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Extended τ-hypergeometric functions and associated properties

Fonctions τ-hypergéométriques étendues et leurs propriétés

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A R T I C L E I N F O

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A B S T R A C T

Recently, an extension of the Pochhammer symbol was used in order to introduce and investigate a family of generalized hypergeometric functions [Srivastava et al. (2014) [11]]. The main object of this paper is to present an extension of the τ-Gauss hypergeometric functions ${}_2R_1^{\tau}(z)$ and investigate its several properties, including, for example, its integral representations, derivative formulas, Mellin transforms and fractional calculus operators. Some interesting special cases of our main results are also pointed out.

R É S U M É

Récemment, une extension du symbole de Pochhammer a été utilisée pour introduire et étudier une famille de fonctions hypergéométriques généralisées [Srivastava et al. (2014) [11]]. L'objet de cette Note est de présenter une extension des fonctions τ-hypergéométriques de Gauss ${}_2R_1^{\tau}(z)$ et d'étudier plusieurs de leurs propriétés, incluant, par exemple, leurs représentations intégrales, les formules de dérivées, les transformées de Mellin et les opérateurs de calcul fractionnaire. Quelques cas particuliers intéressants de nos résultats principaux sont également signalés.

1. Introduction and preliminaries

Throughout this paper, $\mathbb{N}$, $\mathbb{Z}^-$, and $\mathbb{C}$ denote the sets of positive integers, negative integers, complex numbers, respectively, $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$ and $\mathbb{Z}_0^- := \mathbb{Z}^- \cup \{0\}$.

In recent years, several extensions of the well-known special functions have been considered by various authors (see, for example, [2–6,8,13]). In particular, Chaudhry and Zubair [1] (see also [4]) introduced an interesting generalization of the gamma function $\Gamma(z)$ as follows:

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$$\Gamma_p(z) := \begin{cases} \displaystyle\int_0^\infty t^{z-1} \exp\left(-t - \frac{p}{t}\right) \mathrm{d}t & (\Re(p) > 0; \quad z \in \mathbb{C}), \\ \Gamma(z) & (p = 0; \ \Re(z) > 0). \end{cases} \tag{1.1}$$

Very recently, Srivastava et al. [11, p. 487, Eq. (15)] introduced and studied, in a rather systematic manner, the following family of generalized hypergeometric functions:

$${}_rF_s \left[\begin{array}{r} (\alpha_1, p), \alpha_2, \cdots, \alpha_r; \\ \beta_1, \cdots, \beta_s; \end{array} z \right] = \sum_{n=0}^{\infty} \frac{(\alpha_1; p)_n (\alpha_2)_n \cdots (\alpha_r)_n}{(\beta_1)_n \cdots (\beta_s)_n} \frac{z^n}{n!} \tag{1.2}$$

in terms of the generalized Pochhammer symbol $(\lambda; p)_\nu$ [11, p. 485, Eq. (8)]:

$$(\lambda; p)_\nu := \begin{cases} \dfrac{\Gamma_p(\lambda + \nu)}{\Gamma(\lambda)} & (\Re(p) > 0; \ \lambda, \ \nu \in \mathbb{C}), \\ (\lambda)_\nu & (p = 0; \ \lambda, \ \nu \in \mathbb{C}) \end{cases} \tag{1.3}$$

or, equivalently, by means of an integral representation [11, p. 485, Eq. (9)] as follows:

$$(\lambda; p)_\nu = \frac{1}{\Gamma(\lambda)} \int_0^\infty t^{\lambda+\nu-1} \exp\left(-t - \frac{p}{t}\right) \mathrm{d}t$$
$$\big(\Re(p) > 0; \ \Re(\lambda + \nu) > 0 \quad \text{when} \quad p = 0\big). \tag{1.4}$$

Here, and in what follows, $(\lambda)_\nu$ $(\lambda, \nu \in \mathbb{C})$ denotes the Pochhammer symbol (or the shifted factorial) which is defined (in general) by

$$(\lambda)_\nu := \frac{\Gamma(\lambda + \nu)}{\Gamma(\lambda)} = \begin{cases} 1 & (\nu = 0; \ \lambda \in \mathbb{C} \setminus \{0\}), \\ \lambda(\lambda + 1) \cdots (\lambda + n - 1) & (\nu = n \in \mathbb{N}; \ \lambda \in \mathbb{C}), \end{cases} \tag{1.5}$$

it is being understood *conventionally* that $(0)_0 := 1$ and assumed *tacitly* that the Γ-quotient exists (see, for details, [12, p. 21 *et seq.*]).

