obtain: $\bar{u}(x, 0) \le (u_0)^\star(x)$. Similarly, we can also get $\underline{u}(x, 0) \ge (u_0)_\star(x)$, which gives sense to the initial data and proves that $\bar{u}$ and $\underline{u}$ are, respectively, discontinuous viscosity sub- and super-solutions to (1).

2.3. Convergence and existence of weak solution

From estimate (10) and using Simon's Lemma [9, Corollary 4] with the following compactness embedding $BV_{\text{loc}}(\mathbb{R}) \hookrightarrow L^1_{\text{loc}}(\mathbb{R})$, we deduce that, up to extract a subsequence, $(u_\epsilon)_\epsilon$ converges strongly in $C([0, T); L^1_{\text{loc}}(\mathbb{R}))$ to a function u satisfying (6). Furthermore, since the function $u(\cdot, t) \in L^\infty(\mathbb{R}) \cap BV(\mathbb{R})$, we know that it coincides with a right-continuous function almost everywhere in $\mathbb{R}$. Therefore, we will choose among the L^1_{loc} limits a right-continuous function in $\mathbb{R}$.

Now, we give the proof of (7). For the sake of clarity of the presentation, we present it in two steps.

Step 1 (*Local estimates with right and left continuous functions*). First of all, let $T > 0$ and $h > 0$, by the definition of $\bar{u}$, for all $t \in [0, T)$ such that $t + h < T$, we have:

$$\bar{u}(x, t + h) = \lim_{m\to+\infty} u_{\epsilon_m}\big(x_{\epsilon_m}, t^h_{\epsilon_m}\big),$$

where $(\epsilon_m, x_{\epsilon_m}, t^h_{\epsilon_m}) \to (0, x, t + h)$, when $m \to +\infty$.

Then, thanks to the properties of $BV(\mathbb{R})$ functions and using the L^1_{loc} strong convergence of $(u_{\epsilon_m}(\cdot, t))_{\epsilon_m}$, we can show that, for all $a > 0$, $0 < h \le \frac{a}{2}$, there exist a subsequence $u_{\epsilon_n}(\cdot, t)$ and a positive constant $n^h_{a,t}$, such that, for all $n \ge n^h_{a,t}$, we have:

$$u_{\epsilon_n}(y, t) \le 2h + u^1(y + h, t) - u^2(y - h, t), \quad \forall y \in [-a, a], \tag{12}$$

where $u^1(\cdot, t)$ and $u^2(\cdot, t)$ are two bounded, right-continuous and non-decreasing functions (with respect to x) satisfying, for all $t \in [0, T)$, $u(\cdot, t) = u^1(\cdot, t) - u^2(\cdot, t)$.

From (9) and (12), by passing to the limit $n \to +\infty$, we obtain that, for all $T > 0$, for all $h > 0$ such that $h \le \frac{a}{2\gamma_c}$ with $\gamma_c = 2(\|c\|_{L^\infty(\mathbb{R}\times(0,T))} + 1)$ and $t + h < T$,

$$\bar{u}(x, t + h) \le 2h + u^1(x + h\gamma_c, t) - u^2(x - h\gamma_c, t), \quad \text{for all } x \in \left[-\frac{a}{2}, \frac{a}{2}\right]. \tag{13}$$

Similarly, we get:

$$-2h + u^1(x - h\gamma_c, t) - u^2(x + h\gamma_c, t) \le \underline{u}(x, t + h). \tag{14}$$

Since $u^1(\cdot,t)$ and $u^2(\cdot,t)$ are bounded and non-decreasing functions on $\mathbb{R}$, we know that the right and the left limits of these functions exist at every point $x \in \mathbb{R}$. Then, from (13) and (14), we deduce that, for all $T > 0$, $x \in [-\frac{a}{2}, \frac{a}{2}]$, $t \in [0, T)$ and $\alpha > 0$, there exists $\bar{h}^\alpha_{a,t,T}$, such that, for all $0 < h \le \bar{h}^\alpha_{a,t,T}$, we have:

$$\bar{u}(x,t+h) \le \alpha + u^1_r(x,t) - u^2_l(x,t) \quad \text{and} \quad -\alpha + u^1_l(x,t) - u^2_r(x,t) \le \underline{u}(x,t+h), \tag{15}$$

where $u^1_r(\cdot,t)$ and $u^2_r(\cdot,t)$ (resp. $u^1_l(\cdot,t)$ and $u^2_l(\cdot,t)$) denote the right-continuous functions (resp. the left-continuous functions) defined from the right limits (resp. the left limits) of the functions $u^1(\cdot,t)$ and $u^2(\cdot,t)$.