Clearly, when $p = 0$, (1.3) reduces to (1.5) and (1.2) reduces to the familiar generalized hypergeometric function ${}_rF_s$ (see, e.g., [9]).

In 2001, Virchenko et al. [19, p. 90, Eq. (5)] have studied and investigated (see, also, [7]) the following τ-Gauss hypergeometric function:

$${}_2R_1^\tau(z) = {}_2R_1\,(a, \, b; \, c; \, \tau; \, z) = \frac{\Gamma(c)}{\Gamma(b)} \sum_{n=0}^{\infty} \frac{(a)_n \Gamma(b + \tau n)}{\Gamma(c + \tau n)} \frac{z^n}{n!}$$
$$\big(\tau > 0; \ |z| < 1\,; \ \Re(c) > \Re(b) > 0\big). \tag{1.6}$$

They gave the Euler-type integral representation [19, p. 91, Eq. (6)]:

$${}_2R_1\,(a, \, b; \, c; \, \tau; \, z) = \frac{1}{B(b, \, c - b)} \int_0^1 t^{b-1} (1 - t)^{c-b-1} \, (1 - zt^\tau)^{-a} \, \mathrm{d}t.$$
$$\big(\tau > 0; \ |\arg(1 - z)| < \pi\,; \ \Re(c) > \Re(b) > 0\big). \tag{1.7}$$

The special case when $\tau = 1$ in (1.6) and (1.7) yields the familiar representations of Gauss' hypergeometric function [9].

Motivated mainly by some of these aforementioned investigations of the extended generalized hypergeometric function ${}_rF_s$ defined by (1.2), we introduce the extended τ-Gauss hypergeometric functions ${}_2R_1^\tau(z)$. We then systematically investigate certain integral representations, a derivative formula, Mellin transforms and fractional calculus operators of this extended τ-Gauss hypergeometric functions ${}_2R_1^\tau(z)$. Some interesting special cases of our main results are also pointed out. For various other investigations involving generalizations of the hypergeometric function ${}_rF_s$ of r numerator and s denominator parameters, which were motivated essentially by the pioneering work of Srivastava et al. [11], the interested reader may refer to several recent papers on the subject (see, e.g., [14–17] *and* the references cited in each one of these papers).

2. Extended τ-Gauss hypergeometric function

In terms of the generalized Pochhammer symbol $(\lambda, p)_\nu$ $(\lambda, \nu \in \mathbb{C})$ defined by (1.3), we introduce the extended τ-Gauss hypergeometric functions ${}_2R_1^\tau(z)$ as follows: For $a, b, \in \mathbb{C}$ and $c \in \mathbb{C} \setminus \mathbb{Z}_0^-$, we have:

$$
{}_2R_1^\tau(z) = {}_2R_1^\tau((a,p),\, b;\, c;\, z) = \frac{\Gamma(c)}{\Gamma(b)} \sum_{n=0}^{\infty} \frac{(a;p)_n \Gamma(b+\tau n)}{\Gamma(c+\tau n)} \frac{z^n}{n!}
$$
$$
(p \geqq 0;\ \tau > 0,\ |z| < 1;\ \Re(c) > \Re(b) > 0 \quad \text{when} \quad p = 0). \tag{2.1}
$$

Remark 1. The special cases of (2.1) when $\tau = 1$ is easily seen to reduce to the extended Gauss hypergeometric functions [11, p. 487, Eq. (17)]:

$$
{}_2F_1((a,p), b; c; z) = \sum_{n=0}^{\infty} \frac{(a;p)_n\,(b)_n}{(c)_n} \frac{z^n}{n!}.
$$

Also, the special case of (2.1) when $\tau = 1$ and $p = 0$ is seen to yield the classical Gauss's hypergeometric function (see, e.g., [9]).

3. Integral representations and derivative formulae

In this section, we obtain Euler- and Laplace-type integral representations and differential formulae for ${}_2R_1^\tau(z)$ in (2.1).

Theorem 1. *The following integral representation for ${}_2R_1^\tau(z)$ in (2.1) holds true:*

$$
{}_2R_1^\tau((a,p), b; c; z) = \frac{1}{B(b,\, c-b)} \int_0^1 t^{b-1} (1-t)^{c-b-1}\, {}_1F_0\left[(a,p);\, —\, ;\, zt^\tau\right]
$$
$$
\big(\Re(p) > 0;\ \tau > 0,\ \Re(c) > \Re(b) > 0 \quad \textit{when} \quad p = 0\big). \tag{3.1}
$$

Proof. Considering the following elementary identity involving the Beta function $B(\alpha, \beta)$:

$$
\frac{(b)_{\tau n}}{(c)_{\tau n}} = \frac{B(b+\tau n, c-b)}{B(\beta, \gamma-\beta)} = \frac{1}{B(b,\, c-b)} \int_0^1 t^{\beta+\tau n-1} (1-t)^{c-b-1}\, \mathrm{d}t
$$
$$
\big(\Re(c) > \Re(b) > 0\big)
$$

in (2.1) and using the definition (1.2), we get the desired integral representation (3.1). □

Theorem 2. *The following integral representation for ${}_2R_1^\tau(z)$* in (2.1) *holds true:*

$$
{}_2R_1^\tau((a,p), b; c; z) = \frac{1}{\Gamma(a)} \int_0^\infty \mathrm{e}^{-t-\frac{p}{t}}\, t^{a-1}\, {}_1\Phi_1^\tau(b; c; zt)\, \mathrm{d}t
$$
$$
\big(\Re(p) > 0;\ \Re(z) < 1,\ \Re(a) > 0 \quad \textit{when} \quad p = 0\big), \tag{3.2}
$$

where ${}_1\Phi_1^\tau(b;\, c;\, z)$ is the τ-confluent hypergeometric function introduced by Virchenko [18]:

$$
{}_1\Phi_1^\tau(z) = {}_1\Phi_1^\tau(b;\, c;\, z) = \frac{\Gamma(c)}{\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(b+\tau n)}{\Gamma(c+\tau n)} \frac{z^n}{n!} \qquad (\tau > 0;\ \Re(c) > \Re(b) > 0)\,. \tag{3.3}
$$

Proof. Using the integral representation (1.4) of the extended Pochhammer symbol $(a;p)_n$ in (2.1) and using (3.3), we are led to the desired result (3.2). □

Remark 2. The special cases of (3.1) and (3.2) when $\tau = 1$ are easily seen to reduce to the known integral representations of the extended Gauss hypergeometric functions [11, p. 488, Eq. (24)] and [11, p. 488, Eq. (23)]:

$$
{}_2F_1((a,p), b; c; z) = \frac{1}{B(b,\, c-b)} \int_0^1 t^{b-1} (1-t)^{c-b-1}\, {}_1F_0\,[(a,p);\, —\, ;\, zt]\, \mathrm{d}t \tag{3.4}
$$

and

$$ {}_2F_1((a,p),b;c;z) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-t-\frac{p}{t}}\, t^{a-1}\, {}_1F_1(b;c;zt)\,dt, \tag{3.5} $$

respectively. Also, the special cases of (3.1) and (3.2) when $\tau = 1$ and $p = 0$ are seen to yield the classical integral representations of Gauss' hypergeometric function (see, e.g., [9]).

Theorem 3. *Each of the following derivative formula for ${}_2R_1^\tau(z)$ holds true:*

$$ \frac{d^n}{dz^n}\left[{}_2R_1^\tau((a,p),b;c;z)\right] = \frac{(a)_n\Gamma(c)\Gamma(b+\tau n)}{\Gamma(b)\Gamma(c+\tau n)}\, {}_2R_1^\tau[(a+n,p),b+\tau n;c+\tau n;z] \tag{3.6} $$

and

$$ \left(\frac{d}{dz}\right)^n \left[z^{c-1}\, {}_2R_1^\tau((a,p),b;c;\omega z^\tau)\right] = \frac{z^{c-n-1}\Gamma(c)}{\Gamma(c-n)}\, {}_2R_1^\tau((a,p),b;c-n;\omega z^\tau). \tag{3.7} $$

Proof. Differentiating n times both sides of (2.1) with respect to z, we can easily obtain a derivative formula for the extended τ-Gauss hypergeometric function ${}_2R_1^\tau(z)$ asserted by (3.6).