Step 2 (*Link between $\bar{u}$ and $\underline{u}$*). Let $\bar{h}^\alpha_{a,t,T}$, the constant defined above, satisfy (15). We construct a finite covering of $[0, \frac{T}{2}]$ as follows:

$$\bigcup_{0\le i\le N^\alpha_a} \left[\tau^\alpha_{a,i}, \tau^\alpha_{a,i} + \bar{h}^\alpha_{a,\tau^\alpha_{a,i},T}\right] \supset \left[0, \frac{T}{2}\right] \quad \text{with } \tau^\alpha_{a,0} = 0 \text{ and } \tau^\alpha_{a,i+1} = \tau^\alpha_{a,i} + \bar{h}^\alpha_{a,\tau^\alpha_{a,i},T} \text{ for } i = 1,\cdots,N^\alpha_a - 1.$$

Due to the fact that $\mathbb{R} = \bigcup_{a\in\mathbb{Q}}[-\frac{a}{2}, \frac{a}{2}]$, from (15), we show that, for all $x \in \mathbb{R}$, $\tau \in [0, \frac{T}{2}]$ and for every positive constant $\alpha \in \mathbb{Q}$, there exist two indices $a_0 \in \mathbb{Q}$ and $0 \le j \le N^\alpha_{a_0}$, such that:

$$\bar{u}(x,\tau) \le \alpha + u^1_r\big(x, \tau^\alpha_{a_0,j}\big) - u^2_l\big(x, \tau^\alpha_{a_0,j}\big) \quad \text{and} \quad -\underline{u}(x,\tau) \le \alpha - u^1_l\big(x, \tau^\alpha_{a_0,j}\big) + u^2_r\big(x, \tau^\alpha_{a_0,j}\big). \tag{16}$$

Moreover, by the properties of non-decreasing functions, we know that, for all positive constants $\alpha, a \in \mathbb{Q}$ and $0 \le i \le N^\alpha_a$, the functions $u^1_r(\cdot, \tau^\alpha_{a,i})$, $u^1_l(\cdot, \tau^\alpha_{a,i})$ (resp. $u^2_r(\cdot, \tau^\alpha_{a,i})$, $u^2_l(\cdot, \tau^\alpha_{a,i})$) coincide with $u^1(\cdot, \tau^\alpha_{a,i})$ (resp. $u^2(\cdot, \tau^\alpha_{a,i})$), except on a countable set on $\mathbb{R}$, denoted $D^\alpha_{a,i}$. Now, if we define the countable set

$$D = \bigcup_{a,\alpha\in\mathbb{Q}} \bigcup_{0\le i\le N^\alpha_a} D^\alpha_{a,i},$$

thanks to (16), we can deduce that, for all rational numbers $\alpha > 0$, $x \notin D$ and $\tau \in [0, \frac{T}{2}]$

$$0 \le \bar{u}(x,\tau) - \underline{u}(x,\tau) \le 2\alpha.$$

Finally, passing to the limit $\alpha \to 0$ and replacing T by $2T$, we get the following equality:

$$\bar{u}(\cdot,\tau) = \underline{u}(\cdot,\tau) \quad \text{except at most on a countable set in } \mathbb{R}, \text{ for all } \tau \in [0, T]. \tag{17}$$

This, joint to the following inequality $\bar{u} \le u \le \underline{u}$, implies (7).

3. Remarks and conclusion

Note that, as a direct consequence of our main result, we can prove a global existence result of a classical discontinuous viscosity solution to (1), in the case of non-decreasing initial data. This is due to the fact that, the eikonal equation becomes a transport equation in this particular case. Note also that, applying our result, we can show a similar result in the framework of the dislocation dynamics, where the velocity satisfies (4). Let us finally mention that the existence result of a sub- and super-solution, mentioned in Theorem 1.1i), remains valid also in higher dimensions because it uses only the L^∞ bound. However, the *BV* estimate and therefore the equality (7) are valid only in dimension 1, since they require some techniques and properties that are specific to this dimension.

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