Next, according to the uniform convergence of the series (2.1), differentiating term by term under the sign of summation, we have

$$ \begin{aligned} \left(\frac{d}{dz}\right)^n \left[z^{c-1}\, {}_2R_1^\tau((a,p),b;c;\omega z^\tau)\right] &= \frac{\Gamma(c)}{\Gamma(b)} \sum_{m=0}^\infty \frac{(a;p)_m\Gamma(b+\tau m)}{\Gamma(c+\tau m)} \frac{\omega^m}{m!} \left(\frac{d}{dz}\right)^n \left[z^{c+\tau m-1}\right] \\ &= \frac{\Gamma(c)}{\Gamma(b)} \sum_{m=0}^\infty \frac{(a;p)_m\Gamma(b+\tau m)}{\Gamma(c+\tau m-n)} \frac{\omega^m}{m!}\, z^{c+\tau m-n-1} \\ &= z^{c-n-1} \frac{\Gamma(c)\Gamma(c-n)}{\Gamma(b)\Gamma(c-n)} \sum_{m=0}^\infty \frac{(a;p)_m\Gamma(b+\tau m)}{\Gamma(c+\tau m-n)} (\omega z^\tau)^m, \end{aligned} $$

which, in view of the definition (2.1), yields the desired representation (3.7). □

4. Mellin transform

The Mellin transform of a suitable integrable function $f(t)$ with index α is defined, as usual, by

$$ \mathcal{M}\{f(\tau):\tau\to s\} := \int_0^\infty \tau^{s-1} f(\tau)\,d\tau, \tag{4.1} $$

provided that the improper integral in (4.1) exists.

Theorem 4. *The Mellin transform of the function ${}_2R_1^\tau[(a,p),b;c;z]$ defined by (2.1) is given by*

$$ \mathcal{M}\left\{{}_2R_1^\tau((a,p),b;c;z):p\to s\right\} = \Gamma(s)(a)_s\, {}_2R_1^\tau(a+s,b;c;z) \\ (\Re(s)>0 \quad \text{and} \quad \Re(a+s)>0). \tag{4.2} $$

Proof. Using the definition (4.1) of the Mellin transform, we find from (2.1) that

$$ \begin{aligned} \mathcal{M}\left\{{}_2R_1^\tau((a,p),b;c;z):p\to s\right\} &:= \int_0^\infty p^{s-1} \left(\frac{\Gamma(c)}{\Gamma(b)} \sum_{n=0}^\infty \frac{(a;p)_n\Gamma(b+\tau n)}{\Gamma(c+\tau n)} \frac{z^n}{n!}\right) dp \\ &= \frac{\Gamma(c)}{\Gamma(b)} \sum_{n=0}^\infty \frac{\Gamma(b+\tau n)}{\Gamma(c+\tau n)} \frac{z^n}{n!} \frac{1}{\Gamma(a)} \int_0^\infty p^{s-1}\Gamma_p(a+n)\,dp. \end{aligned} $$

Using now the result of Chaudhry and Zubair [4, p. 16, Eq. (1.110)] given by

$$ \int_0^\infty p^{s-1}\Gamma_p(a+n)\,dp = \Gamma(a+s+n)\,\Gamma(s) \qquad (\Re(s)>0), \tag{4.3} $$

we get

$$\mathcal{M}\left\{{}_2R_1^\tau((a,p),b;c;z):p\to s\right\}=\frac{\Gamma(s)\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+s+n)\Gamma(b+\tau n)}{\Gamma(c+\tau n)}\frac{z^n}{n!}$$
$$=\Gamma(s)(a)_s\sum_{n=0}^{\infty}\frac{(a+s)_n\Gamma(b+\tau n)}{\Gamma(c+\tau n)}\frac{z^n}{n!},$$

which, in view of definition (2.1), yields the desired representation (4.2). □

5. Fractional calculus approach

In this section, we consider compositions of the Riemann–Liouville fractional integrals and derivatives $I_{\rho+}$ and $D_{\rho+}$ of order $\alpha\in\mathbb{C}$, $\Re(\alpha)>0$ defined by (see, for details, [10, Sections 2.3 and 2.4]):

$$\left(I_{\rho+}^{\alpha}\varphi\right)(x)=\frac{1}{\Gamma(\alpha)}\int_{\rho}^{x}\frac{\varphi(t)}{(x-t)^{1-\alpha}}\mathrm{d}t\qquad\left(\alpha\in\mathbb{C},\ \Re(\alpha)>0\right)\tag{5.1}$$

and

$$\left(D_{\rho+}^{\alpha}\varphi\right)(x)=\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^{n}\left(I_{\rho+}^{n-\alpha}\varphi\right)(x)\qquad\left(\alpha\in\mathbb{C},\ \Re(\alpha)>0;\ n=[\Re(\alpha)]+1\right).\tag{5.2}$$

Theorem 5. *Let $\rho\in\mathbb{R}_+=[0,\infty)$, $a,\ b,\ c,\ \omega\in\mathbb{C}$ and $\Re(\alpha)>0$, $\Re(c)>0$, $\Re(\tau)>0$. Then, for $x>\rho$, the following relations hold true:*

$$\left(I_{\rho+}^{\alpha}\left[(t-\rho)^{c-1}\,{}_2R_1^\tau((a,p),b;c;\omega(t-\rho)^\tau)\right]\right)(x)=\frac{(x-\rho)^{c+\alpha-1}\Gamma(c)}{\Gamma(c+\alpha)}\,{}_2R_1^\tau((a,p),b;c+\alpha;\omega(x-\rho)^\tau)\tag{5.3}$$

and

$$\left(D_{\rho+}^{\alpha}\left[(t-\rho)^{c-1}\,{}_2R_1^\tau((a,p),b;c;\omega(t-\rho)^\tau)\right]\right)(x)=\frac{(x-\rho)^{c-\alpha-1}\Gamma(c)}{\Gamma(c-\alpha)}\,{}_2R_1^\tau[(a,p),b;c-\alpha;\omega(x-\rho)^\tau).\tag{5.4}$$

Proof. By virtue of the formulas (5.1) and (2.1), the term-by-term fractional integration and the application of the relation [10, 2.44]:

$$\left(I_{a+}^{\alpha}[(t-a)^{\beta-1}]\right)(x)=\frac{\Gamma(\beta)}{\Gamma(\alpha+\beta)}(x-a)^{\alpha+\beta-1}\qquad\left(\alpha,\beta\in\mathbb{C},\ \Re(\alpha)>0,\ \Re(\beta)>0\right)\tag{5.5}$$

yields, for $x>\rho$

$$\left(I_{\rho+}^{\alpha}[(t-\rho)^{c-1}\,{}_2R_1^\tau((a,p),b;c;\omega(t-\rho)^\tau)]\right)(x)=\left(I_{\rho+}^{\alpha}\left[\frac{\Gamma(c)}{\Gamma(b)}\sum_{n=0}^{\infty}\frac{(a;p)_n\Gamma(b+\tau n)}{\Gamma(c+\tau n)n!}\omega^n\,(t-\rho)^{c+\tau n-1}\right]\right)$$
$$=\frac{(x-\rho)^{c+\alpha-1}\Gamma(c)}{\Gamma(c+\alpha)}\,{}_2R_1^\tau[(a,p),b;c+\alpha;\omega(x-\rho)^\tau).\tag{5.6}$$

Next, by (5.2) and (2.1), we find:

$$\left(D_{\rho+}^{\alpha}[(t-\rho)^{c-1}\,{}_2R_1^\tau((a,p),b;c;\omega(t-\rho)^\tau)]\right)(x)$$
$$=\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^{n}\left(I_{\rho+}^{n-\alpha}[(t-\rho)^{c-1}\,{}_2R_1^\tau((a,p),b;c;\omega(t-\rho)^\tau)]\right)(x)$$
$$=\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^{n}\left[\frac{(x-\rho)^{c+n-\alpha-1}\Gamma(c)}{\Gamma(c-\alpha+n)}\,{}_2R_1^\tau[(a,p),b;c-\alpha+n;\omega(x-\rho)^\tau)\right].\tag{5.7}$$

Applying (3.7), we are led to the desired result (5.4). □

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